Application of an incomplete similarity approach in the analysis of impact problems

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Abstract. The assessment of safety to the impact of motor vehicles requires costly experimental tests, such as crash-tests and advanced methods of structural analysis. The design of bus structures in Brazil, for example, does not have adequate calculations methods and regulatory standards, and safety is difficult to assess. Therefore, a methodology is proposed for conducting impact tests on small-scale models that have incomplete similarity, in order to facilitate access to experimental data about vehicle crashworthiness. The methodology is applied to the simplified problem of a drop test on a fully clamped tubular steel beam. The test was numerically simulated using the Abaqus® software, from which it was possible to reproduce results from prototypes and models. Through a methodology presented in the literature, two models were used to correct the incomplete similarity characterized by the parameter regarding the beam thickness. This approach demonstrated a good correlation with the numerical results in the range of the dimensionless group in which the models were made, and started to diverge from the numerical results as it moved away from that range. A second approach, with a second non-similar parameter, was used for comparison with experimental data, which demonstrated a good correlation.

Keywords: Similarity, incomplete similarity, impact, deformable-body explicit dynamics.

1 Introduction

1.1 Contextualization

Dimensional analysis theory is a tool that allows us to have a better understanding of a physical phenomenon before carrying out a numerical or experimental analysis of the problem (Fox et al [1]). It also allows, based on the similarity principle, to estimate the physical quantities that occur in a prototype (which is what we call the real-scale object of study) from an experiment performed in a reduced (or increased) scale model. This theory has been used for many years in the design of aircraft, ships, and buildings, often to analyze the physical effects associated with fluid mechanics. In the present work, an analysis of incomplete similarity approach applied to the behavior of a tubular steel beam under impact loading is made, in order to evaluate its effectiveness and correlation with numerical and experimental results. Such analysis is motivated by contributing to studies to improve the crashworthiness and safety of motor vehicles and aims to provide a methodology that allows the testing of impact problems in reduced scale models which have distorted dimensions.

1.2 Objective

As stated in the contextualization, we seek to propose a methodology that provides theoretical support to carry out tests of a reduced model in impact problems where there is incomplete similarity, in such a way as to allow the investigation of deformations and accelerations present in the metallic structures of vehicles with cheaper and simpler tests. The application of such methodology is restricted to the simplified case of a fully-clamped steel beam. A similarity formulation, as proposed by Barenblatt [2], is applied to the simplified beam drop-test and investigated through two different approaches which are compared to numerical simulations and experimental data.

2 Theoretical foundation

2.1 Dimensional analysis and pi groups theorem

In the study of a given physical phenomenon, we can write the functional relationship between a studied quantity and its governing parameters through the following equation:

$$a = f(a_1,\ldots,a_k,b_1,\ldots,b_m)\,. \tag{1}$$

In this equation, a represents the studied quantity; a_i represent the governing parameters with independent physical dimensions; b_i represent the governing parameters with dependent dimensions (their dimensions can be written as a product of the power of the dimensions of the parameters with independent dimensions); and the function f represents the functional relationship that exists between the parameters and the studied quantity. Barenblatt [2] demonstrates, through mathematical manipulation, that eq. (1) can be rewritten as follows:

$$\Pi = \Phi(\Pi_1,\ldots,\Pi_m)\,, \tag{2}$$

in which:

$$\Pi = \frac{a}{a_1^p \cdots a_k^r} \qquad \text{and} \qquad \Pi_i = \frac{b_i}{a_1^{p_i} \cdots a_k^{r_i}}\,.$$

The exponents p through r of each Π group are determined so that the variables Π and Π_i are dimensionless. The function Φ represents the functional relationship that exists between these variables. Equation (2) states that it is possible to reduce the number of parameters of a given phenomena by using the proper dimensionless numbers; and the result of the phenomena doesn't depends on the choice of the physical dimensions, but on the dimensionless numbers.

2.2 Similarity

In order to state that two given physical phenomena are similar, they must have equal dimensionless numbers Π_1 to Π_m, as defined in eq. (2). The type of similarity defined by this equation is called by Barenblatt [2] as complete similarity. There is an even broader class that lists the dimensionless parameters of a problem. This class is defined, according to Barenblatt (2003), as the equation below:

$$\Pi = \Pi_{l+1}^{\alpha_{l+1}} \cdots \Pi_m^{\alpha_m}\, \Phi_1\left(\frac{\Pi_1}{\Pi_{l+1}^{\beta_1} \cdots \Pi_m^{\delta_1}},\ldots,\frac{\Pi_l}{\Pi_{l+1}^{\beta_l} \cdots \Pi_m^{\delta_l}}\right). \tag{3}$$

The values of Π_{l+1} to Π_m are, particularly, characterized by decreasing the number of parameters of the function Φ_1 in relation to the function Φ of eq. (2), in a way analogous to the role played by the parameters a_1 up to a_k within eq. (2). The exponents α_{l+1} up to α_m and β_i to δ_i are equation parameters that can only be discovered through experimental or numerical analysis, unlike the parameters p to r in eq. (2), which are determined by dimensional analysis. Barenblatt [2] states that there is a special case of eq. (3), which has appeared frequently in scientific research. Such a case is shown in the equation below and occurs when $l+1=m$ and all coefficients β_i to δ_i are equal to zero.

$$\Pi = \Pi_{l+1}^{\alpha}\, \Phi_1(\Pi_1,\ldots,\Pi_l)\,. \tag{4}$$

When some physical phenomenon behaves according to eq. (4), we have the possibility of determining the relationship between model and prototype with similarity only in the values of $\prod_1$ to $\prod_l$, and the parameter $\prod_{l+1}$ does not need to be similar, with the possibility of being corrected by the exponent α, configuring an incomplete similarity.

3 Application

3.1 Description of the fully-clamped beam drop test

The problem that will be analyzed consists of an impact block drop against a fully-clamped beam, with a velocity that is determined by the initial height of the fall subjected to the acceleration of gravity. The physical quantity evaluated is the permanent displacement in the beam after the removal of the mass. Figure 1 shows a schematic figure of the problem under consideration, together with the relevant parameters of the problem.

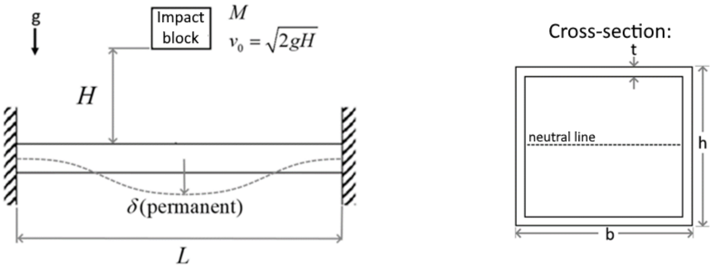

Figure 1. Schematic figure of the beam drop test problem

The parameter δ represents the permanent deformation, in mm; v_0 is the velocity of the impact block at the moment of impact in mm/s; M is the mass of the impact block, in ton; h, b and t are the dimensions of the cross-section of the beam, in mm; The static yield stress of the material is represented by σ_0, given in MPa; and L is the total length, in mm.

The functional relationship between permanent deformation and its parameters was simplified in the following manner:

$$\delta = f(v_0, M, h, t, \sigma_0, b, L). \tag{5}$$

The variables with independent dimensions chosen to represent the problem were v_0, M and h. Thus, the problem $\prod$ groups are as follows:

$$\prod = \frac{\delta}{h}, \qquad \prod_1 = \frac{t}{h}, \qquad \prod_2 = \frac{\sigma_0 h^3}{v_0^2 M}, \qquad \prod_3 = \frac{b}{h}, \qquad \prod_4 = \frac{L}{h}.$$

In order to evaluate the application of eq. (4) and using the $\prod$ groups encountered above, the following functional relationship was defined, which we will call Approach A:

$$\prod = \prod_1^{\alpha} \Phi_1(\prod_2, \prod_3, \prod_4). \tag{6}$$

For the definition of the parameter that does not present similarity, it was taken into account that when a structure like a tube is geometrically scaled it is not always possible for the thickness to obey the same scale factor due to manufacturing limitations. Therefore, it is a practical case of incomplete similarity that can be corrected with the incomplete similarity method expressed in eq. (4).

To apply the incomplete similarity strategy of eq. (6), a model must have the same values of $\prod_2$, $\prod_3$, and $\prod_4$ as the prototype so that the corresponding value of $\Phi_1(\prod_2, \prod_3, \prod_4)$ is also equal. However, in order to determine the value of the exponent α, it is necessary to prepare a second model, with a value of $\prod_1$ different from the first. By equating the numerical results for the two models we can find the values of the constants α and $\Phi_1(\prod_2, \prod_3, \prod_4)$.

For the comparison with the experimental results, another particular case of eq. (3) was considered. We consider that $l+2=m$, all exponents β_i to δ_i are null, and the exponents α_i are not null. We call this case Approach B:

$$\Pi = \Pi_1^{\alpha_1}\,\Pi_2^{\alpha_2}\,\Phi_1(\Pi_3,\Pi_4)\,. \tag{7}$$

Additionally, for Approach B, two scale factors are defined and will be used later to relate the results obtained by similarity with the experimental ones. According to the definition of the scale factor and replacing the values of δ with eq. (7), the scale factor of δ is obtained as follows:

$$\beta_\delta = \frac{(\delta)_M}{(\delta)_P} = \frac{(h)_M}{(h)_P}\left(\frac{(\Pi_1)_M}{(\Pi_1)_P}\right)^{\alpha_1}\left(\frac{(\Pi_2)_M}{(\Pi_2)_P}\right)^{\alpha_2}. \tag{8}$$

The time scale factor is also defined:

$$(v_0)_P = \frac{(\Delta L)_P}{(\Delta T)_P} = \frac{(\Delta L)_M/\beta}{(\Delta T)_M/\beta_t} = \frac{\beta_t}{\beta}(v_0)_M \rightarrow \beta_t = \beta\frac{(v_0)_P}{(v_0)_M} \rightarrow \beta_t = \frac{(h)_M}{(h)_P}\frac{(v_0)_P}{(v_0)_M}. \tag{9}$$

The scale factor β represents the geometric scale factor of dimensions except for thickness; ΔL is an arbitrary displacement and ΔT is an arbitrary time interval.

3.2 Numerical model

Figure 2 shows the boundary conditions applied to the numerical model used to represent the drop-test, with a 1/4 geometry of the beam. A converged mesh consisting of quadrilateral shell elements with 4 nodes and full integration was positioned on the middle surface of the tube thickness. The simulations were conducted using Abaqus® explicit dynamics solver, with double floating-point precision and automatic time step. The impactor block is modeled as a rigid shell coupled to a point mass that, due to symmetry, is equal to 1/4 the total mass of the problem. At the beginning of the simulation, the impactor block is adjacent to the beam, with an initial velocity corresponding to the drop height. The contact between the block and the beam was defined as being of the “hard” type and without friction. Gravitational acceleration is defined as 9800 mm/s^2.

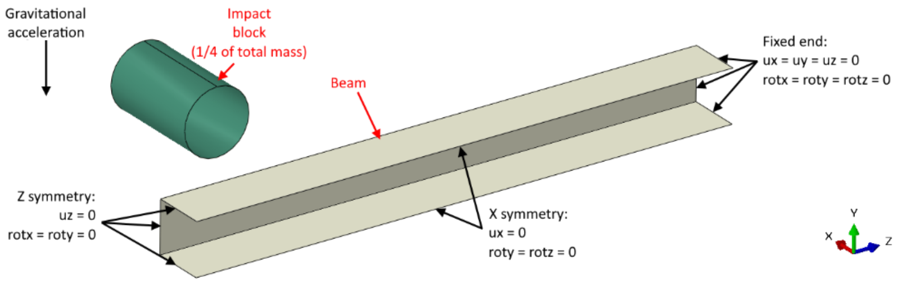

Figure 2. Boundary conditions applied to the beam

The beam material was modeled as a steel with a modulus of elasticity of 210 GPa and a Poisson's coefficient of 0.3. In the plastic portion, a static hardening curve was defined with an initial static yield stress of 335 MPa. The specific weight was defined as 7.85 x 10^{-9} ton/mm^3. The hardening was defined as being sensitive to strain rate using the Cowper-Symonds [3] model, which is defined by the following equation:

$$\sigma_d = \sigma_0\left[1+\left(\frac{\dot{\varepsilon}}{D}\right)^{1/p}\right]. \tag{10}$$

Where σ_0 is the static yield stress, in MPa and $\dot{\varepsilon}$ is the strain rate, in s^{-1}. The material constants p = 5 and D = 40.4 s^{-1} were used to represent mild steel, as recommended by Jones [4].

3.3 Correlation of results of similarity methods with numerical simulations

In the application of Approach A, two models were used to assess the results of a prototype. Equating the results generated by the two models, the coefficients α = - 1.629 and $\Phi_1(\prod_2 = 22.6, \prod_3 = 2, \prod_4 = 20)$ = 0.00294 were found. The function obtained by inserting these coefficients into eq. (6) is shown in Fig. 3 and compared with the numerical values generated by simulating nine prototypes with values of $\prod_1$ ranging between 0.033 and 0.187.

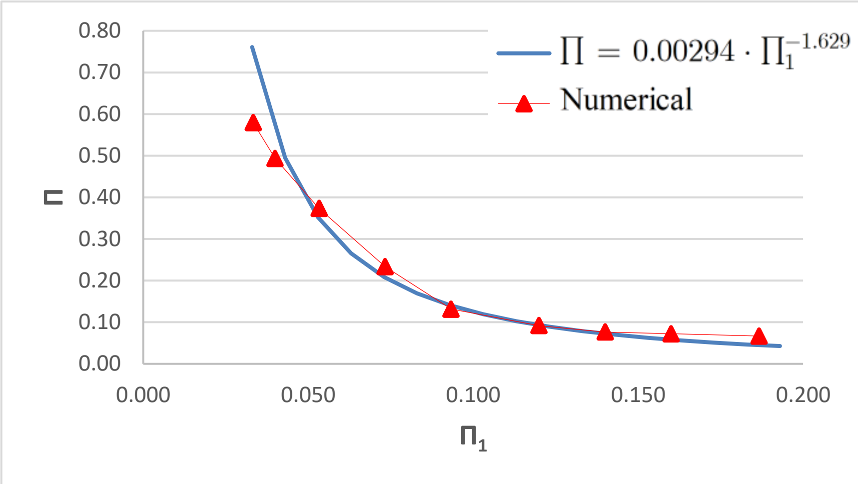

Figure 3. Comparison between results obtained by similarity using Approach A with numerical results obtained by simulation

For the value of $\prod_1$ = 0.073, there is an 11.2% deviation from the value obtained by similarity in relation to the numerical value. For values of $\prod_1$ less than 0.05 and greater than 0.14, the two curves begin to show a trend of divergence, reaching deviations of 29.2%, when $\prod_1$ = 0.033, and 32.3%, when $\prod_1$ = 0.187, from the values obtained by similarity in relation to the numerical models. Taking into account that the models used to calculate the coefficients in eq. (6) were made at $\prod_1$ = 0.05 and 0.10, it is naturally expected that in this range the results predicted by the similarity will have a more precise fit. Analyzing the deformation profile of the beams, it can be seen that the permanent deformation of the prototype with $\prod_1$ = 0.033, shown in Fig. 4, is mostly constituted by a localized kneading of the profile and demonstrates that a different physical phenomenon dominates the problem, which can justify the divergence in values from there.

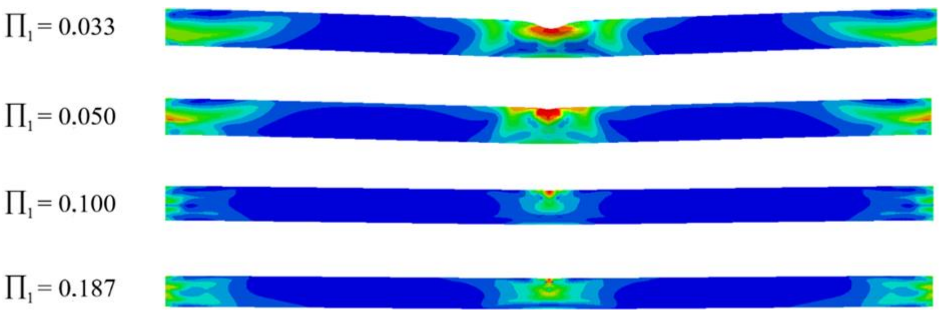

Figure 4. Permanent deformation profile of the beam for different values of $\prod_1$

For Approach B, the scaling process between model and prototype is analogous to that for Approach A. The function obtained by inserting the coefficients in eq. (7) is compared with the numerical values generated by simulating seven prototypes with values of $\prod_1$ ranging between 0.017 and 0.189, keeping constant $\prod_2$ = 16.96 and other six prototypes in which it remained constant $\prod_1$ = 0.078 and $\prod_2$ varied between 7.54 and 49.84. Figure 5 shows the comparison between the results obtained by similarity and those obtained numerically for the case where $\prod_2$ is constant.

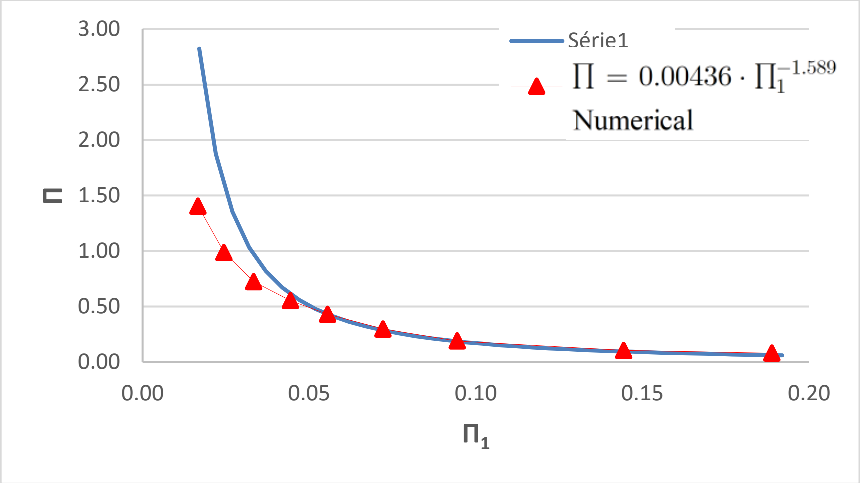

Figure 5. Comparison between results using Approach B considering a fixed value of $\prod_2$ = 16.96

Figure 6 shows the results of the group $\prod$ for the cases in which it remained constant $\prod_1$ = 0.078 and the value of $\prod_2$ varied. It is observed that, from $\prod_2$ = 30, the results start to show divergence, while smaller values of $\prod_2$ have a good correlation.

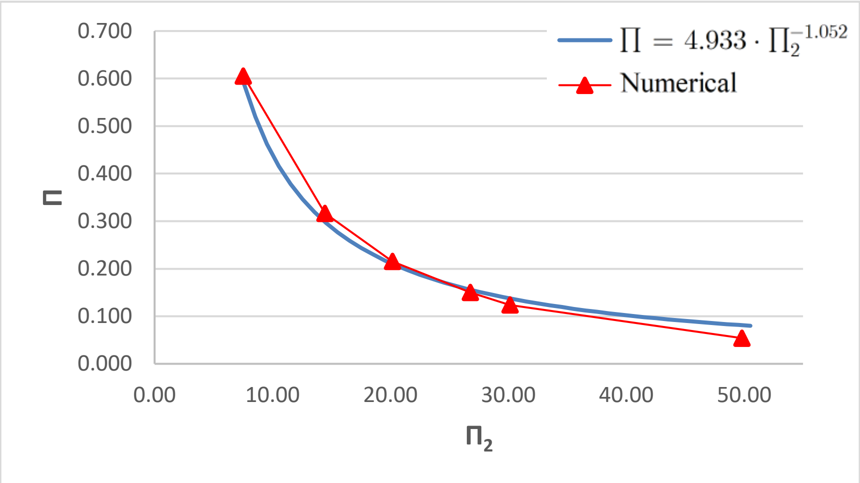

Figure 6. Comparison between results obtained by similarity using Approach B with numerical results obtained by simulation, considering a fixed value of $\prod_1$ = 0.078

3.4 Comparison with experimental results

A beam experimentally tested by Sordi [5], with cross-section dimensions 80x40x1.95 mm and length of 800 mm, was taken as a prototype. It was subjected to an impact load of a mass of 0.162 ton. From numerical simulation, three models were developed according to Approach B. From the analytical equation, a value of $\prod_1$ = 1.14 was found for the prototype. The experimental test of the beam presented a value of permanent deformation δ = 37.2 mm, which corresponds to a value of $\prod_1$ = 0.93. Comparing the values, there is an error of 25.8% of the result obtained by similarity in relation to the experimental result.

The comparison of another variable was made in which the displacement-time curve of the impactor block in the model and in the prototype was analyzed, right after contact with the beam. Then, the displacement of the model's impact block was corrected with the deformation δ scale factor, and the time was also corrected using its scale factor. In this comparison, the maximum value of the displacement of the corrected model's impact block presents an error of 5% in relation to the experimental value. Figure 7 shows the comparison between these results.

Using eqs. (8) and (9), the displacement and time of the model were corrected as follows:

$$t_{\text{CORRECTED}} = \frac{t_{\text{MODEL}}}{\beta_t} = 0.6444 \cdot t_{\text{MODEL}} \,. \tag{11}$$

$$\delta_{\text{CORRECTED}}(0.644 \cdot t_{\text{MODEL}}) = \frac{\delta_{\text{MODEL}}(t_{\text{MODEL}})}{\beta_\delta} = 3.3475 \cdot \delta_{\text{MODEL}}(t_{\text{MODEL}}) \,. \tag{12}$$

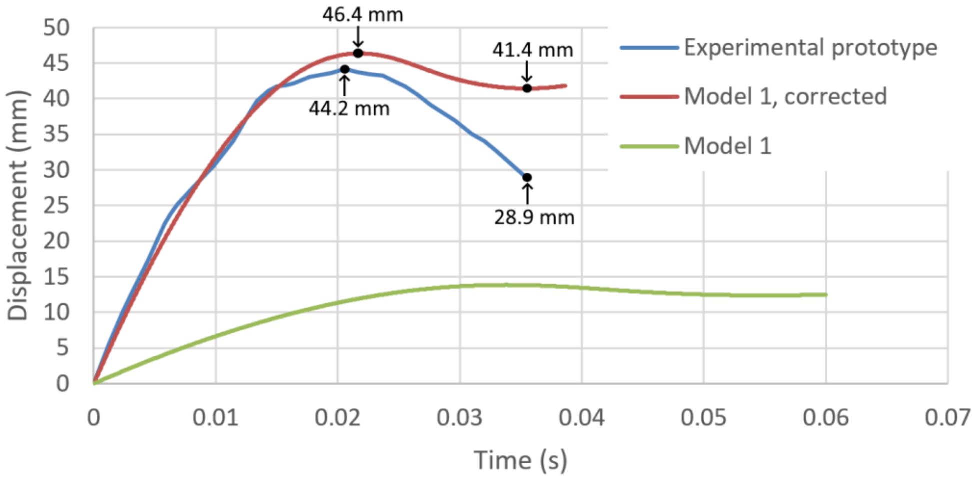

Figure 7. Impactor block displacement curve for Model 1, Experimental prototype, and corrected Model 1

4 Conclusions

In this work, the possibility of applying an incomplete similarity methodology in the context of impact problems was explored. When comparing the analytical approach with results generated through simulation a good fit was obtained, with the requirement that the non-similar parameters of the models must be adopted in such a way that, in that interval, they have the same physical mechanisms governing the response of the system as the prototype. It was observed that there was a good correlation with experimental results, showing errors of 25.8% in relation to the experimental results when the permanent deformation was evaluated and an error of 5% when comparing the maximum impact block's displacement. Through this work, it was shown that simple formulas can allow for correction in the distortions that occur in the reduced scale model and, therefore, the research can be further developed to allow its utilization in more complex impact scenarios, like crash-tests, which can lead to improvements in crashworthiness and safety of motor vehicles.

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