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Fatigue life estimation in a beam subject to random loads using probabilistic methods in the frequency domain

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Abstract. Mechanical structures subjected to alternating cyclic loads are subject to failure due to fatigue, a resulting from the accumulation of damage caused by alternating cycles. The aim of this work is the use of theoretical random excitations of the stationary and Gaussian type in the computational study of probabilistic criteria in the frequency domain for fatigue analysis in beams. Using the Finite Element Method to perform a dynamic analysis of the system, a theoretical random load is applied to the structure and the Frequency Response Function, PSD (Power Spectrum Density) and spectral parameters are calculated. With these structural characteristics, the fatigue life of the requested structure is estimated using several probabilistic criteria, comparing the values obtained and analyzing the difference of each methodology. From these results, it is possible to verify the importance of fatigue life calculation of dynamic systems, the efficiency of modeling procedures as a tool for fatigue analysis of random dynamic systems and also the influence of the variation of the results obtained in this type of structure.

Keywords: Random Vibration Fatigue, Frequency Domain, Rainflow

1 Introduction

Despite decades of study on the subject, the analysis and prediction of failure in elements subject to random loading is often inaccurate. According to Fu [1], the level of damage due to fatigue depends mainly on operational conditions, material properties, manufacturing processes, structural defects and environmental factors. As a result, the theoretical values differ from the practical values of a given structure.

In general, the fatigue phenomenon is characterized by the deterioration of the material subject to external loads that vary over time. These cyclic loads result in localized plastic deformations, resulting in microscopic cracks that extend and join together forming large cracks. Nucleation and crack growth are the basic causes of fatigue failure [2].

Estimating fatigue life in systems with random loads is an important step in structural design. Two methodologies stand out in the calculation of final life in fatigue: analysis in the time domain and analysis in the frequency domain [3]. Time domain analyzes are simpler, but are limited to specific cases and require a high computational cost. The frequency domain analyzes model the stresses in the structure as a random and stationary process, using the parameters of the spectral density function (PSD) to estimate damage in the structure [4].

The use of the damage accumulation criterion for life prediction is very common due to the ease of implementing algorithms that calculate the number of cycles from the historical stress data. In the work of Hu et al. [5], a methodology based on this criterion is defined, in order to consider uncertainties present in structures subject to known cyclic loads. The method proposed in a cantilever beam and a door connection component was used, obtaining precise results from Monte Carlo simulations. The development of specifications for life fatigue tests using the time domain damage accumulation criterion was also developed by Moon et al. [6]. Several tests varying the acceleration were performed, defining the optimal value for the vibration test for the analyzed components.

The comparison between results of temporal and spectral methods shows the differences and limitations of each methodology. The numerical analysis of a mooring line is performed by Xu et al. [7], initially considering a statistical analysis of the stresses in simplified methods, and later using a spectral analysis by the Rayleigh distribution. The results show that the spectral method can be underestimated by about 10-30%. A multiaxial analysis is performed by Reytier et al. [8], also comparing the results in the time and frequency domain of an aircraft.

Methods that use the spectral approach are more computationally efficient, in addition to the possibility of modeling structures when there is no access to the stress history. The modeling of a structural ship component subject to operational loads was performed by Wang [9]. Uncertainties were inserted in the model, in addition to recommendations related to the random loads modeling and choice of structural parameters. Deterministic and random vibrations are considered in the methodology of Zheng et al. [10] , which determines the fatigue life of a drilling column in axial and torsional modes of vibration, based on the criterion of maximum shear stress. However, Yustiawan et al. [11] estimates a life of 11 years for an ocean cable measuring buoy, also using a spectral methodology.

The same damage accumulation criterion is also used by Rafiee [12] in the development of a probabilistic methodology, evaluating tubes subject to internal hydrostatic pressures. The stochastic modeling focus on the operational simulation of the tubes. In addition to the use of the spectral method in the calculation of fatigue life, Rokni and Tabeshpour [13] performs vibration control modeling by shock absorbers, analyzing their influence on the fatigue life of offshore platform structures.

Finally, some authors insert damping in the structure to analyze the increase in fatigue life. In the work of Gonçalves et al. [14], the effect of coupling viscoelastic materials on the structure in the final result of fatigue life is analyzed. The multiaxial analysis is based on the methodology proposed by Lambert et al. [15], which includes the loading path through the use of a prismatic shell, better representing the second invariant of the stress tensor.

2 Fatigue from random loads

The fatigue analysis of a structure consists of calculating the number of cycles necessary for the structure to collapse, and the estimated life time can also be calculated based on the test parameters. For loads with constant amplitudes, this calculation is summarized by counting the cycles and comparing these cycles with the material's limit values. However, for real structures, requests are random in nature, requiring a non-deterministic and probabilistic analysis. In all approaches to analyse fatigue life, the criterion of damage accumulation is used, which is calculated by the damage estimate that a given level or probability of tension causes to the structure [4].

2.1 Damage accumulation criteria

According to Miner [16], the damage caused by sinusoidal cyclical stresses is calculated by eq. (1):

$$E[AD] = \sum_i \frac{n(\sigma_{a_i})}{N(\sigma_{a_i})}, \tag{1}$$

with $E[AD]$ as the expected value for the damage accumulation, $n(\sigma_{a_i})$ as the number of cycles with alternating stress amplitude σ_{a_i} and $N(\sigma_{a_i})$ as the number of cycles for fatigue failure with alternating stress amplitude σ_{a_i}.

Inserting the experimental characteristics of cycles for fatigue according to the $sigma - N$ curve of the material, we have in eq. (2) the damage expectation for a continuously distributed amplitude distribution:

$$E[D] = \frac{T \cdot E[P]}{K} \cdot \int_0^\infty \sigma_a^m \cdot p(\sigma_a)\mathrm{d}\sigma_a, \tag{2}$$

with T as the test duration, $E[P]$ the peak expectation and $p(\sigma_a)$ the probability of alternating stress amplitude σ_a (PDF function).

2.2 Time domain

The estimation and calculation of damage in a structure by random request is calculated by counting stress cycles, in the time domain. The objective is to relate the variable amplitudes with values of cycles and constant amplitudes, obtained experimentally. The use of different counting methods for the same stress history can change the result of the estimate [17].

The Rainflow counting method, proposed by Matsuichi and Endo [18], consists of transforming a set of floating stresses into a simplified set of stress inversion data. It gets its name from the allusion to runoff water on a roof. The results of the cycle count can be represented in a histogram of constant intervals, which can be used to obtain the Probability Density Function (PDF). Despite its simplicity, it still has a lot of application in the uniaxial analysis of fatigue life of simpler structures.

2.3 Frequency domain

The methodologies based on the frequency domain start from the statistical and probabilistic premise contained in the Spectral Density Function (PSD). The values obtained from stress cycles, calculated by extracting the PSD parameters from the structural response, consist mostly of empirical or analytical approximations from simulations and comparisons with experimental tests [4].

There are two types of main signals, taking into account the frequency range of the spectral values. If the peak values are concentrated in a small frequency window, this signal is said to be narrow band. If, instead, the main values are separated by a large frequency range, then the signal is characterized as broadband (or wideband). The importance of this signal differentiation lies in the conservatism of the solution. Due to the nature of the signal, which is discrete over time and is called beat effect, the narrowband signal has large cycles of high stress value, thus resulting in greater damage accumulation than a wideband signal [19].

The methodologies used in the present work will be described below. The methods were chosen based on the results obtained by the authors and also by the repercussion of the scientific content of each one.

Petrucci e Zuccarello

This method allows the estimation of fatigue life without the Probability Density Function $p(\sigma)$, for structures subject to uniaxial wideband stress.

For the general case, there is the accumulation of damage calculated by eq. (3) [2, 20]:

$$D = \frac{N}{K} \cdot \int_m \int_r p(\sigma) \cdot \left(\frac{1}{1 - \frac{m}{\sigma_u}} \right)^m \mathrm{d}m\mathrm{d}\sigma, \tag{3}$$

with K and m as the constant values of the Wöhler curve ($\sigma - N$), and σ_u the tensile strength.

From the equivalent tension of Goodman σ_e, defined by Shigley [21], the eq. (4) can be rewritten as follows:

$$D = \frac{N}{K} \cdot \int_{\sigma_e} p(\sigma_e) \cdot \sigma_e^m \mathrm{d}\sigma_e = \frac{N}{K} \cdot \chi_\mu, \tag{4}$$

with χ_μ as the m-th order moment of the probability density function. The eq. (5) corresponds to the durability equation, with m_i corresponding to the i-th spectral moment and D_r the accumulated damage to failure:

$$T_f = \sqrt{\frac{m_2}{m_4}} \cdot \frac{2 \cdot \pi \cdot K \cdot D_r}{\chi_\mu}. \tag{5}$$

In order to facilitate the calculation of χ_μ, an approximation is done to a function $g(\alpha_x, \beta_x, \alpha_{\dot{x}}, \beta_{\dot{x}}, m, \gamma)$, making it possible to calculate using only material properties, variance and PSD parameters, according to eq. (6):

$$\chi_\mu = m_0 \cdot \frac{m}{2} \cdot g(\alpha_x, \beta_x, \alpha_{\dot{x}}, \beta_{\dot{x}}, m, \gamma), \tag{6}$$

with $\gamma = \frac{x_{max}}{\sigma_u}$ and x_{max} as the maximum x value or $3 \cdot \sqrt{m_0}$.

From various computer simulations, Petrucci and Zuccarello [22] defined a second degree polynomial approximation $\Psi(\alpha_x, \beta_x, m, \gamma)$ for the previous function, resulting in eq. (7) and eq. (8):

$$\Psi = \frac{(\Psi_2 - \psi_1)}{6} \cdot (m-3) + \Psi_1 + \left[\frac{2}{9} \cdot (\Psi_4 - \Psi_3 - \Psi_2 + \Psi_1) \cdot (m-3) + \frac{4}{3} \cdot (\Psi_3 - \Psi_1) \right] \cdot (\gamma - 0.15), \tag{7}$$

$$\begin{aligned}
\Psi_1 &= -1.994 - 9.381 \cdot \alpha_x + 18.349 \cdot \beta_x + 15.261 \cdot \alpha_x \cdot \beta_x - 1.483 \cdot \alpha_x^2 - 15.402 \cdot \beta_x^2, \\
\Psi_2 &= 8.229 - 26.510 \cdot \alpha_x + 21.522 \cdot \beta_x + 27.748 \cdot \alpha_x \cdot \beta_x + 4.338 \cdot \alpha_x^2 - 20.026 \cdot \beta_x^2, \\
\Psi_3 &= -0.946 - 8.025 \cdot \alpha_x + 15.692 \cdot \beta_x + 11.867 \cdot \alpha_x \cdot \beta_x + 0.382 \cdot \alpha_x^2 - 13.198 \cdot \beta_x^2, \\
\Psi_4 &= 8.780 - 26.058 \cdot \alpha_x + 21.628 \cdot \beta_x + 26.487 \cdot \alpha_x \cdot \beta_x + 5.379 \cdot \alpha_x^2 - 19.967 \cdot \beta_x^2.
\end{aligned} \tag{8}$$

From these equations, it is possible to estimate the fatigue life given by the formula shown in eq. (9):

$$T_f = \sqrt{\frac{m_2}{m_4}} \cdot \frac{2 \cdot \pi \cdot K \cdot D_r}{\sqrt{m_0^m} \cdot \exp\left[\Psi(\alpha_x, \beta_x, m, \gamma)\right]}. \tag{9}$$

Tunna

The main purpose of this methodology is to reduce intrinsic conservatism based on a narrow band empirical process, using the inverse Fourier transform (FFT) of the PSD function, and performing the cycle counting by the Rainflow method [19]. The result is an approximation of the probability density function (PDF), exposed in eq. (10):

$$p(\sigma)_{tunna} = \left[\frac{\sigma}{4 \cdot \alpha_x^2 \cdot m_0} \cdot \exp\left(-\frac{\sigma^2}{8 \cdot \alpha^2 \cdot m_0}\right)\right]. \tag{10}$$

Dirlik

This methodology is based on a numerical simulation of the signal via Monte Carlo, to obtain an approximation of the probability density function (PDF) for stress amplitudes σ_a. It consists mainly in the sum of an exponential probability density and two Rayleigh distributions. In eq. (11) and eq. (12), it is shown the methodology described by Benasciutti and Tovo [23]:

$$p(\sigma_a)_{dirlik} = \frac{\frac{D_1}{Q} \cdot \exp\left(-\frac{Z}{Q}\right) + \frac{D_2 \cdot Z}{R^2} \cdot \exp\left(-\frac{Z^2}{2 \cdot R^2}\right) + D_3 \cdot Z \cdot \exp\left(-\frac{Z^2}{2}\right)}{\sqrt{m_0}}, \tag{11}$$

$$\begin{aligned}
Z &= \frac{\sigma_a}{\sqrt{m_0}}, & x_m &= \frac{m_1}{m_0} \cdot \sqrt{\frac{m_2}{m_4}} = \beta_x \cdot \alpha_x, \\
R &= \frac{\alpha_x - x_m - D_1^2}{1 - \alpha_x - D_1 + D_1^2}, & D_1 &= \frac{2 \cdot (x_m - \alpha_x^2)}{1 + \alpha_x^2}, \\
D_2 &= \frac{1 - \alpha_x - D_1 + D_1^2}{1 - R}, & D_3 &= 1 - D_1 - D_2, \\
Q &= \frac{1.25 \cdot (\alpha_x - D_3 - D_2 \cdot R)}{D_1}.
\end{aligned} \tag{12}$$

Wirsching

This methodology is also based on a modified empirical process, using a narrow band solution with a damage correction factor $E[D]_{NB}$. The calculation of the expected damage to the structure is done by eq. (13) and eq. (14), as described by Bishop et al. [19], Su [24]:

$$E[D]_{wirsch} = E[D]_{NB} \cdot (a(m) + (1 - a(m)) \cdot (1 - \epsilon)^{c(m)}), \tag{13}$$

$$\begin{aligned}
a(m) &= 0.926 - 0.033 \cdot m, & c(m) &= 1.587 \cdot m - 2.323, \\
\epsilon &= \sqrt{1 - \gamma^2}, & E[D]_{NB} &= \frac{T \cdot E[P]}{K} \cdot (\sqrt{2} \cdot \sqrt{m_0})^m \cdot \Gamma\left(1 + \frac{m}{2}\right).
\end{aligned} \tag{14}$$

3 Numerical example

For comparison of the described methods, a numerical simulation was carried out using the finite element method on a cantilever beam of rectangular section, with dimensions $15 \times 1.5 \times 300\ mm$, with fixed support in one of the ends. The other end was considered free, and two load cases were analysed: one with the application of a dynamic load $F(t) = A \cdot \sin(\omega \cdot t)$, and the other with a random Gaussian load, both applied at the end of the beam. The solution of the dynamic system was performed by the Newmark method, which calculates the fields of acceleration, speed and displacements from the discretization of the analysis time in an iterative process.

The material considered for this beam was structural aluminum type 2219-T851, with the following characteristics: $K = 1.08 \cdot 10^{22}\ MPa$, $m = 7.3$, $\sigma_u = 446\ MPa$, $E = 73.1\ GPa$, $\nu = 0.33$ and $\rho = 2840\ kg/m^3$ [22, 25]. The time considered for the numerical analysis was $10\ s$, with an amplitude $A = 0.5\ N$ and frequency $\omega = 10\ Hz$ in the sinusoidal case. The random load had an maximum amplitude of $0.5\ N$. In addition to the loads, a numerical noise was added to the force signal, in order to simulate experimental processes, with amplitude of $0.01\ N$ and normal distribution. The signal of the applied force and the result of the dynamic analysis is shown

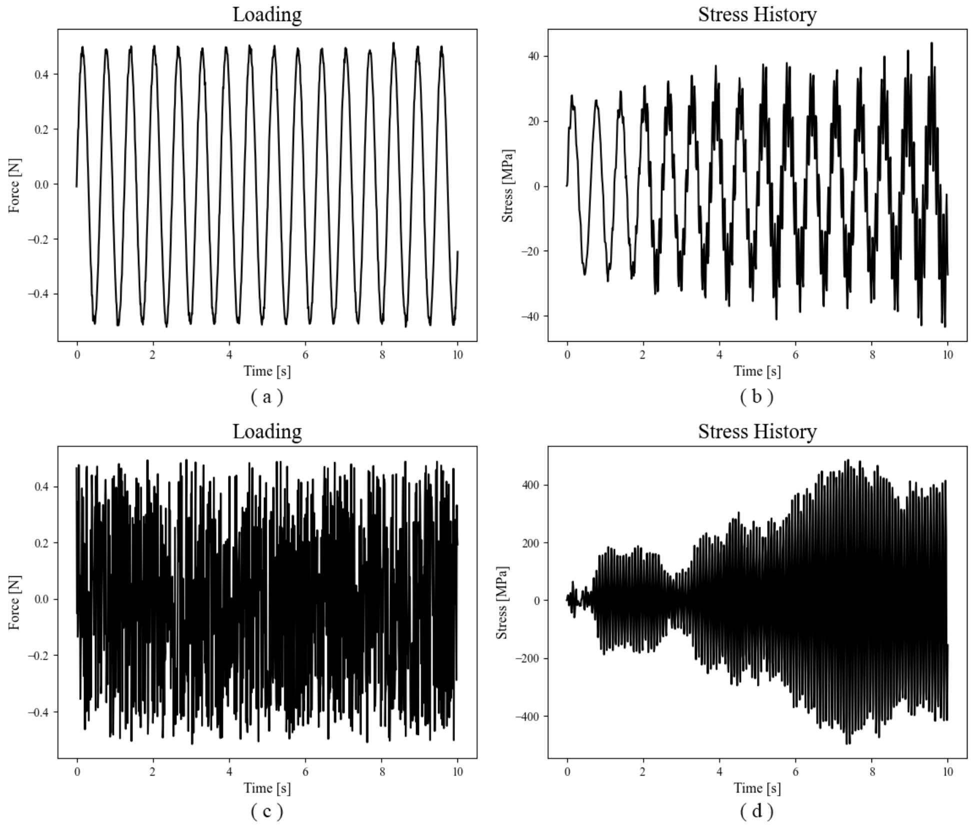

Figure 1. (a) Load signal at the end of the beam with sine load, (b) Stress history of the element with the highest amplitudes with the sine load, (c) load signal at the end of the beam with random load and (d) Stress history of the element with the highest amplitudes with the random load

in Fig. 1, relative to the result of the most requested element, which in this case consists of the element closest to the fixed base.

Due to the nature of the stress response signal, it is possible to perceive a wideband signal, due to the shape of the curve shown in Fig. 1. This signals were used in the calculation of cycle counting by the Rainflow method, used to calculate life in fatigue by the criterion of damage accumulation from discrete signals over time. However, as explained previously, the analysis of life in fatigue in the frequency domain allows a less computationally expensive analysis with fewer limitations. To obtain the signal in the frequency domain, the Fourier Transform is applied to the stress history, as described by Bishop et al. [19]. The reverse path can also be performed using the Inverse Fourier Transform. Thus, for the studied structure, the FFT was applied to the Fig. 1 signal and the spectral density graphs of Fig. 2 were obtained.

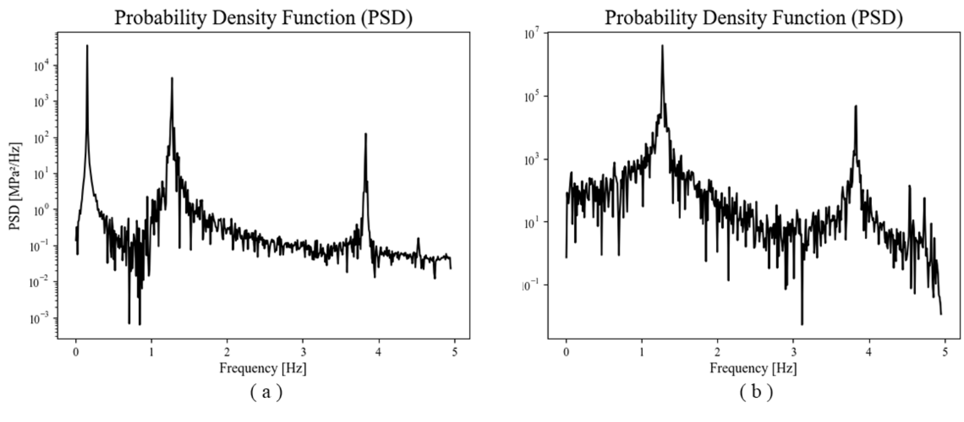

Figure 2. PSD calculated from the stress history: (a) Sinusoidal and (b) Random

The values of the spectral density function make it possible to obtain the PSD parameters necessary for calculating fatigue life, such as spectral moments and peak expectations. These values are found in Table 1. Inserting these values in the equations previously described, according to each specific procedure of the method, it is possible to calculate the damage accumulation as well as the fatigue life expectancy of the structure element, in seconds. The results are shown in Table 2.

Table 1. Results of spectral parameters

Load Type	m_0	m_1	m_2	m_4	$E[P]$	α_x	β_x
Sinusoidal	65.111	21.964	21.112	87.931	2.041	0.279	0.592
Random	8255.591	10947.24	15579.076	56641.109	1.907	0.72	0.965

Table 2. Results of accumulated damage and fatigue life

Method	Sinusoidal		Random	
	Damage	Lifetime [s]	Damage	Lifetime [s]
Rainflow	6.34e-08	1.58e+08	1.03e+01	9.68e-01
Petrucci	1.10e-11	9.05e+11	2.61e-03	3.83e+03
Dirlik	1.07e-11	9.39e+11	2.17e-03	4.60e+03
Tunna	2.01e-12	4.98e+12	9.06e-02	1.10e+02
Wirsching	9.73e-13	1.03e+13	4.31e-05	2.32e+05

In the sinusoidal case, the accumulated damage in all methods is below 1, thus concluding that in these experimental conditions, the structure would not fail. It is noticed that the Rainflow method is the most conservative method, showing the divergence that was already expected due to the limitations of this method. The other methods, on the other hand, have discrepancies, but with greater proximity to each other compared to the one in the time domain. The results are consistent despite the divergences, since the spectral methods are based on empirical processes. Analysing the random case, we can se a greater amount of cycles, causing more damage to the structure. By the Rainflow method, the structure fail within this experiment because it's above 1. This discrepancy of values between the methods shows the limitations of the Rainflow, due to its conservatism when applied to cases with many variations in the direction of the load and with high load cycles. In the PSD graph, the peaks have greater values, increasing the spectral parameters and decreasing the lifetime.

With this results, it's possible to see that the amplitude plays little part in the fatigue analysis, since both cases have the same amount ($0.5\ N$), and the final values have great difference in life expectation. The cycle and the frequency, on the other hand, dictates the amount of counted cycles and the values of the PSD, changing drastically the results of the accumulated damage. It is clear the differance between the time domain and the spectral domain fatigue analysis, so each case should be analysed to see which approach is the most suitable in relation to the nature of the load and the type of the structure.

4 Conclusion

Fatigue failure prediction and fatigue life estimation are not analytical processes, thus requiring an empirical approach to perform the calculations. Five methodologies were presented in the present work, one in the time domain and the rest in the frequency domain. These methodologies have advantages and limitations, depending on the type of signal and structure to be considered.

From the results obtained in the numerical analyzes, it is possible to conclude that there are divergences between the results, indicating that there is no definitive method for determining the fatigue life of structures subject to random loads. It should be noted that, due to the ease of implementation of the methods, all of which are based on the criterion of damage accumulation, the most interesting option for structures subject to uniaxial loads is the combination of the various methods presented. In addition, it is recommended to carry out several loading scenarios, varying the nature of the applied force, the boundary conditions and the characteristics of the material.

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