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Numerical strategies for complete assessment of the moment-curvature relationship of steel, concrete and steel-concrete composite sections

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Abstract. In the non-linear evaluation of the behavior of steel, concrete and steel-concrete composite cross sections, increment and iteration strategies are necessary for the complete construction of the moment-curvature relationship. For the correct simulation of sections composed of strain softening materials, a strategy of constant force increments fails to capture the downward stretch of the moment-curvature relationship. The present study aims to couple path-following strategies to the Strain Compatibility Method (SCM) to pass load limit points in the construction of the relations that describe the mechanical behavior of the section. In this context, the generalized displacement technique, depending on the generalized stiffness parameter, and the minimum residual displacement method, will be adapted to the cross sectional problem. Concomitantly, strain control strategies will be implemented as a comparison parameter, since when using it, the load limit points do not prevent the complete construction of the cross section equilibrium paths. The constitutive relationships will be addressed explicitly as well as the residual stresses present in the steel profiles. To validate the proposed numerical formulation, the results obtained are compared with numerical and experimental data available in the literature.

Keywords: Moment-curvature relationship, non-linear evaluation, cross-sectional analysis, SCM

1 Introduction

The analysis of the cross sectional behavior is important to measure the parameters of its stiffness and bearing capacity, directly impacting the structural element behavior. Considering the nonlinear stress-strain relationships of materials usually used structurally, the analysis procedures must be able to accurately capture such effects.

Some authors developed studies that focused on the analysis of the nonlinear behavior of cross sections. Li et al. [1] made a numerical study of rectangular tubular cross-sections and welded I cross-sections. The authors used a methodology based on quasi-Newton methods for the cross-sectional analysis [2]. Zubydan [3] made a specific study of UB and UC steel cross-sections under minor axis bending. In this study, the author proposed an empirical formulation for calculating the flexural stiffness degradation of these sections. In other words, the author developed an equation for the tangent modulus of elasticity of a section along the load history. Chiorean [4] made a brief study of the behavior of a steel I section totally encased in concrete considering the AISC LRFD [5] and ECCS [6] residual stress models. However, the author worked with generalized stiffness (using materials constitutive relations). More recently, Lemes et al. [7, 8] used the strain compatibility method to assess the strength and also axial and bending stiffness within the context of concentrated plasticity-based formulations. For the analysis, the standard Newton-Raphson method was coupled to the strain compatibility method where the constitutive relationships of the materials were explicitly used.

In the Lemes et al. [7, 8] researches, simplified increment and iteration strategies were adopted, which were interrupted when finding the load limit point of the moment-curvature relationship. In other words, the softening parts of these relationships were not obtained. Once using constitutive relations disregarding the materials strain-softening effect, such a stretch would be negligible. In this sense, Caldas [9] pointed out that a simple solution to obtain the stretches with negative rigidity of the moment-curvature relationship can be found using an increment strategy based on deformations. Chiorean [4] presented a formulation for the complete construction of the moment-curvature diagrams that were determined such that axial force and bending moment ratio was kept constant. A strain-driven algorithm has been developed, the solution of the nonlinear equilibrium equations was controlled by the assumed strain values in the most compressed point and by solving just two coupled nonlinear equations.

The purpose of this work is to use path-following methods to pass through critical points in the moment-curvature relationship of cross sections composed of steel and concrete. For corrections and control in the load increment, adaptations were made in the Generalized Stiffness Parameter (GSP) [10] to the variables present in the problem addressed here. During the iterative process, the minimum residual displacement norm strategy [11] was adapted.

2 Cross sectional analysis

The SCM is an EBT-based approach for the evaluation of compact cross-sections. Under external loads, a structure will gradually deform until it reaches equilibrium [5]. Once the internal and external forces are equal, the deformation stops, and, at the cross-section level, is studied by SCM [7].

2.1 Moment-curvature relationship

The discretization shown in Fig. 1 is used to find the axial strain, ε_i, in plastic centroid (PC) of each cross-sectional sub-area. Thus, through the material constitutive relationship, it is possible to obtain the respective stress, σ_i. In Figure 1, the deformed shape of an I section is illustrated for a combination of normal efforts (axial force and bending moment). Thus, the axial strain in the ith sub-area can be written as follows:

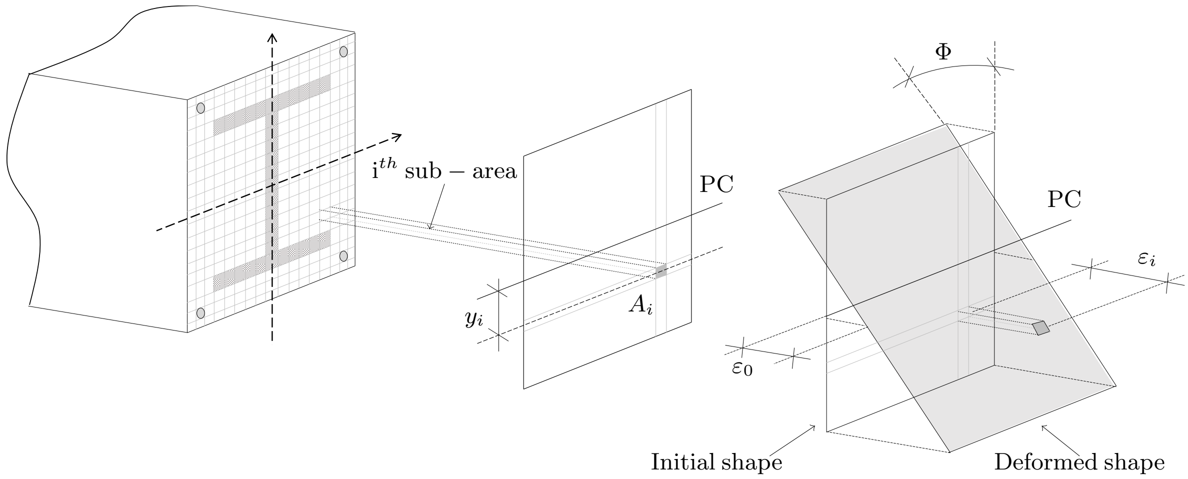


Figure 1. Linear strain field in major axis bending

$$\varepsilon_i = \varepsilon_0 + \Phi y_i + \varepsilon_{ri} \tag{1}$$

where y_i is the distance between the plastic centroids of the analyzed sub-area and the cross section, ε_0 is the axial strain of the PC section, ε_r is the strain due to residual stress, and Φ is its curvature. The variables ε_0 and Φ are terms of the strain vector $\mathbf{X} = [\varepsilon_0 \quad \Phi]^T$.

Numerically, it can be said that the section's equilibrium is obtained when the following relation between the external ($\mathbf{f}_{ext}$) and internal ($\mathbf{f}_{int}$) forces vector is satisfied:

$$\mathbf{F}(\mathbf{X}) = \mathbf{f}_{ext} - \mathbf{f}_{int} \cong 0 \tag{2}$$

The internal forces vector is calculated as a function of the strain vector ($\mathbf{X}$) by means of classical integrals. Thus, the Eq. 2 can be written as follows:

$$\mathbf{F}(\mathbf{X}) = \begin{bmatrix} N \\ M \end{bmatrix} - \begin{bmatrix} N_{int} = \displaystyle\int_A \sigma\left[\varepsilon\left(\varepsilon_0, \Phi\right)\right] dA \\ M_{int} = \displaystyle\int_A \sigma\left[\varepsilon\left(\varepsilon_0, \Phi\right)\right] y dA \end{bmatrix} \cong 0 \tag{3}$$

being N the axial nodal effort and M the bending moment at the analyzed node, obtained in structural analysis; N_{int} and M_{int} are the internal forces.

To solve the previous nonlinear equation, Chiorean [4] pointed out that by adopting $\mathbf{X} = \mathbf{0}$ at the process start, the convergence is achieved quickly. However, the convergence will only be satisfied in the first iteration if external forces are null. Thus, for the next iteration $(k + 1)$, the strain vector is calculated as follows:

$$\mathbf{X}^{k+1} = \mathbf{X}^k + \left[\mathbf{F}'\left(\mathbf{X}^k\right)\right]^{-1}\mathbf{F}\left(\mathbf{X}^k\right) \tag{4}$$

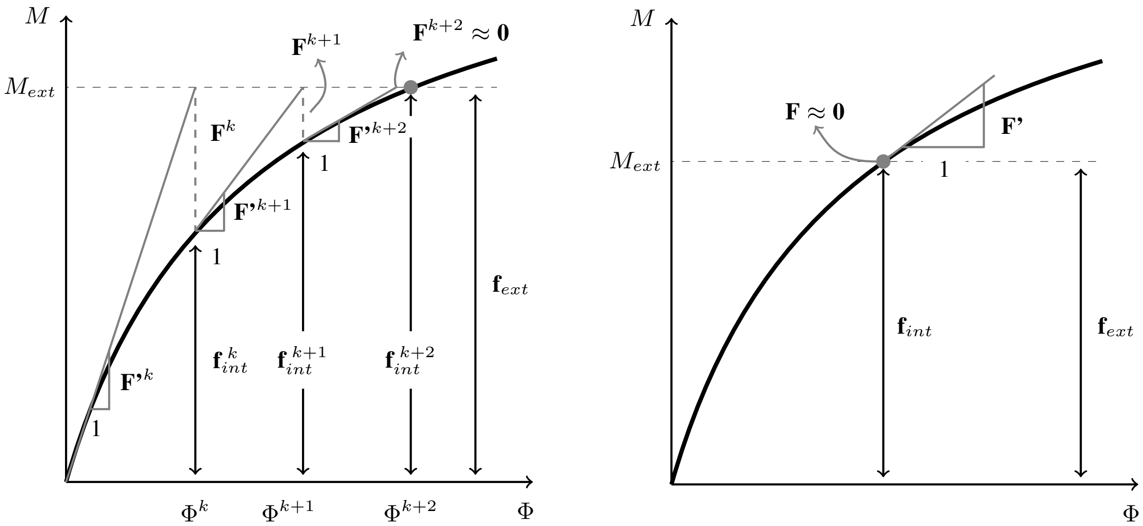
where $\mathbf{F}'$ is the Jacobian matrix of the nonlinear problem expressed by the Eq. (2), , that is:

$$\mathbf{F}' = \left(-\frac{\partial \mathbf{F}}{\partial \mathbf{X}}\right) = \begin{bmatrix} f_{11} = \dfrac{\partial N_{int}}{\partial \varepsilon_0} & f_{12} = \dfrac{\partial N_{int}}{\partial \Phi} \\ f_{21} = \dfrac{\partial M_{int}}{\partial \varepsilon_0} & f_{22} = \dfrac{\partial M_{int}}{\partial \Phi} \end{bmatrix} \tag{5}$$

Further details of the moment-curvature construction process can be found in Lemes et al. [7].

2.2 Cross sectional stiffness

When the cross-section equilibrium is reached, the external and internal forces vectors are numerically equal. Thus, the deformed configuration of the cross section, described by strain vector **X**, is found. For this condition, the parameters of cross sectional stiffness are determined. In turn, the axial strains in the sub-areas are used to calculate the Jacobian matrix at this point, as illustrated in Fig. 2(b).



(a) Iterative process

(b) Equilibrium point

Figure 2. $M \times \Phi$ relationship: stiffness parameters

Considering the incremental form of the cross-section problem, we have the following equation:

$$\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon \\ \Delta\Phi \end{Bmatrix} = \begin{Bmatrix} \Delta N \\ \Delta M \end{Bmatrix} \tag{6}$$

Using the stiffness concept, the differentiation of the force by its respective strain defines the stiffness of the analyzed degree of freedom. As the problem has two degrees of freedom, to obtain the axial stiffness the bending moment is kept constant ($\Delta M = 0$). Therefore, the system resolution to obtain the ratio of the force increment ΔN by the axial strain increment $\Delta\varepsilon$ defines the section axial stiffness EA_T. The same process can be adapted to obtain flexural stiffness EI_T. The calculated stiffnesses are presented below:

$$EA_T = \left.\frac{\Delta N}{\Delta \varepsilon}\right|_{\Delta M=0} = f_{11} - \frac{f_{12} f_{21}}{f_{22}} \tag{7}$$

$$EI_T = \left.\frac{\Delta M}{\Delta \Phi}\right|_{\Delta N=0} = f_{22} - \frac{f_{12}f_{21}}{f_{11}} \tag{8}$$

with f_{ij} being the constitutive matrix terms defined by Eq. (5).

3 Path-following strategies

In finite element method context, the nonlinear static solver consists of obtaining the equilibrium between internal and external forces for each load increment as described in Eq. (2) and modified as follows [8]:

$$\mathbf{f}_{ext} - \mathbf{f}_{int} \cong 0 \rightarrow \underbrace{(\mathbf{f}_{fix} + \lambda \mathbf{f}_r)}_{\mathbf{f}_{ext}} - \mathbf{f}_{int} \cong 0 \tag{9}$$

where $\mathbf{f}_{fix}$ is fixed forces vector, λ is the bending moment increment factor and $\mathbf{f}_r$ is the reference load vector.

To solve the nonlinear problem, load increment and iteration strategies are used.

The initial increase of the load parameter, $\Delta\lambda^0$, is automatically determined by the modified technique of generalized displacement [10]. Thus, $\Delta\lambda^0$ is calculated as:

$$\Delta\lambda^0 = \pm\Delta\lambda_1^0 \sqrt{\left| \frac{\left({}^1\delta\mathbf{X}_r^T\right)\left({}^1\delta\mathbf{X}_r\right)}{\left({}^t\delta\mathbf{X}_r^T\right)\left(\delta\mathbf{X}_r\right)} \right|} = \pm\Delta\lambda_1^0\sqrt{|GSP|} \tag{10}$$

where index 1 indicates the $\Delta\lambda^0$ and $\delta\mathbf{X}_r$ (tangential strains) values obtained in the first loading step, and GSP represents the Generalized Stiffness Parameter.

In the traditional scheme of the Newton-Raphson method, the load parameter λ is kept constant throughout the iterative process. Thus, the equilibrium path can be obtained until a limit point and/or a bifurcation point is reached. The variation of λ during the iterative cycle enables the full equilibrium path to be traced. In this work, the minimum residual displacement norm strategy proposed by Chan [11] was used. In this strategy, the correction of the load parameter $\delta\lambda^k$ is given by the equation:

$$\delta\lambda^k = -\frac{\left(\delta\mathbf{X}_r^k\right)^T \delta\mathbf{X}_g^k}{\left(\delta\mathbf{X}_r^k\right)^T \delta\mathbf{X}_r^k} \tag{11}$$

with $\delta\mathbf{X}_g^k$ being the correction obtained from the application of the Newton-Raphson method with the conventional strategy of λ increment, and $\delta\mathbf{X}_r^k$ the iterative vector displacements resulting from the application of $\mathbf{f}_r$.

4 Numerical example

Chiorean [4] presented a study of the residual stress models influence on the steel-concrete composite section behavior shown in Fig. 3. This is a W12x120 section totally encased in concrete and reinforced by four 20 mm diameter bars. The strengths f_{yd}, f_{yrd} and f_{cd} are taken equal to 30, 40 and 2 kN/cm^2, respectively. The modulus of elasticity of the steel section and the reinforcement are equal to 20000 kN/cm^2. In addition, it is noted that the concrete limit strains, ε_{ci} and ε_{cu} were taken as 0.002 and 0.0035, respectively, as was the steel ultimate strain, ε_u, taken as 0.01. The concrete softening was simulated by the parameter γ = 0.15 and for the concrete tensile behavior was considered α_1 = 1 and α_2 = 0.75.

5 Conclusions

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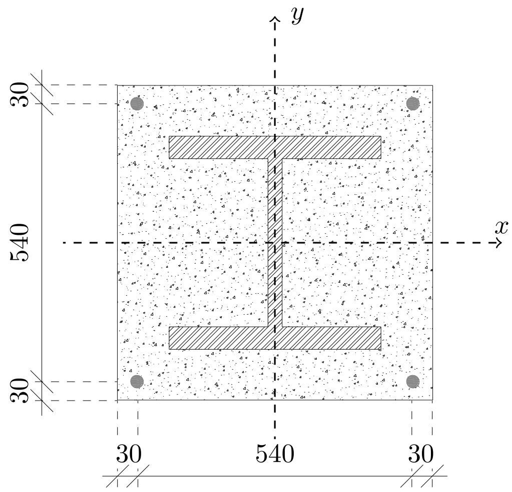

Figure 3. Composite section (dimensions in mm)

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