INELASTIC-LARGE DISPLACEMENT ANALYSIS OF STRUCTURES WITH CONTACT CONSTRAINTS

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Abstract. This work presents a numerical methodology based on inelastic-large displacement approach to simulate the 2D nonlinear behavior of structures under contact constraints imposed by the soil/rock. Non-linearities sources are addressed, such as: second-order effects, plasticity and contact. It is worth mentioning the adoption of a nonlinear finite element formulation based on the explicit separation between rigid body movements and those that cause strain (co-rotational approach). The finite element formulation also considers the nodal concentrated plasticity in which the material nonlinear behavior is represented by an explicit constitutive relationship by using the Strain Compatibility Method (SCM). The SCM is also applied here to define any cross-sections typology strength under axial force and bending moment. Thus, this numerical formulation can be used to obtain the non-linear response of problems involving structure-support interaction. The contact constraints imposed by the soil/rock can be considered as bilateral and unilateral. In case of unilateral constraints, a penalty method is applied in each load increment and during the iterative process. Numerical modeling of a structural circular ring under contact constraints are presented.

Keywords: Advanced numerical analysis, Co-rotational formulation, SCM-RPHM coupling, Contact constraints

1 Introduction

In many practical situations, structural elements are supported by other bodies or geological medium that offer resistance to their movements only in certain directions. Problems where the structure can lose contact other bodies, or even slip on its support, are usually found in the literature under the name “Contact Problems” [1]. Among the engineering problems where this structure-medium interaction can be found, the following stand out:

rails supported on railway sleepers, floors, buried pipes, foundation column piles, lateral bracing of columns in buildings, and the contact problem between the plates that make up a steel profile.

Soil/rock-structure interaction problems, in particular, can be characterized as unilateral or bilateral contact problems. Bilateral contact problem considers that the contact medium reacts to both traction and compression demands. A more realistic modeling of the soil/rock can be obtained considering in its formulation the reaction only to compression requests, which characterizes the contact problem as unilateral.

The analysis of unilateral contact problems is complex, as these problems have a non-linear behavior even when considering small displacement and strain, and linear elastic material. Thus, the unilateral contact problem can be formulated as an optimization problem with constraints imposed by other bodies or geological medium. The optimal conditions of the constrained structural problem are obtained by solving the equilibrium equations attending to the constraints and the contact complementarity condition.

In this context, Sun and Natori [2] presented a numerical study of stability problems of beams with large strains, as well as the post-buckling behavior, associated with unilateral contact constraints. Contact conditions were introduced via the penalty method. Unilateral contact problems were also studied in Simo et al. [3], where a numerical solution for large deflection structural problems, subject to contact constraints, and exhibiting an inelastic constitutive response was presented.

This work aims to study the nonlinear response of systems involving the soil/rock-structure interaction, always looking for the most realistic numerical modeling of the engineering problem (structural/geotechnical). The discrete model is used to represent the soil/rock behavior. For the structure modelling, non-linearity sources such as second order effects and inelasticity are addressed. It is worth emphasizing the adoption of the co-rotational framework in geometric nonlinear finite element (FE) formulation, based on the explicit separation between rigid body movements and those that cause strain. The FE formulation also considers the nodal concentrated plasticity. Thus, the simulation of the material nonlinear behavior of materials is approached through the Strain Compatibility Method (SCM), where the constitutive relationships of the materials are used explicitly [5]. The SCM is applied in determining the cross-section strength capacity as well. Furthermore, the present approach is not limited to a specific cross-section typology.

The CS-ASA will be the computational basis used in this work, which was initially developed for nonlinear static and dynamic analysis of steel structures [4]. More recently, Lemes [5] introduced the possibility of advanced static analysis of concrete and composite structures.

2 Numerical Formulation

For the consideration of nonlinear geometric effects, the co-rotational formulation (CRF) is used to describe the movement of the structure, which according to Hsiao et al. [6], Battini [7] and Santana [8], can be adapted to both the total Lagrangian formulation (TLF) and the updated Lagrangian formulation (ULF). In CRF, the movement is decomposed into two parts: one related to the element's rigid body movement and the other associated with pure strain. The element undergoes two movements: the first is the rigid body movement including rotation and translation of the element, and the second consists of the relative deformation in a local coordinate system that produces energy. Thus, it is necessary to obtain the relationship between the local and global coordinate system for the update necessity in the analysis of structural systems.

The co-rotational approach is convenient to stabilize the relationship between local and global variables, according to Alhasawi et al. [9]. Starting from the Virtual Works Principle (VWP), it is possible to describe the relationship between the forces between the two referential systems, and making the differentiation of the global forces in relation to the vector of global displacements gives the global stiffness matrix, as follow [10]:

$$\mathbf{K}_g = \frac{\Delta \mathbf{f}_g}{\Delta \mathbf{u}_g} = \mathbf{B}^T \mathbf{K}_s \mathbf{B} + \frac{\mathbf{z}\mathbf{z}^T}{L_f} N + \frac{1}{L_f^2}\left[\mathbf{r}\mathbf{z}^T + \mathbf{z}\mathbf{r}^T\right](M_i + M_j) \tag{1}$$

where L_f is the finite element length; $\mathbf{K}_s$ is the element stiffness matrix in the local system; N, M_i and M_j are the internal forces referring to the local degrees of freedom and **B** is the displacement transforming matrix from the global to the local system and vice versa, and:

$$\mathbf{r} = \begin{bmatrix} -c & -s & 0 & c & s & 0 \end{bmatrix}^T \tag{2}$$

$$\mathbf{z} = \begin{bmatrix} s & -c & 0 & -s & c & 0 \end{bmatrix}^T \tag{3}$$

where s is sin α, c is cos α. More details about this formulation can be seen in Silva [11].

For systems where the interaction between geological medium and structure is evaluated, the stiffness matrix of the structure must be added to the stiffness matrix of the foundation (soil/rock), $\mathbf{K}_b$, and the same must be done with the internal forces vector. Therefore:

$$\mathbf{K}_s = \mathbf{K}_{lc} + \mathbf{K}_b \tag{4}$$

in which $\mathbf{K}_{lc}$ is the structure stiffness matrix in the local system; $\mathbf{K}_b$ is the foundation matrix which, by considering the discrete spring model, is obtained by the contribution of each spring stiffness matrix ($\mathbf{K}_{bi}$). The springs are considered with an elastic behavior and no interaction between them is considered, and the stiffness matrix of each spring is given by:

$$\mathbf{K}_{bi} = \begin{bmatrix} k_{xi} & 0 & 0 \\ 0 & k_{yi} & 0 \\ 0 & 0 & k_{\theta i} \end{bmatrix} \tag{5}$$

where k_{xi}, k_{yi}, and $k_{\theta i}$, are respectively, the spring translations and rotation stiffnesses. In the FEM discretization, the structure mesh nodal points should coincide with the spring positions.

The beam-column finite element is used in the structure discretization, and the Refined Plastic Hinge Method (RPHM) is adopted, which is a concentrated plasticity model. By submitting a structural element to external efforts, it deforms, generating internal forces to balance the system. This strain, at the level of the cross-section, is addressed by the SCM, where it is assumed that the deformation field is linear, and that the section remains flat after the strain, as illustrated in Fig. 1 [12]. Discretization of cross sections is performed on fibers, where residual stresses are explicitly applied, and the element evaluation of axial stiffness and bending stiffness is based on the tangent to the moment-curvature relationship.

The representation of the behavior of a given material under the influence of a tension or compression force is given by its constitutive relationship. The bilinear stress-strain diagram (Fig. 2) was adopted as constitutive model to the steel material, with the following equations [12]:

$$\sigma = \begin{cases} -f_y, & \text{se} \quad -\varepsilon_u \geq \varepsilon > -\varepsilon_y \\ E_s\varepsilon, & \text{se} \quad -\varepsilon_y < \varepsilon < -\varepsilon_y \\ f_y & \text{se} \quad \varepsilon_y \leq \varepsilon > \varepsilon_u \end{cases} . \tag{6}$$

where σ is the axial stress; ε is the axial strain; f_y is the yield strength; E_s is the steel elasticity modulus; ε_y is the yield axial strain; and, ε_u is the ultimate axial strain.

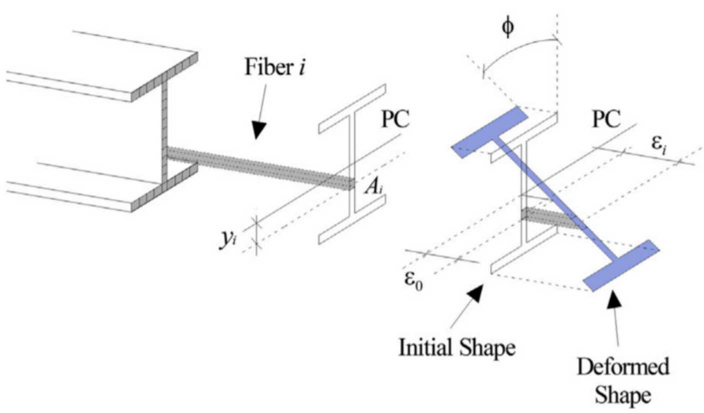

Figure 1. Linear field of deformations

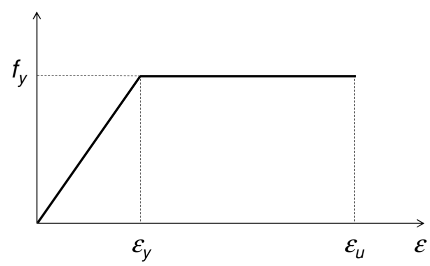

Figure 2. Steel constitutive relationship

During the sectional analysis, the Newton-Raphson iterative method is adopted to obtain the moment-curvature relationship, and as result the element axial stiffness and bending stiffness. So, starting from a given axial effort (N), the bending moment (M) is incrementally amplified until the last resistive moment is reached. The cross-section balance is given by:

$$\mathbf{F}(\mathbf{X}) = \mathbf{f}_{ext} - \mathbf{f}_{int} \cong \mathbf{0} \rightarrow \mathbf{F}(\mathbf{X}) = \begin{bmatrix} N_{ext} \\ M_{ext} \end{bmatrix} - \begin{bmatrix} N_{int} \\ M_{int} \end{bmatrix} \cong \mathbf{0} \tag{7}$$

in which $\mathbf{X} = [\varepsilon_0 \quad \Phi]^T$ is the position vector where ε_0 is the axial strain in the plastic centroid (PC) of each finite element and Φ is the respective curvature. The vector of external forces $\mathbf{f}_{ext}$ is composed, respectively, of the external axial efforts (N_{ext}) and bending moment (M_{ext}). The components of the internal force vector $\mathbf{f}_{int}$ is composed, respectively, by the internal axial efforts (N_{int}) and bending moment (M_{int}), which are obtained from the cross-section deformed configuration.

According to Chiorean [14], it is efficient to start the process by doing the position vector null ($\mathbf{X} = \mathbf{0}$), but convergence is only reached in the first iteration if the external efforts are null. Thus, for the next iteration (k+1) the strain vector is given by:

$$\mathbf{X}^{k+1} = \mathbf{X}^k + \left[\mathbf{F}'\left(\mathbf{X}^k\right)\right]^{-1} \mathbf{F}\left(\mathbf{X}^k\right) \tag{8}$$

where **F** is the Jacobian matrix of the nonlinear sectional analysis, expressed by:

$$\mathbf{F}' = \left(-\frac{\partial \mathbf{F}}{\partial \mathbf{X}}\right) = \begin{bmatrix} f_{11} = \dfrac{\partial N_{int}}{\partial \varepsilon_0} & f_{12} = \dfrac{\partial N_{int}}{\partial \Phi} \\ f_{21} = \dfrac{\partial M_{int}}{\partial \varepsilon_0} & f_{22} = \dfrac{\partial M_{int}}{\partial \Phi} \end{bmatrix} \tag{9}$$

At the end of iterative loop, the cross-section shows the deformed configuration related to the equilibrium condition, and at this point, the generalized stiffness parameters are calculated. The fibers axial deformation is used to calculate the Jacobian matrix (**F**) in terms of the axial rigidity (EA_T) and flexural rigidity (EI_T), as follows ([14], [12]):

$$EA_t = f_{11} - \frac{f_{12} f_{21}}{f_{22}} \tag{10}$$

$$EI_t = f_{22} - \frac{f_{12} f_{21}}{f_{11}} \tag{11}$$

where the terms fij are the component of the constitutive matrix of the cross-section, Eq. 9.

When, for a given axial force, the maximum moment of the moment-curvature relationship is reached, there is the total plastification of the section. This pair of efforts is a point of the *Normal Force x Bending Moment* capacity curve.

3 Unilateral Contact Problem Treatment

The unilateral contact problem is a non-linear problem since the region of contact between the bodies is unknown a priori. This nonlinearity comes from the imposed boundary conditions, which are given in the form of inequalities restrictions, corresponding to the kinematic condition of non-penetration between the bodies and the static condition of the contact pressure being compressive. Such restrictions can be expressed in the Kuhn-Tucker complementary form [15].

The unilateral contact problem is formulated here as an optimization problem with constraints imposed by other bodies or geological medium, where the optimal conditions of the problem are obtained by solving the system equilibrium equations attending to the constraints and the complementary contact condition. For the imposed unilateral restriction treatment, the contact problem is transformed into an unrestricted minimization problem, through the Penalty Method [15].

After the system discretization, the equivalent minimization problem can be rewritten as follows:

$$\text{Min } \Pi\left(\mathbf{U},\mathbf{U}_b\right) \tag{12}$$

$$\text{Subject to } -\varphi\left(\mathbf{U},\mathbf{U}_b\right) \leq 0 \quad \text{on } S_c \tag{13}$$

where **U** is the structure nodal displacements vector; $\mathbf{U}_b$ is the nodal displacements vector of the base; and φ is the non-penetration condition between the bodies in the contact region S_c.

With the Penalty Method, the contact region S_c is initially approximated and, subsequently, an iterative process as the Newton-Raphson method is used to correct it. With this, it is possible to quantify approximately the participation of the elastic medium in obtaining other unknowns of the problem. At each iteration, through a new evaluation of S_c, the participation of the elastic medium is corrected and a new base stiffness matrix ($\mathbf{K}_b$) is obtained. This iterative process is completed, indicating that the solution of the unilateral contact problem has been reached, when a certain convergence criterion is satisfied.

4 Numerical Application

This item presents the unilateral contact problem solution of a circular ring which is pressed against a rigid base by a point load applied at its top (Fig. 3). This problem was initially proposed by Simo et al. [3] who focused on the numerical solution of structural problems with large deflection, subject to contact constraints and unilateral boundary conditions, exhibiting inelastic behavior. Their inelastic response was obtained using an elasto-viscoplastic constitutive model, formulated directly in terms of resultant stresses. For the modeling of contact and unilateral restrictions, they employed a penalty procedure, as this work.

The following data (dimensionally compatible units) were considered: radius (R) equal to 100; steel elasticity modulus (E_s) equal to 10^9; material yield stress (f_y) equal to $365.18.10^2$; momentum of inertia (I) equal to 10^{-6}; and cross-section area (A) equal to 0.1. For symmetry reasons, only half of the ring was discretized, and using 50 finite beam-column elements. The rectangular cross section was divided into 10 fibers. The rigid flat base was described by discrete springs with a high stiffness value (k_y = 1200).

This system was studied considering the linear elastic and inelastic material behavior, and good agreement was obtained in terms of equilibrium path (load-displacement curves, Fig. 4) when compared to the results presented by Simo et al. [3]. It is noteworthy that in the equilibrium path, considering the inelastic material behavior, there is a slight difference between the results presented by [3] and the present work. This slight difference can be explained by the different treatment used in the inelastic material modeling.

The deformed configurations for different load levels are shown in Fig. 5. As it can be seen, the elastic and inelastic behavior of the circular ring obtained by this work (carried out in CS-ASA system) is in a good agreement with the ones provided by [3].

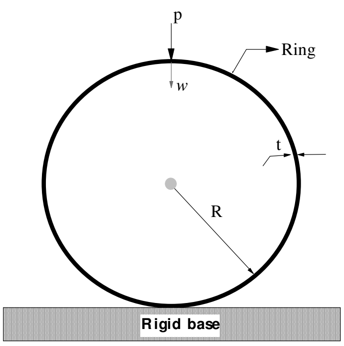

Figure 3. Circular ring and rigid base

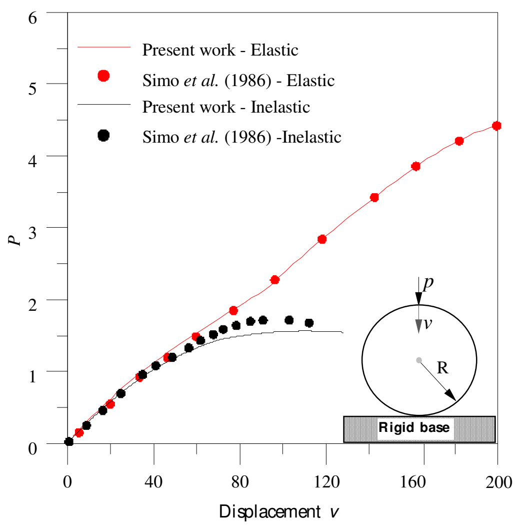

Figure 4. Elastic and inelastic load-displacement curves

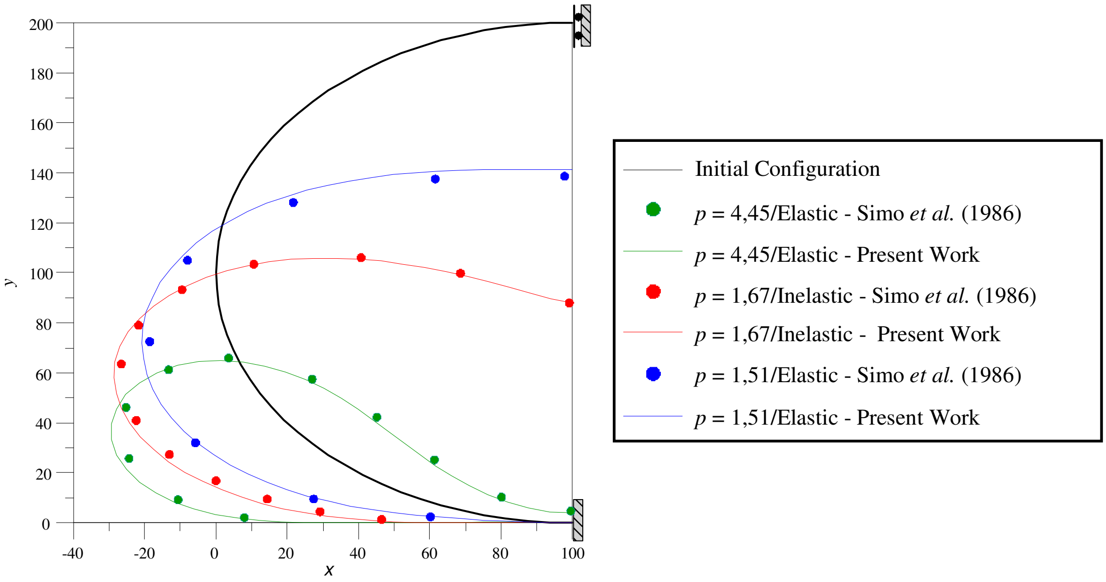

Figure 5. Circular ring deformed configurations

5 Final Comments

In this work, a generalized numerical methodology for elastic and inelastic second order analysis of systems involving the structure-geological medium (soil/rock) interaction was presented. Part of the numerical strategy proposed is based on the SCM-RPHM coupling, and the geometric nonlinearity was considered through a co-rotational formulation. To the unilateral contact problem, where the contact region is unknown a priori, it was proposed to transform the constraint problem into an unconstraint minimization problem through the penalty method. The mathematical model used to represent the elastic medium or foundation was the discrete spring model.

In the example presented, the proposed methodology efficiently represented the second-order elastic and inelastic behavior of a circular ring-rigid base interaction problem, since it was observed a good approximation with the literature results.

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References

[1] J. F. A. C. Abreu. Estudo computacional via ANSYS de vigas, arcos e anéis com restrições de contato impostas por bases elásticas. Dissertação de Mestrado. Programa de Pós-Graduação em Engenharia Civil, Universidade Federal de Ouro Preto, Ouro Preto, MG, Brasil, 2016.
[2] S. M. Sun and M. C. Natori. Numerical solution of large deformation problems involving stability and unilateral constraints. Computers & Structures, Vol. 58(6), pp. 1245-1260, 1996.
[3] J. C. Simo, P. Wriggers, K. H. Schweizerhof, R. L. Taylor. Finite deformation post-buckling analysis involving inelasticity and contact constraints. International Journal for Numerical Methods in Engineering, Vol. 23, pp.779-800,1986.
[4] A. R. D. Silva. Sistema computacional para análise avançada estática e dinâmica de estruturas metálicas. Tese de Doutorado. Programa de Pós-Graduação em Engenharia Civil, Universidade Federal de Ouro Preto, Ouro Preto, MG, Brasil, 2009.
[5] Í. J. M. Lemes. Análise Avançada via MRPR de Estruturas Mistas de Aço e Concreto. Dissertação de Mestrado, Programa de Pós-Graduação em Engenharia Civil, Universidade Federal de Ouro Preto, Ouro Preto, MG, Brasil, 2015.

[6] K. M. Hsiao H. J., Horng, Y. R. Chen. A co-rotational procedure that handles large rotations of spatial beam structures. Computers & Structures, vol. 27 n° 6, pp. 769-781, 1987.
[7] J. M. Battini. Co-rotational beam elements in instability problems. Ph.D Thesis, Royal Institute of Technology – Department of Mechanics, Stockholm / Sweden, 2002.
[8] M. V. B. Santana. Desenvolvimento de sistema computacional via MATLAB/GUI para análise geometricamente não linear de estruturas. Dissertação de Mestrado, Programa de Pós-Graduação em Engenharia Civil, Deciv/EM/UFOP, Ouro Preto/MG, Brasil, 2015.
[9] A. Alhasawi, P. Heng, M. Hjiaj, S. Guezoulli e J. M. Battini. Co-rotational planar beam element with generalized elasto-plastic hinges. Engineering Structures, v. 151, p. 188-205, 2017.
[10] S. Chang, J. M. Battini, M. Hjiaj. Energy-momentum method for co-rotational plane beams: A comparative study of shear exible formulations. Finite Elements in Analysis and Design, v. 134, p. 41-54, 2017.
[11] J. L. Silva. Formulações corotacionais 2D para análise geometricamente não linear de estruturas reticuladas. Dissertação de Mestrado, Programa de Pós-Graduação em Engenharia Civil, PROPEC/EM/UFOP, Ouro Preto/MG, Brasil, 2016.
[12] Í. J. M. Lemes, R. A. M. Silveira, A. R. D. Silva e P. A. S. Rocha. Nonlinear analysis of two-dimensional steel, reinforced concrete and composite steel-concrete structures via coupling SCM/RPHM. Engineering Structures, vol. 147, pp. 12-26, 2017.
[13] EC3, 2005. Cen eurocode 3 design of steel structures. vol. part 1. European Committee for Standardization.
[14] C. G. Chiorean. A Computer Method for Nonlinear Inelastic Analysis of 3D Composite Steel-Concrete Frame Structures. Engineering Structures, vol. 57, pp. 125-152, 2013.
[15] D.G. Luenberger. Introduction to Linear and Nonlinear Programming, Addison-Wesley Publishing Company, Inc. 1973.