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Structural reliability for the design of cold-formed steel I-sections members undergoing web crippling under interior loading conditions

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Abstract. Cold-formed steel sections (CFS) are subjected to different buckling modes. This work presents the results of structural reliability analysis of web crippling strength expression currently used in the Brazilian Standard, with a particular focus on I-sections (Stiffened Flanges) beams subjected to the loading conditions of Interior-One-Flange (IOF). Previous research studies of web crippling showed that I-sections members fastened to the bearing plate (support), subjected to IOF load case, presented inadequate reliability indexes (β) when compared to the target reliability index. Experimental data of I-sections members reported in the literature were analyzed to obtain the professional factor statistics. The First-Order Reliability Method (FORM) was used for the standard calibration procedure. The reliability analysis was developed for load combinations found in the North American and the Brazilian standards, in addition to the nominal live-to-dead load ratio, L_n/D_n, equal to 3 and 5. Target reliability indexes usually used in the calibration of the main international standards for CFS members were considered. It was found that the resistance factor present in the Brazilian standard should be increased for the I-sections members. A revision is necessary to standardize the level of safety of the design to the limit state of the web crippling.

Keywords: cold-formed sections, FORM method, web crippling.

1 Introduction

This work aims to evaluate the safety of cold-formed steel sections subjected to concentrated load in regions without transverse web stiffeners (web crippling), with an initial focus on I-sections with stiffened flanges. Research related to the application of structural reliability in design standards has been carried out in Brazil. Regarding the standards for cold formed sections, the works by Freitas et al. [1, 2], Brandão [3, 4] , Brandão et al. [5, 6, 7] and Toledo [8].

The design criteria used in this work were obtained from NBR 14762 [9]. The reliability index (β), obtained by the FORM method, was used in the calibration of resistance factors and will be compared with target reliability indexes (β_o), obtained in the calibration of the North American Specification (NAS) [10]. The reliability index was also evaluated by the Second-Order Reliability Method (SORM) and by the Monte Carlo Simulation (SMC) by computational routines with the application of CalREL software [11]. It is noteworthy that the NAS was calibrated by First Order Second Moment Method (FOSM). In FOSM, the probability distribution function is ignored, unlike more precise methods such as FORM, SORM and SMC used in this work.

A new proposal for formatting the design methodology provided in NBR 14762 (2010) will be presented, defining different resistance factors for different load cases.

2 Structural Reliability

Structural reliability is evaluated by the relationship between the measures of the probability of failure, P_f, and the reliability index, β, (Ditlevsen and Madsen [12]), and can be solved using approximate analytical methods, such as the First-Order Reliability Method (FORM) and the Second-Order Reliability Method (SORM) or simulation methods such as Monte Carlo Simulation (SMC).

FORM is based on an iterative procedure for determining the probability of failure and sensitivity measures of the structure. The main challenges of the method are the transformation of the basic variables into standard normal statistically independent random variables and the approximation of the failure surface by a linear surface at the point with the shortest distance to the origin. This point is called the design point and corresponds to the most probable point of failure. The reliability index β represents the minimum distance from the design point to the origin. Thus, the higher the reliability index, the more safe the structure.

After obtaining β, it is possible to estimate the probability of failure, according to eq. (1), where Φ(.) represents the standard normal cumulative distribution.

$$P_f = \Phi(-\beta) \tag{1}$$

In the case of SORM, instead of approximating the failure surface by a linear surface, it is made an approximation by a quadratic surface at the design point, as shown in Fig. 1. The second-order reliability method improves the FORM result by including additional information on limit state curvature (Haldar and Mahadevan [13]).

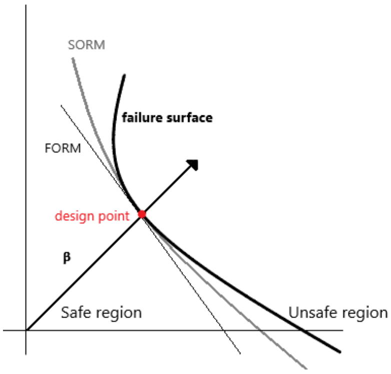

Figure 1. Comparison between FORM and SORM

The analysis using the Monte Carlo Simulation Method is based on the generation of random variables according to a specific probability distribution. The evaluation of the structural response is given from the probability of failure, calculated by the ratio between the number of trials n for which the limit-state function is less than zero and the total number of simulations (Melchers and Beck [14]).

Considering the limit state equation, the failure function describes the relationship between the resistance (R) and load (S) variables. The resistance of a structural element is typically a function of the strength of the material, the geometry of the section and its dimensions. The load can be expressed in terms of dead and live loads arising from use and occupation. This function can be represented by eq. (2) (Hsiao [15]):

$$G(.) = R_n MFP - (D + L) \tag{2}$$

Regarding resistance R_n is the nominal resistance determined in the project, and the variables M, F, and P are dimensionless random variables. The random variable M, called the "material factor", reflects the uncertainties of the material. The "fabrication factor" F is related to the variability of geometric properties. The professional factor P is a variable that reflects the uncertainties in determining the strength of a structural element. As for the loads, the gravitational actions, D and L, represent the variables of the dead and live loads. Table 1 presents the statistical parameters and probability distribution functions of random variables (Ellingwood et al [16]).

Table 1. Statistical parameters and probability distributions of random variables

Random variables	Mean Value/ nominal value	Coefficient of variation	Probability distribution
M	1.10	0.10	Lognormal
F	1.00	0.05	Lognormal
D	1.05	0.10	Normal
L	1.00	0.25	Largest Extreme Value

Table 2 summarizes the data on gravitational actions and target reliability indexes (β_o) used in the calibration of the NAS Specification (2016). The combination of $1.25D_n+1.5L_n$ shares is the same adopted by the Brazilian standard. As there is no definition of procedures and specific calibration parameters for NBR 14762 (2010), the nominal live-to-dead load ratio (L_n/D_n) and the target reliability indexes showed in table 2 will be adopted.

Table 2. NAS Specification calibration data

	$\gamma_D D_n + \gamma_L L_n$	L_n/D_n	β_o
LRFD	(1) $1.2D_n+1.6L_n$	5	2.5
LSD	(2) $1.25D_n+1.5Ln$	3	3.0

2.1 Professional Factor

The professional factor P is a random variable that reflects the uncertainties arising from the analysis methods. This variable can be obtained by the relationship between results obtained experimentally and theoretically. In this work 192 web crippling tests of I-section taken from experimental studies [17, 18, 19] were used. As a result of the statistical analysis of variable P, the mean of this variable (P_m), the standard deviation (σ_P), and the coefficient of variation (COV) are obtained. The mean of the variable indicates the accuracy of the theoretical model. Thus, P_m is equal to unity if the theoretical model exactly represents the experimental result.

2.2 Web Crippling

CFS are thin-walled and highly slender, so they are subject to web crippling due to concentrated load or reactions directly into the web(s). However, the theoretical analysis of web crippling is complex, due to a large number of variables involved in the behavior of these sections. For this reason, the normative provision is based on a wide experimental investigation (NAS Specification [10]). In 2008, the American Iron and Steel Institute (AISI) published a standard test procedure for testing web crippling [20]. However, the experimental studies considered as reference for this work are before this standard and did not follow the requirements of AISI S909.

The nominal strength F_n, per web, should be calculated by eq. (3) (NBR 14762 [11]; NAS [12]). It is noteworthy that the formulation of the design strength, F_{Rd}, is the same for both standards. However, the codes are differentiated by the presentation of resistance factors. In the Brazilian standard, γ is the resistance factor, greater than unity, which divides the nominal strength (Fn /γ), while in the NAS format, there is a factor ϕ, smaller than unity, multiplying the nominal strength (ϕ Fn).

$$F_n = \alpha t^2 f_y sen\theta \left(1-\alpha_r\sqrt{\frac{r_i}{t}}\right)\left(1+\alpha_c\sqrt{\frac{c}{t}}\right)\left(1-\alpha_h\sqrt{\frac{h}{t}}\right) \tag{3}$$

where,

θ is the angle between plane of web and plane of bearing surface, in degrees;

r_i is the inside bend radius;

c is the bearing length; whose minimum value is equal to 20 mm;

h is the flat dimension of web measured in plane of web;
t is the web thickness;
f_y is the design yield stress;
α is a coefficient associated with web crippling;
α_r is a coefficient associated with the inside bend radius;
α_c is a coefficient associated with is the bearing length;
α_h is a coefficient associated with web slenderness.

The flange support condition corresponds to the case where a flange is fastened or unfastened in the region of concentrated loads, to restrict the flange rotation, as shown in the Fig.2.

a) unfastened b) fastened

Figure 2. Web crippling on a web unfastened and fastened (Janarthanan et al. [21])

As established by the NAS (2016), there are 4 different load cases covered by the NBR 14762 (2010). These cases depend on whether the concentrated load is acting on two flanges or just one profile flange. Furthermore, load cases depend on the position of the concentrated load, that is, at the end of the bar or applied at some intermediate point in the span of the bar. These four loading cases can be defined as:

-EOF, End One Flange Loading;
-IOF, Interior One Flange Loading;
-ETF, End Two Flange Loading.
-ITF, Interior Two Flange Loading;

Comparing the Brazilian standard NBR 14762 (2010) with the NAS (2016) it is observed that the formulation of the design strength, F_{Rd}, is the same in both cases, however, the NBR 14762 (2010) has only one resistance factor, γ, equals 1.35 for all load cases while the resistance factors of the NAS vary according to the load case and support condition. Considering the relationship of the resistance factor with the probability of failure and, consequently, with the safety of the structure, this standardization of the resistance factor does not allow the user of the Brazilian standard to perceive the implicit level of safety in the different loading cases.

It is important to note that the coefficient calibrations of the current NAS were analyzed in a Final Report by Beshara and Schuster [22] to establish the resistance factors, ϕ, for each case of web crippling. However, in the specific case of the I-section IOF with the flange fastened, without mentioning the references, the North American standard reports a review on safety and resistance factors carried out in 2005 aiming at a more consistent calibration. Nonetheless, the coefficients and the safety factor adopted in the American standard generate reliability indexes lower than those obtained by the methodology proposed by Beshara and Schuster [22].

3 Results

In this item the results of the reliability analysis are presented based on the results of experimental tests [17, 18, 19] of I-sections whose failure occurred by web crippling. The reliability indexes (β) were obtained using CalREL [11] by FORM, SORM and SMC methods.

Table 3 shows the results of the analysis of the professional factor variable (P) and the number of specimens N per data grouping. Data were grouped based on the type of loading and the support condition. Adjustments for the best probability distribution for a P variable were performed based on the Anderson Darling test with the aid of the Minitab 17 software [23].

Table 3. Professional factor statistics for I-section

Load cases	Support condition	N	P_m	*COV*	Probability distribution
IOF	fastened	18	0.646	0.058	Smallest Extreme Value
EOF	unfastened	44	1.079	0.224	Largest Extreme Value
IOF	unfastened	23	0.879	0.147	Largest Extreme Value
ETF	unfastened	49	1.012	0.226	Normal
ITF	unfastened	58	0.991	0.269	Lognormal

Table 4 presents the results of the reliability analysis with the reliability indexes obtained with γ= 1.35 defined by the Brazilian standard. Overall, the reliability indexes obtained showed small deviations from the target β_o=2.5. However, for the IOF case, that is, for sections subjected to Interior-One-Flange loading, where the flange is fastened, the experimental resistance is much lower than the nominal theoretical resistance (P_m= 0.645).

Table 4. Professional factor statistics, reliability index β, resistance factor γ for web crippling I-sections

Method of NBR 14762 (2010)				$1.2D_n$+$1.6L_n$ (1)		$1.25D_n$+$1.5L_n$ (2)	
				L_n/D_n = 5	L_n/D_n = 3	L_n/D_n = 5	L_n/D_n = 3
	n	18	β_{FORM}	1.66	1.66	1.48	1.49
IOF	P_m	0.646	β_{SORM}	1.60	1.59	1.41	1.42
Fastened	*COV*	0.058	β_{SMC}	1.60	1.59	1.41	1.41
			$\gamma_{(2.5)}$	1.71	1.69	1.80	1.76
			$\gamma_{(3.0)}$	1.98	1.94	2.08	2.02
	n	44	β_{FORM}	2.80	2.84	2.65	2.70
EOF	P_m	1.079	β_{SORM}	2.81	2.84	2.65	2.70
Unfastened	*COV*	0.224	β_{SMC}	2.81	2.84	2.65	2.71
			$\gamma_{(2.5)}$	1.22	1.22	1.29	1.27
			$\gamma_{(3.0)}$	1.44	1.42	1.52	1.48
	n	23	β_{FORM}	2.47	2.50	2.30	2.35
IOF	P_m	0.879	β_{SORM}	2.49	2.52	2.32	2.37
Unfastened	*COV*	0.147	β_{SMC}	2.50	2.52	2.33	2.38
			$\gamma_{(2.5)}$	1.36	1.35	1.44	1.41
			$\gamma_{(3.0)}$	1.59	1.56	1.67	1.63
	n	49	β_{FORM}	2.26	2.24	2.16	2.15
ETF	P_m	1.012	β_{SORM}	2.17	2.15	2.06	2.06
Unfastened	*COV*	0.226	β_{SMC}	2.18	2.16	2.09	2.08
			$\gamma_{(2.5)}$	1.51	1.53	1.59	1.60
			$\gamma_{(3.0)}$	1.51	2.04	2.12	2.13
	n	58	β_{FORM}	2.23	2.22	2.10	2.10
ITF	P_m	0.991	β_{SORM}	2.20	2.19	2.07	2.07
Unfastened	*COV*	0.269	β_{SMC}	2.21	2.19	2.08	2.08
			$\gamma_{(2.5)}$	1.50	2.50	1.57	1.56
			$\gamma_{(3.0)}$	1.81	1.79	1.90	1.87

The resistance factors presented in Tab. 4, $\gamma_{(2.5)}$ and $\gamma_{(3.0)}$, were obtained using the FORM method and the target reliability index of 2.5 and 3.0. It is observed that the load combination (1) generates as results higher reliability indexes than for the combination (2). Furthermore, it is observed that the reliability index obtained by FORM is a good approximation when compared to the SMC, although the SORM method is more accurate.

Table 5 shows the summary of the calculated resistance factors using FOSM and FORM, assuming the target reliability index of 2.5 and the L_n/D_n ratio equal to 3 and 5. Except for the EOF case, that is, sections subject to external force applied to one flange in which the flange is unfastened, all other defined cases suggest that the resistance factors need to be greater than the value of "1.35" specified by NBR 14762 (2010).

Table 5. Resistance factors calculated with combination (2) for I-sections

Support and Flange Conditions	Load Cases		γ_{FOSM}		γ_{FORM}	
			L_n/D_n 5	L_n/D_n 3	L_n/D_n 5	L_n/D_n 3
Fastened to Support	One-Flange Loading or Reaction	End	-	-	-	-
		Interior	1.78	1.74	1.80	1.76
Unfastened	One-Flange Loading or Reaction	End	1.31	1.30	1.29	1.27
		Interior	1.43	1.40	1.44	1.41
	Two-Flange Loading or Reaction	End	1.40	1.39	1.59	1.60
		Interior	1.54	1.53	1.57	1.56

As observed in previous work by Brandão [4] that used the FOSM method, the same method used in NAS calibration, it was found low reliability indexes for the I-sections, defined as the IOF with the flange fastened. The results are influenced by the professional factor statistics, mainly the mean, P_m, well below the unity. In this case, the required strength factors are much higher than recommended by the Brazilian standard, which is confirmed in this work when using more precise methods such as FORM, SORM and SMC.

4 Conclusions

Reliability analyzes of I-sections subject to concentrated loads in regions without transverse stiffeners, which failed by web crippling, were organized according to the support condition of the flange and the loading case.

The Brazilian standard adopts only one resistance factor for all cases, unlike the American standard which varies according to the load case and support condition. However, it was found that the resistance factor present in the Brazilian standard should be significantly increased for the bars of I-section with stiffened flanges since in practically all cases analyzed the calculated results were higher than $\gamma = 1.35$, specified by NBR 14762 (2010). It is noteworthy that the calibration of the American standard was performed using the FOSM method, however, from the analysis using more accurate methods, such as the FORM method, the need to review the resistance factors present in the current standard is confirmed.

A review is needed to standardize the safety level to the web crippling limit state, with the specification of a set of resistance factors resulting from the calibration procedures. In addition, it is necessary to define the target reliability indexes (β_o) and the nominal live-to-dead load ratio (L_n/D_n) by the Brazilian standard review committees.

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