Comparative study between structural systems in prestressed concrete

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Abstract. This work presents a theoretical comparison of distinct modeling of a model structure through linear analysis using the commercial structural analysis and design software, CSiBridge. The selected structure was a 20.0m spar beam. The central idea was to compare its behavior when varying parameters in the models that characterize the adopted prestressing system. Aspects such as the structural system, the steel class used, which is a function of the type of reinforcement, the level and type of prestressing system and the geometric design of the reinforcement, stand out. The response values investigated in all lines of study were displacements, bending moments and axial and shear stress, aiming to present a complete and critical analysis, pointing out possible reasons for the perceived differences.

Keywords: Prestressed Concrete, Prestressed Losses, Bridge Modeling, CSiBridge.

1 Introduction

According to Pfeil [1], prestressing can be defined as the artifice of introducing, in a structure, a previous state of tension, in order to improve its resistance or its behavior, under the action of several requests. In prestressed concrete, the so-called active reinforcement is tensioned, conveniently placed on the structural element, in order to compress the regions tensioned by external loads. Through this strategy, many advantages are given to the project, such as: greater slenderness of the pieces (span/section height), that is, the ability to overcome a larger span with the same section; limitation or elimination of cracks; decrease in the possibility of corrosion of the reinforcement; greater fatigue strength of steel, as well as greater resistance to tangential stresses (shear).

Assessing the context based on practice in project offices, the use of commercial software for dimensioning bridges, such as CypeCAD and CSiBridge, is widespread in order to optimize the quality of projects, as well as the time needed for its elaboration. The use of these computational resources is extremely important in practice, for example, due to the high level of complexity in the structural analysis phase of the models, in turn, a consequence of the large volume and variety of elements, as well as the varied sets of loads.

Since the application of prestressed structural systems is increasingly used, it is necessary to understand the impact of the solutions presented in the scope of calculation from the change of coefficients and parameters still in the design phase. In this context, the objective of this work is to investigate factors that, when altered, can cause significant impacts on the general response of a prestressed concrete model. Among them, the geometry of the prestressing tendon, the boundary conditions of the model, the prestressing level - that is, the proportion between active and passive reinforcement adopted - the treatment of tension losses in the strands, etc., stand out.

2 Strategies For Numerical Modeling of Prestressed Concrete

The simulation of active reinforcements can be done mainly through 'loads' or 'elements'. In the first, the reinforcement is simply converted to an equivalent load that acts on the structure, while in the second, the object is considered as an element whose rigidities are accounted for in the system. Regarding the prestressing force applied to the structural model, there are different approaches. The established value can represent the load in the act of prestressing or after occur losses (immediate, progressive or both). These forms are differentiated by the previous definition of the loss coefficients or the drop in the reinforcement stress.

The most adequate way to simulate the type of prestressing system depends primarily on the constructive strategies of the model, and on the type of approach regarding the loss coefficients. For the case of pre-stressed systems, friction losses, whobble of the reinforcement curvature and anchorage accommodation must be null because, according to Buchaim [2], the steel will already be stretched during the process of concreting process. However, this is not valid for the case of post-tensioned reinforcement in which indications about immediate losses must be considered for the correct simulation.

2.1 Application to CSIBridge

The software adopted for comparisons is the CSiBridge v. 22.0.0. The software has two different ways of modeling prestressing tendons, as shown in Figure 1. It is worth noting that when using the 'element' class, the program automatically calculates the loss due to elastic shortening, with redundancy of losses if there is a previous value attributed to this loss.

In order to characterize the type of structural system adopted, losses must be correctly quantified based on the formulations of the current Normative Code. For this purpose, the Bridge allows the introduction of seven values divided into two groups: three for ‘Friction and Anchorage Losses’ and four to ‘Other Losses’. In the same form, the points where the reinforcement will have active anchorage are attributed (in practice, this information reflects on which ends the reinforcement will be stretched) and the initial value of the prestressing load, as force or stress.

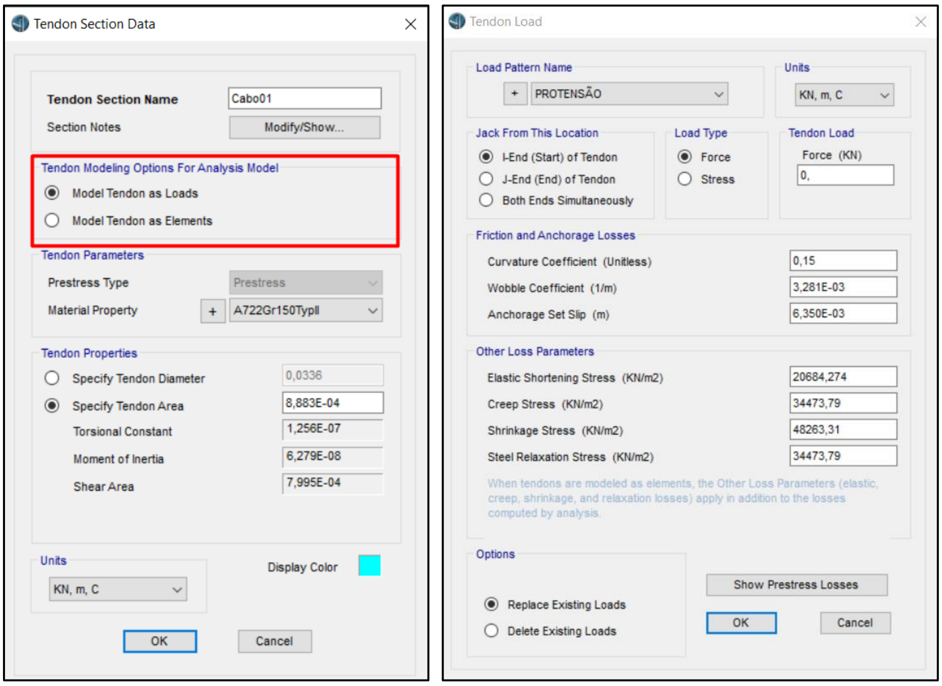

Figure 1. Forms for defining tendon properties with emphasis on the modeling form (load x element) and prestressing loads (default values)

2.2 Applications

The number of variables and combinations to be studied in models is large, regardless of the calculation program. To assist in this study, it was decided to evaluate the behavior of a beam with a span a 20m. For this, some possible variations were defined in view of prestressed concrete systems and ways of modeling them, highlighted in the diagram in Figure 2.

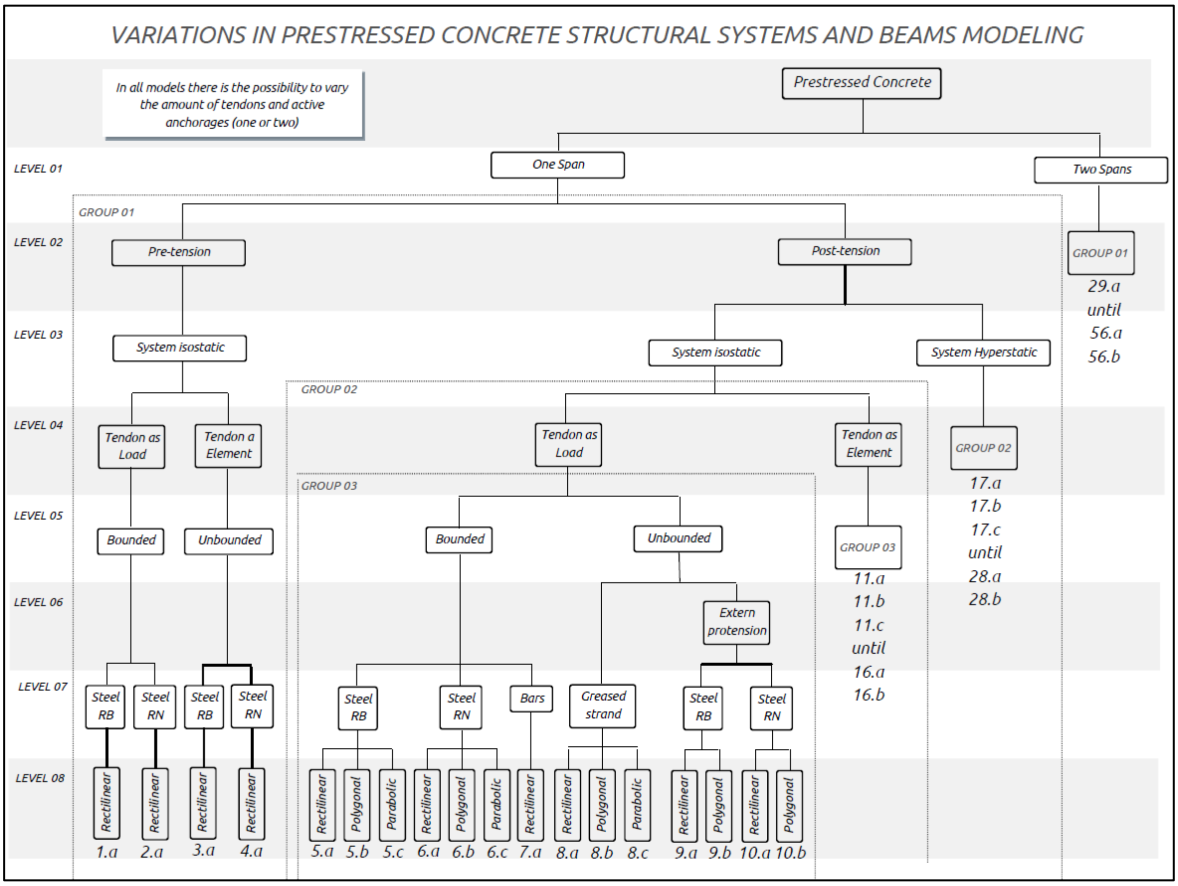

Figure 2. Variations of structural systems and prestressed concrete beam modeling

In practical cases there are limitations to a set of admissible values for deflections, stresses and cracks, conditioning the strength intensity and position (eccentricity) of the reinforcement. It is noteworthy that, in these models, the efficient pre-dimensioning of the reinforcement and prestressing force was not prioritized, based on the limit states relevant to each case. Table 1 shows the selected arrangements.

The beam, with rectangular section, was modeled with the following properties:

- Dimensions: Base $b = 30cm$; Height $h = 100cm$; Length $L = 20.0m$;
- Concrete $f_{ck} = 40MPa; E_{cs} = 28.7\ GPa$;
- Steel area 7φ12,7 $A_p = 690.9mm^2$ (simply supported) or $A_p = 1184mm^2$ 12φ12,7 (continuous);
- Steel CP190RB → $E_p = 195GPa\ ; f_{ptk} = 190kN/cm^2;\ f_{pyk} = 171kN/cm^2$.

Table 1. Synthesis of cases selected for study

Study	Case	System	Nº Span	Type beam	Nº tendons	Anchor	Tendon geometry	Tendon modeling	P_0 (kN)
1	1.a	Pre-tension	1	Simply supported	1	Both	Rectilinear	Element	1000
	3.a	Pre-tension	1	Simply supported	1	Both	Polygonal	Load	1000
	11.a	Bonded tendons	1	Simply supported	1	Both	Parabolic	Element	960

Study	Case	System	Nº Span	Type beam	Nº tendons	Anchor	Tendon geometry	Tendon modeling	P_0 (kN)
2	17.a	Pre-tension	2	Simply supported	1	Both	Rectilinear	Load	1000
	21.b	Bonded tendons	2	Simply supported	1	Both	Polygonal	Load	960
	21.c	Bonded tendons	2	Simply supported	1	Both	Parabolic	Load	960
	24.c	Unbonded tendons	2	Simply supported	1	Both	Parabolic	Load	1040
	19.a	Pre-tension	2	Simply supported	1	Both	Rectilinear	Element	1000
	27.b	Bonded tendons	2	Simply supported	1	Both	Polygonal	Element	960
	27.c	Bonded tendons	2	Simply supported	1	Both	Parabolic	Element	960
	30.c	Unbonded tendons	2	Simply supported	1	Both	Parabolic	Element	1040
3	33.c	Bonded tendons	2	Hyperstatic	1	Both	Parabolic	Load	1650
	33.c’	Bonded tendons	2	Hyperstatic	2	Both	Parabolic	Load	1650
	33.c”	Bonded tendons	2	Hyperstatic	1	Start	Parabolic	Load	1650
	36.c	Unbonded tendons	2	Hyperstatic	1	Both	Parabolic	Load	1780
	39.c	Bonded tendons	2	Hyperstatic	1	Both	Parabolic	Element	1650
	39.c’	Bonded tendons	2	Hyperstatic	2	Both	Parabolic	Element	1650
	39.c”	Bonded tendons	2	Hyperstatic	1	Start	Parabolic	Element	1650
	42.c	Unbonded tendons	2	Hyperstatic	1	Both	Parabolic	Element	1780

The Table 2 brings the values of efforts and displacements relevant to each case.

Table 2. Synthesis of responses from structural models.

Case	PROTENSION (PROT)				SELF WEIGHT (SW)				COMB. 1 - SW + PROT.			
	Ap = 6.91cm² CP190 RB - Span 20m											
	max. deflection	**max. moment**	**max. axial**	**max. shear**	**max. deflection**	**max. moment**	**max. axial**	**max. shear**	**max. deflection**	**max. moment**	**max. axial**	**max. shear**
11.a	*0.0183*	*-291.5*	*-971.5*	*0*	*-0.0196*	*372.1*	*-18.6*	*75.5*	*-0.0013*	*80.6*	*-990.1*	*75.5*
3.a	*0.0188*	*-300*	*-1000*	*0*	*-0.0196*	*375*	*0*	*75*	*-0.0008*	*75*	*-1000*	*75*
11.a	*0.0164*	*-262.3*	*-874.5*	*0*	*-0.0196*	*372.1*	*-18.6*	*75.5*	*-0.0032*	*109.8*	*-893.1*	*75.5*

Case	PROTENSION (PROT)					SELF WEIGHT (SW)					COMB. 1 - SW + PROT.				
	Ap = 6.91cm² CP190 RB - Span 10m														
	max. deflection	**M max (1/2 Span)**	**M max (central support)**	**max. axial**	**max. shear**	**max. deflection**	**M max (1/2 Span)**	**M max (central support)**	**max. axial**	**max. shear**	**max. deflection**	**M max (1/2 Span)**	**M max (central support)**	**max. axial**	**max. shear**
17.a	*0.0047*	*-300,0*	*-300,0*	*-1000,0*	*0,0*	*-0.001221*	*93.75*	*0,0*	*0,0*	*-37.5*	*0.003479*	*-206.25*	*-300,0*	*-1000,0*	*-37.5*
21.b	*0.00133*	*-256.3*	*246.9*	*-841.02*	*102.7*	*-0.001221*	*93.75*	*0,0*	*0,0*	*-37.5*	*0.000109*	*-162.55*	*246.9*	*-841.02*	*102.7*
21.c	*0.00259*	*-252,0*	*230.6*	*-828.6*	*184.7*	*-0.001221*	*93.75*	*0,0*	*0,0*	*-37.5*	*0.001369*	*-158.25*	*230.6*	*-828.6*	*147.2*
24.c	*0.00293*	*-283.7*	*267.4*	*-931,0*	*214.1*	*-0.001221*	*93.75*	*0,0*	*0,0*	*-37.5*	*0.001709*	*-189.95*	*267.4*	*-931,0*	*176.6*
19.a	*0.00456*	*-291.5*	*-291.5*	*-971.53*	*0,0*	*-0.001242*	*93.01*	*0,0*	*-4.6*	*-37.75*	*0.003318*	*-198.49*	*-291.5*	*-976.13*	*-37.75*
27.b	*0.00144*	*-247.03*	*241.08*	*-826.6*	*100.9*	*-0.001248*	*93,0*	*0,0*	*-4.65*	*-37.75*	*0.000192*	*-154.03*	*241.08*	*-831.25*	*99.5*
27.c	*0.002665*	*-244.9*	*226.2*	*-803.1*	*174.6*	*-0.001248*	*93,0*	*0,0*	*-4.65*	*-37.75*	*0.001417*	*-151.9*	*226.2*	*-807.75*	*136.85*
30.c	*0.00301*	*-275.4*	*261.8*	*-903.33*	*202,0*	*-0.001248*	*93,0*	*0,0*	*-4.65*	*-37.75*	*0.001762*	*-182.4*	*261.8*	*-907.98*	*164.25*
	Ap = 11.84cm² CP190 RB - Spans 2 x 10m														
Case	**max. deflection**	**M max (1/2 Span)**	**M max (central support)**	**max. axial**	**max. shear**	**max. deflection**	**M max (1/2 Span)**	**M max (central support)**	**max. axial**	**max. shear**	**max. deflection**	**M max (1/2 Span)**	**M max (central support)**	**max. axial**	**max. shear**
33.c	*0.0033*	*-386.4*	*510.73*	*-1406.1*	*333,0*	*-0.00051*	*52.8*	*-93.08*	*0,0*	*45.5*	*0.00279*	*-336.6*	*417.65*	*-1406.1*	*304.8*
33.c'	*0.0031*	*-354.1*	*476.8*	*-1430.74*	*287.6*	*-0.00051*	*52.8*	*-93.08*	*0,0*	*45.5*	*0.00259*	*-304.02*	*383.72*	*-1430.74*	*259.4*
33.c"	*0.0033*	*-389.2*	*503.9*	*-1406.1*	*333.7*	*-0.00051*	*52.8*	*-93.08*	*0,0*	*45.5*	*0.00279*	*-338.6*	*410.82*	*-1406.1*	*305.5*
36.c	*0.00375*	*-436.2*	*589.3*	*-1636.8*	*383,0*	*-0.00051*	*52.8*	*-93.08*	*0,0*	*45.5*	*0.00324*	*-386.6*	*496.22*	*-1636.8*	*354.8*
39.c	*0.00344*	*-367,0*	*484.7*	*-1328.9*	*316.5*	*-0.000536*	*52.3*	*-91.7*	*-8.1*	*47.2*	*0.002904*	*-316.9*	*393,0*	*-1401.5*	*288.04*
39.c'	*0.00324*	*-337.8*	*455.8*	*-1367,0*	*275.5*	*-0.000536*	*52.3*	*-91.7*	*-8.1*	*47.2*	*0.002704*	*-284.4*	*364.1*	*-1414.4*	*250,0*
39.c"	*0.00349*	*-369.4*	*478.3*	*-1329,0*	*317.1*	*-0.000536*	*52.3*	*-91.7*	*-8.1*	*47.2*	*0.002954*	*-318.6*	*386.6*	*-1401.5*	*289.7*
42.c	*0.00387*	*-413.8*	*558.6*	*-1545.08*	*636.8*	*-0.000536*	*52.3*	*-91.7*	*-8.1*	*47.2*	*0.003334*	*-363.8*	*466.9*	*-1553.2*	*335.4*

2.2.1 Tendon modeled as element and as load

Comparation of cases 1.a, 3.a and 11.a. (see Figure 2 and Table 2)

For the first study (Study 1), the geometry was established with the straight tendon, eccentricity constant e=-30cm from the neutral line and active anchorage at both ends.

The first two cases, using pre-tension, served to highlight the intern treatment to tendons of the program as ‘elements’ and as ‘loads’, and third (even though it is not usual to adopt straight tendons in post-tension) for show how immediat losses influence in model. The fields of long-term losses (shrinkage, creep and relaxation) and elastic shortening are null to make the purpose of this case study more evident. The curvature friction coefficient is equals to 0.2 and whobble friction coefficient 0.02. The limit values for maximum applied stresses are, according to Brazilian norm NBR6118 [3], $0.77f_{ptk}$ or 0.85 f_{pyk}, for pre-tension, and $0.74f_{ptk}$ or 0.82 f_{pyk}, for bounded post-tension. Being f_{ptk} the minimum characteristic tensile strength and f_{pyk} characteristic steel yield strength.

It can be concluded:

- Although the input values are equals for Cases 1.a and 3.a, the response obtained are different. This is due to the prestressing loss of elastic shortening which is automatically considered in the case of reinforcement modeled as 'element'. The difference of approximately 29kN corresponds exactly to the application of eq. (1) (represents the product between the elasticity modular ratio and the initial stress in concrete):

$$\Delta\sigma_{p,enc} = \frac{\Delta P_{enc}}{A_p} = \alpha_p \sigma_{ci}. \quad (1)$$

- In case 11.a the drop was even more intense due to the calculation of the factors that characterize the losses in the anchoring process, precisely what differentiates post-tension with adherence from pre-tension at the modeling level.
- When calculating the immediate elastic shortening, the Bridge did not realize any kind of correction in the elastic modulus of the concrete to anchorage date. This behavior was calculated based on the value inserted in the material properties, which consists of $E_{c,28}$.

2.2.2 Tendon geometry and prestressing systems in simply supported beams

Comparation of Cases 17.a, 21.b, 21.c, 24.c, 19.a, 27.b, 27.c and 30.c (see Figure 2 and Table 2)

The scenery in Study 2 is the same as in the first study, but now with division into 2 spans of 10m (inclusion of a central support) and variation in the geometry of the reinforcement, as summarized in Table 2. The values of losses coefficients and maximum stress are the same shown in the previous study, except for the greased strand system which has higher limits imposed by Norm [3] for the applied stress ($0.80f_{ptk}$ or 0.88 f_{pyk}).

It can be concluded (see diagrams of Figure 3):

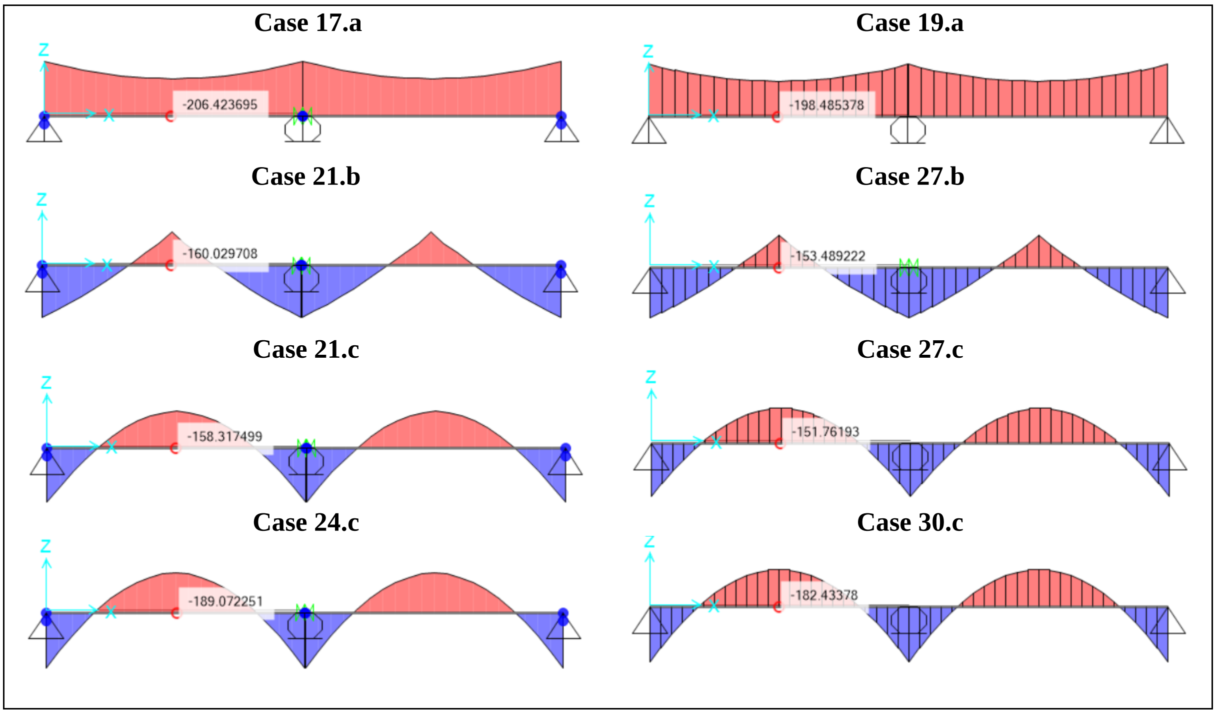

Figure 3. Bending moments diagrams (SW+PROT) for the beams of the Cases of Study 2 described in Table 1.

- The bending moment diagrams (or any others) are independent in each span due to the disconnection between the spans in the central support, the position that defines the symmetry axis of the diagrams;
- The moments on the supports are those generated solely by prestressing in the case of isostatic spans. In straight tendons, the variation along the length comes from the 2nd degree function that describes the bending moment generated by the self-weight;
- In polygonal tendons, the equivalent force acts on the tendon bisector, characterizing the abrupt change in inclination of the bending moment diagram of cases 21.b and 27.b;
- In models whose reinforcement was modeled as an element, there is a portion of compression in the concrete for the "Self-Weight" load case (see Table 2). This behavior arises from the reaction to the tendon pull resulting from the beam deformation. Also in these models, prestressing efforts are generally smaller due to the reduction in the effective prestressing load caused by the internal elastic shortening calculation, as discussed in the previous study.

2.2.3 Cable geometry and bonded and unbonded systems in continuous beams

Comparation of Cases 33.c, 36.c, 39.c and 42.c. (see Figure 2 and Table 2) Continuous beam and tendon with parabolic sections starting from the ends with e=30cm.

The intention, when dealing with these cases, is to assess the sensitivity of the model by increasing the amount of tendons and/or active anchorages in the beam. When using two different tendons (each with half of the supposedly pre-dimensioned total steel area) the general layout is changed, with a change in the distribution of internal efforts. Post-tensioned systems, with and without adherence, differ from each other by the interaction between elements and by the maximum prestressing values, as shown in previous studies.

The losses coefficients are function of the prestressing system, with friction coefficient equal 0.2 and wobble 0.002 for post-tension and 0.1 e 0.001 for pre-tension. All with 6mm for accommodate the anchors.

It can be concluded:

- The applied loads exceeded the recommended (the deflection against gravity caused by prestressing was more than 6 times greater than the deflection caused by self-weight) and did not balance the self-weight as desired;
- By alternating the position of the tendon along the outline, it is possible to combat both the positive moments in the middle of the spans and the negative ones in the central support;
- There are bending efforts at the ends due to the tendon eccentricity at the start nonzero;
- In the system with greased sheath (42.c and 36.c) there is a prestressing gain because it is allowed to apply higher prestressing values, in addition to reducing the magnitude of immediate losses;
- For the case of the tracing with two tendons (33.c' and 39.c'), the axial force was greater because the output of the second tendon at the end had a lower inclination and, consequently, a larger horizontal projection. On the other hand, considering a single theoretical tendon represented by the sum of the steel areas and located at the center of gravity of this area, the bending moments are reduced, since its eccentricity is smaller;
- Hyperstatic effects appear in continuous elements and can be seen in this study through the reaction in the central support - R_{hiper} - as shown in Figure 4. It shows the characteristic shearing force diagram for this type of situation, in which there is a "jump" referring to to the hyperstatic effect of prestressing.

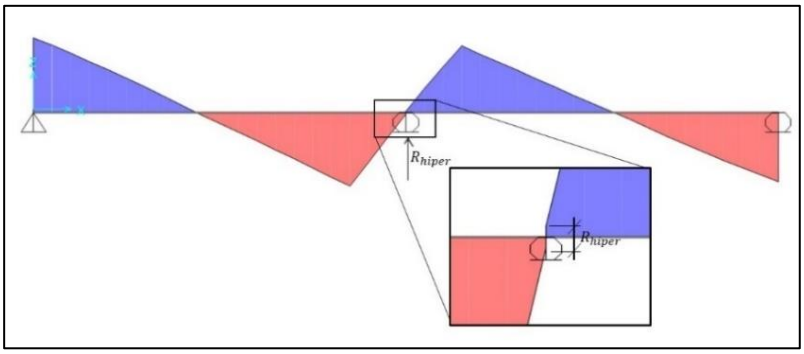

Figure 4. Characteristic shear diagram (PROT only) for hyperstatic beams of Study 03 described in Table 1.

3 Conclusions

In this work, a comparative study was made between rectangular prestressed concrete beams with different boundary conditions, adopting variations in the prestressing system, quantity and geometry of the tendons, number of active anchors and prestressing modeling strategies. The relevant results of each model are presented in Table 2.

It was seen in the use of Bridge that the immediate losses are automatically calculated with the same formulation of Norm [3], except for the immediate shortening of the concrete, which depends on the user. Long-term losses also represent values entered directly by the user, so the creep, shrinkage and relaxation processes must be defined prior to performing the analysis. To define the final value of the acting force, that is, after all losses, the program works directly with the load variation established by user in a phase preceding the data processing.

An excellent active reinforcement modeling tool of program is presented by giving it “load” or “element” aspects, as shown in item 2.2.1. When modeling it as an element, the tendon is discretized into smaller segments and their respective axials deformations are calculated and then converted into equivalent stresses that occur along its length. These stresses are transferred to the structure, reflecting the shortening loss arising from the deformation. This approach gives physical properties to the object and actually treats it as an element belonging to the set, unlike the first option. As, for example, in study 2, this simulation was more real due to the appearance of axial efforts and the reduction of the effective prestressing load, as an identifier in item 2.2.2.

As observed in the values of Study 3 in Table 2, by alternating the position of the cable along its path, it is possible to combat both the positive moments in the middle of the spans and the negative ones above the central support. Also considering Study 2, systems with greased sheaths have a prestressing gain because it is allowed to apply higher prestressing values, in addition to reducing immediate losses.

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References

[1] W. Pfeil. *Prestressed Concrete: Introduction.* 2 ed. Rio de Janeiro: Technical and Scientific Books Pub., 1984. 205 p.
[2] R. Buchaim. *Prestressed Concrete: axial traction, simple and shear force*. Londrina: Eduel, 2007, 256p.
[3] Brazilian Association of Technical Standards. Design of concrete structures - Procedure - NBR6118. Rio de Janeiro, ABNT, mar/2014, 237p.