Reliability of perforated cold-formed steel channel-section beams using FORM method

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Abstract. Cold-formed steel (CFS) members can be designed with the AISI Direct Strength Method (DSM) which utilizes the local, distortional, and global elastic buckling properties of a cross-section to predict ultimate strength. The Brazilian standard also includes the DSM, but does not consider the presence of holes in the section. This paper develops the reliability assessment of beams, designed based on available methods in the literature adapted for sections with perforation. Such methods have their origin in the Direct Strength Method. An experimental database of cold-formed steel channel-section beams was established. Using this database, a statistical analysis of test results was conducted to determine the probability distribution function that provides the best fit to the professional factor data and the distribution parameters. The statistical parameters of professional factor, material strength, geometrical properties, load type, and load ratio were considered in the reliability evaluation. Then, the structural reliability indexes of beams subject to the local and distortional buckling modes were evaluated using the first order reliability method (FORM). The results showed that the target reliability index of 2.5 was not reached when the load combination of the Brazilian code was applied for profiles susceptible to the distortional buckling mode.

Keywords: cold formed sections; FORM method; perforated beams.

1. Introduction

According to ABNT NBR 14762 [1] cold-formed steel (CFS) members are shapes manufactured by press-braking blanks sheared sheets, cut lengths of coils or by roll forming cold-rolled coils or sheets, both forming operations being performed at ambient room temperature. There are three buckling modes observed in CFS beams: lateral-torsional or global, local and distortional. Such modes must be considered when calculating their resistance.

CFS members, being light, easy to transport, handling, manufacturing and also providing several alternative sections, offer several features in construction (Yu et al., [2]). CFS members are increasingly present in various sectors of civil construction and are widely used as structural elements. In order to obtain specific functionalities, it is usually manufactured with perforations in the web or flange to facilitate the passage of pipes in general. Such perforations have effects on the failure modes and strength of these structural members.

This work shows the study of the analysis of the structural reliability of lipped C-section CFS members with web perforations submitted to simple bending. The elements of this study were laterally contained to avoid the occurrence of lateral-torsional buckling. Load and resistance factors provided in ABNT NBR 14762 [1] and AISI S100 [3] are applied in the analysis, verifying if they lead to the target reliability index (β_0). In the calibration of the AISI S100 [3], for limit state designs in Load and Resistance Factor Design (LRFD) and Limit State Design (LSD) formats, β_0 is equal to 2.5 for LRFD and equal to 3.0 for LSD.

LRFD and LSD have the same philosophy in which resistance moment must always be greater than or equal to the requesting moment. But they use different load factors, load combinations, L_n / D_n ratio and target reliability indexes (Tab. 1).

Table 1. AISI standard calibration data.

Format	$\gamma_D D_n + \gamma_L L_n$	L_n/D_n	ϕ (AISI)	β_o
LRFD	$(i) 1.2D_n + 1.6L_n$	5	0.90	2.5
LSD	$(ii) 1.25D_n + 1.5L_n$	3	0.90	3.0

Initially, the reliability analysis was performed using the calibration data from the AISI S100 [3]. Subsequently, the analysis was performed using combination (ii), which is equivalent to the last combination present in ABNT NBR 14762 [1]. CFS structural members with perforations are not covered in the Brazilian code, as well as the L_n/D_n ratio adopted in the calibration is not defined. Then the AISI S100 [3] L_n/D_n ratios equal to 5 and 3 were adopted. The load factor (γ) for CFS structural members subjected to simple bending in the Brazilian code is equal to 1.10. The coefficient γ is related to the load factor (ϕ) present in the American code so that $\gamma=1/\phi$.

Reliability indexes were obtained using the First Order Reliability Method (FORM). Based on a database created from 101 tests on CFS beams, carried out by several authors, statistical information was obtained to the professional coefficient (P), which is defined by the ratio between experimental resistance moment and theoretical resistance moment. The theoretical resistance moment was obtained by applying the Direct Strength Method (DSM). The elastic stability analysis needed to apply DSM was done with CUFSM software (Schafer and Ádàny [4]).

2. DSM for members with perforations

The Direct Strength Method, described in Annex C of ABNT NBR 14762 [1] and present in the main body of AISI S100 [3], offers a procedure for determining the forces of different buckling failure modes for CFS members. DSM analyzes the buckling of the entire cross section rather than individual elements. For a beam, the resistance curves related to the global, distortional and local buckling modes, as well as the slenderness limits, can be seen in Fig. 1. The characteristic value of the resistant bending moment (M_n) is given as the smallest value calculated for global (M_{ne}), local (M_{nl}) and distortional (M_{nd}) buckling. For the application of the DSM, the elastic stability analysis must be performed to obtain the bending moments of elastic buckling or critical moments global (M_{cre}), local (M_{crl}) and distortional (M_{crd}).

Developed initially by Schafer and Pekoz [5], DSM had its extension to cold-formed profile bars with perforations studied by Moen and Schafer [6] in six different formulations. In this work, five formulations applicable to beams with perforations were addressed:

MRD-1: Method 1 has formulations identical to the original method. However, in determining the critical moments of buckling, the influence of the perforations must be considered;

MRD-2: the yield moment of the gross section is replaced by the yield moment of the liquid section. The influence of perforations in determining critical moments must also be considered;

MRD-3: the nominal local and distortional resistive moments are limited to the flow moment of the liquid section. A local-global interaction in M_{ne} is assumed. Considers the influence of perforations at critical moments;

MRD-4: the nominal resistance moment related to local buckling is limited to the flow moment of the liquid section. The determination of critical moments takes into account the perforations. It assumes local-global interaction in the determination of M_{ne} and a transition between elastic buckling and liquid section flow is included in the determination of the nominal moment resistance related to distortional buckling;

MRD-5: the same information as in method 4 is observed. However, it is considered a non-linear transition in the determination of the nominal moment resistance related to the local buckling.

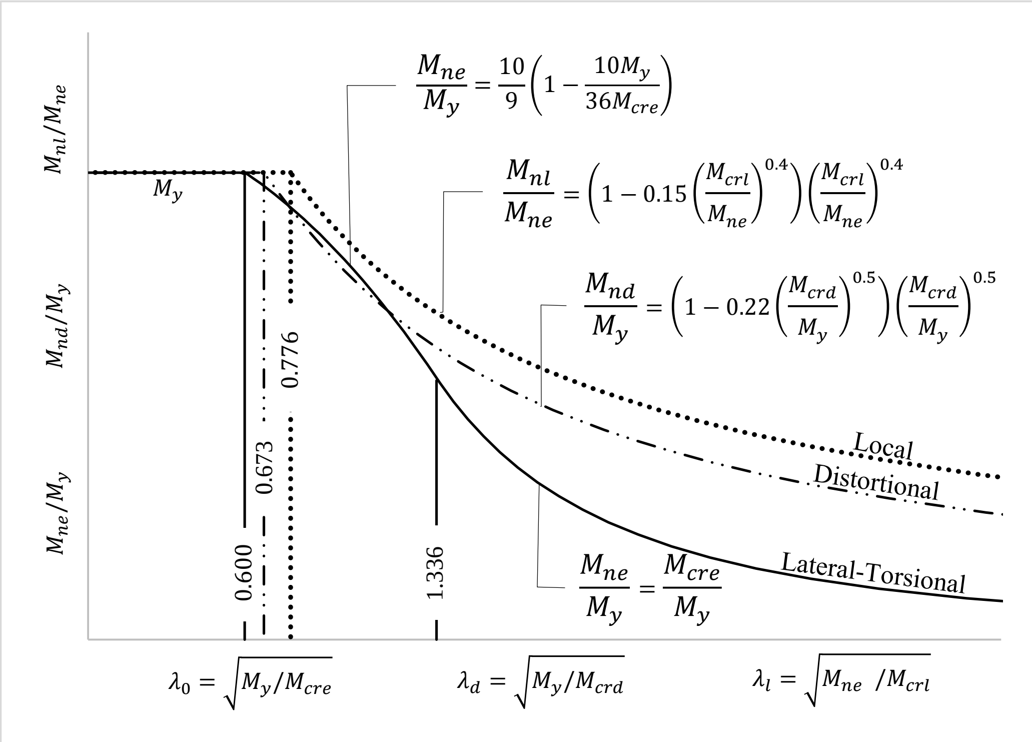

Figure 1. DSM strength curves for a beam

3. Professional factor

As with experiments, uncertainties are present in models. Such uncertainties are the result of theoretical approximations of the real behavior of materials and simplifications in considering actions and their consequent effects. During analysis, comparisons are made between experimental results (M_{exp}), obtained in laboratories by tests, and theoretical results (M_{teo}), obtained by models in accordance with normative predictions.

The professional factor or model error (P) is a random variable that expresses the uncertainties inherent to the variables considered in the model under analysis. The random variable P is obtained by the ratio between M_{exp} and M_{teo}, considering the average strength values of the materials and disregarding resistance factors. By statistical analysis of variable P, its mean (P_m) and the coefficient of variation (V_P) are obtained. The MINITAB® software [7] was used to calculate the mean values and standard deviations, as well as to obtain the best fit probability distribution function.

The database created for the study has 101 results from different experimental programs by the following authors: Shan & Laboube [8], Rad [9], Moen et al. [10], Zhao et al. [11] and Chen et al. [12]. It was verified that 49 have local buckling mode as dominant and 52 have distortional buckling as dominant mode. The results of the statistical analysis of the professional factor can be seen in Tab. 2. The information was grouped according to each formulation and failure mode.

Table 2. Professional Factor statistics for DSM Method.

Group	Nomenclature	P Statistics				Probability density function
		N	P_m	σ_P	V_P	
DSM-1; Local mode	L1	49	1.062	0.059	0.055	Lognormal
DSM-1; Distortional mode	D1	52	0.975	0.073	0.075	Normal
DSM-1; All data	T1	101	1.017	0.079	0.078	Normal
DSM-2; Local mode	L2	49	1.067	0.058	0.055	Lognormal
DSM-2; Distortional mode	D2	52	1.023	0.070	0.069	Type I Extreme Value
DSM-2; All data	T2	101	1.045	0.068	0.065	Lognormal
DSM-3; Local mode	L3	49	1.063	0.058	0.055	Lognormal
DSM-3; Distortional mode	D3	52	0.975	0.073	0.075	Normal
DSM-3; All data	T3	101	1.018	0.079	0.078	Normal
DSM-4; Local mode	L4	49	1.063	0.058	0.055	Lognormal
DSM-4; Distortional mode	D4	52	0.989	0.064	0.065	Normal
DSM-4; All data	T4	101	1.025	0.072	0.070	Normal
DSM-5; Local mode	L5	49	1.063	0.058	0.055	Lognormal
DSM-5; Distortional mode	D5	52	0.989	0.064	0.065	Normal
DSM-5; All data	T5	101	1.025	0.072	0.070	Normal

4. Performance function

The concept of a limit state is used to define failure in the context of structural reliability analysis. A limit state is a boundary between the desired and undesired performance of a structure. This limit is often mathematically represented by a failure or performance function. The performance function can be a function of many variables: load, material properties, dimensions and geometry of the section, analysis factor and so on. Gravitational loads can be described in terms of permanent (D) and variable (L) loads. The performance function used in this work is defined by:

$$g(.) = R_n MFP - c'(D + L). \tag{1}$$

The variable R_n is the nominal resistance based on the model used, M (material factor), F (fabrication factor) and P (professional factor) are variables that reflect the uncertainties of material properties, cross section geometry and analysis methods used, respectively. Finally, c' is a deterministic coefficient. The statistical parameters given in Tab. 3 are used for CFS members and such data are based on Ellingwood et al. [13], Hsiao [14] and AISI S100 [3].

Table 3. Statistics of load and resistance parameters.

Variable	Mean-to-nominal value ratio	Coefficient of Variation	Distribution
M	1.10	0.10	Lognormal
F	1.00	0.05	Lognormal
Dead Load (D)	$D_m/D_n = 1.05$	0.10	Normal
Live Load (L)	$L_m/L_n = 1.00$	0.25	Type I Extreme Value

5. FORM method

The function of a limit state can be explicit or implicit from the basic random variables and can have a simple or complicated form. The joint probability density function of the random variables for this type of problem is in general difficult to obtain. FORM is an analytical method used to calculate the probability of failure. The failure function is rewritten in terms of standard normal space variables or reduced variables. The search for the design point is a fundamental step to obtain the probability of failure by the reliability index (β). In this work, to obtain the design point, an algorithm developed by Hasofer and Lind [15] and improved by Rackwitz and Fiessler [16] was applied. The reliability index is defined as the distance between the design point and the origin of the reduced variables space. The probability of failure is then calculated by:

$$p_f = \Phi(-\beta). \quad (2)$$

Where Φ(.) is the cumulative distribution function of the standard normal variable. The FORM method also provides significant measures regarding the contribution importance of each variable in relation to the probability of failure.

6. Results

For reliability analysis, four situations were considered as shown in Tab. 4. Situations 1A and 1B represent the formats used in AISI S100 [3]. As previously mentioned, the Brazilian code does not define the live load and dead load ratio used for calibration. Therefore, the analysis was performed for values equal to 5 and 3, represented in situations 2A and 2B, respectively.

Table 4. Situations involved in reliability analysis.

Situation	Load Combination	Ln/Dn	Resistance Factor	β_o	Code
1A	$1.2D_n + 1.6L_n$	5	$\emptyset = 0.90$	2.5	AISI S100
1B	$1.25D_n + 1.5L_n$	3	$\emptyset = 0.90$	3.0	
2A	$1.25D_n + 1.5L_n$	5	$\gamma = 1.10$	2.5	NBR 14762
2B	$1.25D_n + 1.5L_n$	3	$\gamma = 1.10$	2.5	

It is possible to observe in Fig. 2 the reliability indexes obtained by the FORM method for situations 1A and 1B, which represent the LRFD and LSD formats, respectively. For the local mode, the LRFD reliability index presented values higher than the target value of 2.5. In the case of the distortion mode, only the LRFD - D2 showed a satisfactory result in relation to the target index. For the analysis that involves all data, in relation to the LRFD format, in general, the reliability index converges to the target value. Regarding the LSD format, of the values obtained for the reliability index, none of the groups presented satisfactory results. The values in this case were well below the target value of 3.0 indicating the need for a more conservative resistance factor.

Figure 2. Comparison of reliability indexes β for situations 1A and 1B.

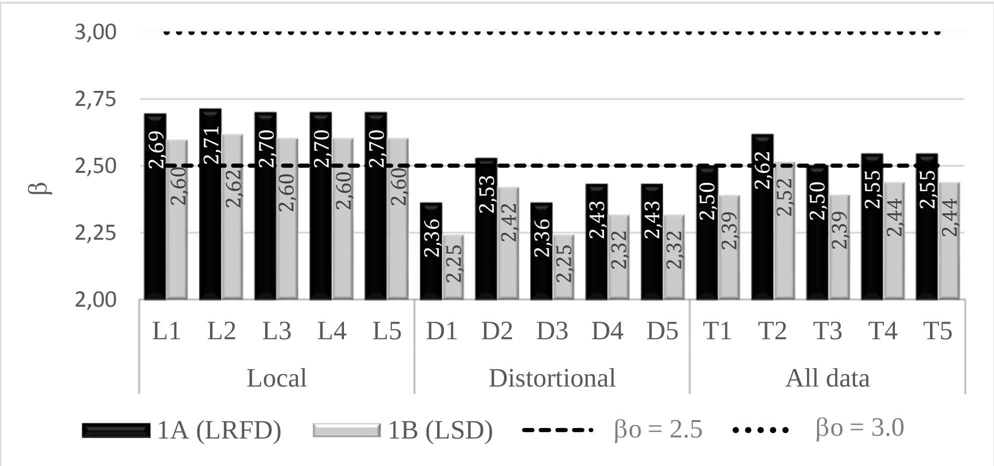

Figure 3 shows the reliability index values obtained for situations 2A and 2B. For the local mode, both situation 2A and 2B presented satisfactory values for the target index 2.5. Situation 2B presented results above the target index. Regarding the distortional mode, both 2A and 2B obtained reliability index results that were much lower than the target. Results with this characteristic were observed by Toledo [17] analyzing the reliability of cold formed profile elements without perforations subject to simple bending. For the analysis involving all data, only situation 2B-T2 presented a reliability index value close to the target.

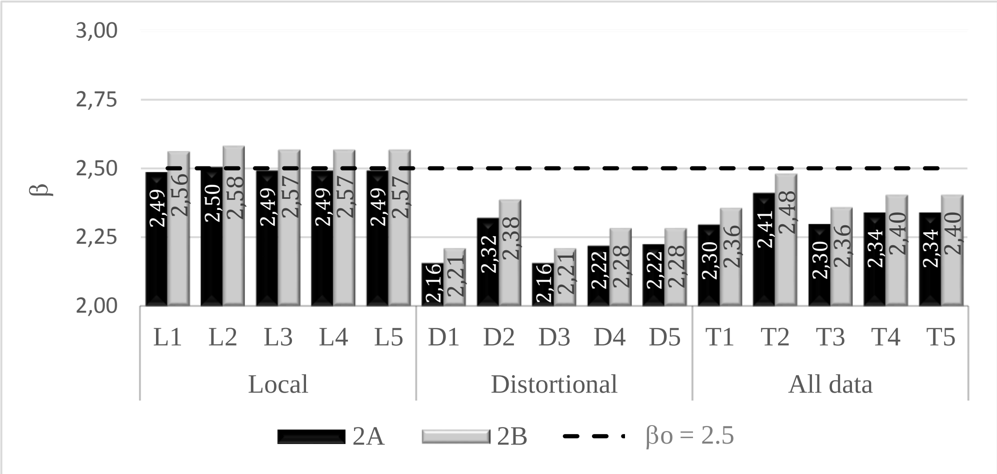

Figure 3. Comparison of reliability indexes β for situations 2A and 2B

7. Conclusions

The objective of this work was to evaluate the reliability of cold-formed elements, with core perforations, submitted to simple bending, designed according to the MRD from formulations proposed by Moen and Schafer [6]. Using the load factor $\phi = 0.90$ of AISI S100 [3], the LRFD format achieved a satisfactory result only for the local mode. The reliability indexes obtained for LSD showed results well below the target value of 3.0. Using the load factor $\gamma = 1.10$ together with the load combination $1.25D_n + 1.5L_n$ of ABNT NBR 14762 [1], and L_n/D_n ratio equal to 5 or 3, the reliability index only reaches satisfactory values for the target value equal to 2.5 in the occurrence of the local mode. The resistance factor of the Brazilian code for situations involving a distortional mode is not sufficient to determine a minimally safe structural element. A study involving the distortional mode is necessary to obtain a resistance factor that reach the reliability target value.

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Authorship statement. The authors confirm that they are solely responsible for the authorship of this work, and that all material that has been included here as part of this article is the property of the authors or has the permission of the fit be included here.

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