Comparison of two open-source codes of the boundary element method for acoustic problems

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Abstract. A comparison between two competing free and open-source codes of the boundary element method (BEM) for internal and external acoustic problems is carried out in this work. The BEM programs used were Bempp-cl and BB. While Bempp-cl is an well established Python program developed at University College London and the University of Cambridge, BB is a more recent code, developed at the University of Brasília, written in Julia language. For the mesh generation, an also open-source program, Gmsh, was used. Initially, internal problems on a cubic geometry were solved, both in terms of response analysis in the resonant frequencies as well as frequency response analysis for the three first modes of the problem. For the same problem, a processing time analysis was also conducted, in order to assess the efficiency of the mentioned programs. The solutions obtained by the two programs were compared with each other as well as with analytical solutions, when available. The results for the cubic geometry shown that both programs were able to successfully predict the behavior for the modes and for the frequency analysis, with a satisfactory time consuming. Therefore, the use of BEM and, specially, of both programs studied in this article are very powerful tools, freely available, to solve large and complex engineering problems, with great efficiency and accuracy.

Keywords: Boundary element method, Acoustics

1 Introduction

Acoustics are present daily in the lives of most people. What we humans perceive as sound is nothing more than oscillations in the field of pressure in the air surrounding our ears. Such phenomenon, so present in the environment, could not fail to be an object of study in engineering, whether to ensure acoustic comfort for people or in high-tech applications. Examples are found in the study of automobiles, aircraft, concert halls, submarines and rockets. There are also less usual applications, such as the acoustic levitator, developed by Marzo et al. [1].

In the acoustic analysis of engineering problems, numerical methods are often used. As engineering deals with real problems, often purely analytical solutions are impossible to be obtained or are very imprecise due to formulation simplifications, leading to the need of using numerical methods. According to Brancati [2], in the field of acoustics, the BEM stands out as being the most efficient, specially for exterior problems, and is used by the main commercial packages present in the industry.

In this paper, a comparison between two open-source BEM programs, BB[1] and Bempp[2], is conducted. Both programs are compared in terms of precision (using analytical solutions) and time consumption.

BB is an open-source implementation of the BEM by collocation, written in the Julia language, which allows acceleration by hierarchical matrices and the use of parallel processing, developed by the group of boundary elements of UnB (UnBEM). The program is capable of solving the Laplace and Helmholtz equations for two and three dimensions. To allow an easily replicable work methodology, which uses only open-source programs, BB allows integration with Gmsh, FreeCad and ParaView.

Bempp is another open-source BEM program, written entirely in Python, developed at University College London and the University of Cambridge. It uses Just in Time (JIT) compilers to make the Python language

[1]Available at: `https://github.com/alvarocafe/BB`.
[2]Available at: `http://bempp.com`.

faster. PyOpenCL is used by default, allowing code parallelization using the CPU or GPU. Alternatively, it is also possible to use numba. The software uses the indirect formulation of the BEM. In addition, the potential operators discretization is done by the symmetrical Galerkin method. It is also possible to use the FMM through external routines interface. The program also allows integration with the finite element method (FEM), using an interface to the FEM program FEniCS.

2 Boundary Element Method

The BEM is a numerical method used to solve partial differential equations (PDE) using bounday integral equations (BIE). The formulation used in this work is the direct boundary element method for the Helmholtz equation, described in more details in Brebbia and Dominguez [3], Kirkup [4] and Liu [5].

The propagation of acoustic waves through a fluid medium Ω is described by the wave equation. Assuming that the motion is time-harmonic, the BEM can be used to solve the non-homogeneous Helmholtz equation:

$$\nabla^2 u + k^2 u = f, \qquad \forall \mathbf{x} \in \Omega, \tag{1}$$

where u is a reduced velocity potential, $k = \omega/c$ is the wave number, c is the speed of sound, ω is the angular frequency and **x** is the source point.

For an harmonic wave, u is related to the pressure, p, by:

$$p = ue^{-j\omega t}, \tag{2}$$

where t is the time and $j = \sqrt{-1}$. The physical pressure of the problem is obtained by considering only the real part of the equation.

The two main boundary conditions are Dirichlet (known pressure), $u = \overline{u}, \forall \mathbf{x} \in \Gamma$, and Neumann (known speed), $\partial u/\partial n = \overline{u}_n = j\omega\rho v_n, \forall \mathbf{x} \in \Gamma$, where ρ is the density of the fluid medium and v_n the known speed.

The boundary integral equation can be found using Green's Second Identity:

$$\int_\Omega (\phi\nabla^2\phi^* - \phi^*\nabla^2\phi)d\Omega = \int_\Gamma \left(\phi\frac{\partial\phi^*}{\partial n} - \phi^*\frac{\partial\phi}{\partial n}\right)ds, \tag{3}$$

where ϕ and ϕ^* are generic functions. Considering $\phi = G$ and $\phi^* = u$, it is obtained:

$$\int_\Omega [G\nabla^2 u - u\nabla^2 G]d\Omega = \int_\Gamma \left[G\frac{\partial u}{\partial n} - u\frac{\partial G}{\partial n}\right]d\Gamma, \tag{4}$$

where G is the fundamental solution. To obtain the fundamental solution of the Helmholtz equation, it is necessary to apply a unit concentrated source (pulsing point), represented by the Dirac delta δ, in the source point **x**, so that the mathematical representation of the answer in another point **y**, called field point, is the Green's fundamental solution. Therefore, the fundamental solution must satisfy the fallowing equation:

$$\nabla^2 G + k^2 G + \delta(\mathbf{x}, \mathbf{y}) = 0. \tag{5}$$

Solving eq. (5), the fundamental solution for three-dimensional acoustic problems is obtained:

$$G(\mathbf{x}, \mathbf{y}, \omega) = \frac{1}{4\pi r}e^{jkr}, \tag{6}$$

where r is the distance between the source and the field points. Now, applying eq. (1) and eq. (5) on eq. (4), it is possible to obtain:

$$u(\mathbf{x}) = \int_\Gamma \left[G\frac{\partial u}{\partial n} - \frac{\partial G}{\partial n}u \right] d\Gamma + \int_\Omega [Gf] d\Omega \qquad \forall \mathbf{x} \in \Omega. \tag{7}$$

Taking **x** to the boundary Γ, and considering that, for scattering problems, $u = u^S + u^I$, where u^S is scattered wave and u^I is the incident wave, the conventional boundary integral equation (CBIE) is:

$$c(\mathbf{x})u(\mathbf{x}) = \int_\Gamma \left[G\frac{\partial u(\mathbf{y})}{\partial n} - \frac{\partial G}{\partial n}u(\mathbf{y}) \right] d\Gamma(\mathbf{y}) + u^I(\mathbf{x}) + \int_\Omega f d\Omega. \tag{8}$$

where $c(\mathbf{x}) = 1/2$ for smooth boundary on Γ.

The CBIE can be also presented by using boundary and potential operators. This approach is referred by Liu [5] as being the indirect BEM formulation. The operators used in this formulation are:

- Single-layer

$$[V_k\phi](\mathbf{x}) = \int_\Gamma G_k(\mathbf{x},\mathbf{y})\sigma(\mathbf{y})d\Gamma(\mathbf{y}), \qquad \mathbf{x} \in \Gamma; \tag{9}$$

- Double-layer

$$[K_k\phi](\mathbf{x}) = \int_\Gamma \frac{\partial G_k(\mathbf{x},\mathbf{y})}{\partial n(\mathbf{y})}\sigma(\mathbf{y})d\Gamma(\mathbf{y}), \qquad \mathbf{x} \in \Gamma; \tag{10}$$

- Adjoint double-layer

$$[K'_k\phi](\mathbf{x}) = \int_\Gamma \frac{\partial G_k(\mathbf{x},\mathbf{y})}{\partial n(\mathbf{x})}\sigma(\mathbf{y})d\Gamma(\mathbf{y}), \qquad \mathbf{x} \in \Gamma; \tag{11}$$

- Hypersingular

$$[W_k\phi](\mathbf{x}) = -\int_\Gamma \frac{\partial^2 G_k(\mathbf{x},\mathbf{y})}{\partial n(\mathbf{x})\partial n(\mathbf{y})}\sigma(\mathbf{y})d\Gamma(\mathbf{y}), \qquad \mathbf{x} \in \Gamma. \tag{12}$$

Now, the CBIE, eq. (8), can be written in terms of the potential operators:

$$u_\Omega = \nu_k u_n - \kappa_k u_\Gamma + u^I. \tag{13}$$

By taking the trace on both sides of eq. (13), it can be obtained, for Dirichlet and Neumann problems:

$$V\frac{\partial u}{\partial n} = (\pm\frac{1}{2}I + K)\overline{u}, \tag{14}$$

$$Wu = (\pm\frac{1}{2}I - K')\overline{u}_n. \tag{15}$$

where the identity sign is positive for interior problems and negative for exterior problems.

According to Smigaj et al. [6], problems in which a combination of Dirichlet and Neumann boundary conditions are imposed simultaneously in different parts of the boundary can be solved with:

$$V\frac{\partial u}{\partial n} - Ku = (\frac{1}{2}I + K)\overline{u} - V\overline{u}_n, \qquad \mathbf{x} \in \Gamma_1; \tag{16}$$

$$Wu + K'\frac{\partial u}{\partial n} = (\frac{1}{2}I - K')\overline{u}_n - W\overline{u}, \qquad \mathbf{x} \in \Gamma_2. \tag{17}$$

3 Results

3.1 Internal acoustic response for the modes in a closed cubic cavity

The first comparisons were conducted on the internal problem of a cube, with size equal to 10. The response obtained by BB and Bempp were compared with analytical solutions obtained using the transfer matrix method. More details on the method and analytical solutions can be found at Gilbert [7], Morais [8] and Ferreira [9].

The mesh was generated using the open-source software Gmsh[3]. It has 1476 triangular elements and 740 nodes. The solution for the internal points were calculated on 40 points, located in a line in the center of the cube.

Three problems with different boundary conditions were solved: closed-closed, with Neumann closed conditions imposed to every surface, open-closed, with Neumann closed conditions imposed to 5 surfaces and 1 face with Dirichlet open condition, and open-open, with with Neumann closed conditions imposed to 4 surfaces and 2 faces with Dirichlet open conditions. The analytical solution is known for all those problems. The problems were solved for the three fist modes. Figure 1 shows the comparison between BB, Bempp and analytical solutions for the second mode of the closed-closed case.

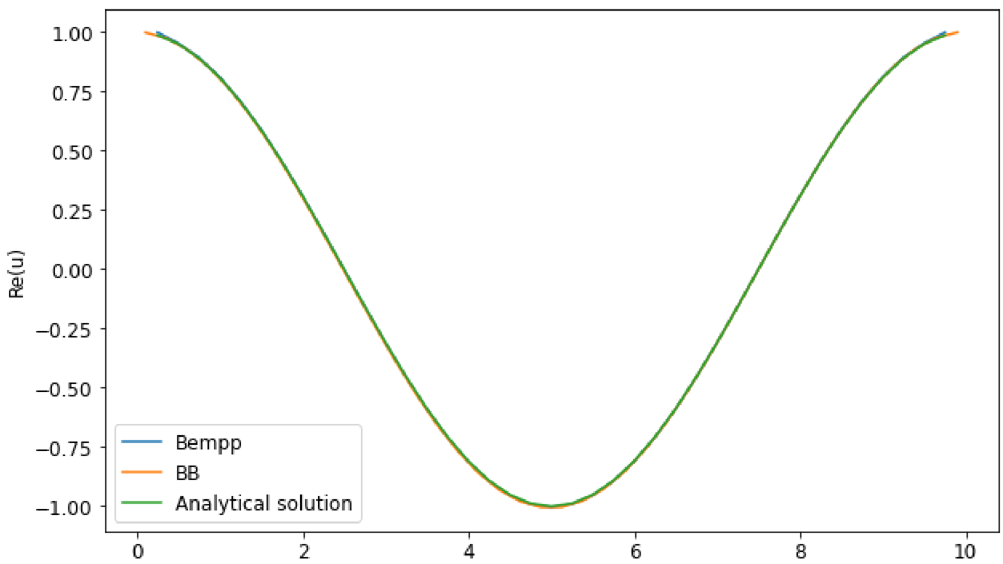

Figure 1. Comparison of the response inside the cube for the second mode with closed-closed boundary conditions

The numerical solution error in relation to the analytical was evaluated using the root mean square value (RMS) of the normalized error. The error obtained for the three problems can be seen on Table 1.

[3]Available at: `https://gmsh.info/`

Table 1. RMS error for the first three nodes of the cubic geometry problems

		n=1	n=2	n=3
BB	closed-	0,0005017	0,0024834	0,0102628
Bempp	closed	0,0017108	0,0018608	0,0018774
BB	open-	0,0010254	0,0012278	0,0074104
Bempp	closed	0,0052056	0,0007647	0,0057125
BB	open-	0,0033344	0,0028213	0,0265669
Bempp	open	0,0003359	0,0004193	0,1237878

3.2 Frequency response analysis in a closed cubic cavity

Besides the analysis of the modes inside the cube, a frequency response analysis was also conducted, in order to quantify the precision of both software in identifying the resonant frequencies for this geometry. The response was obtained for 400 frequency points, in a range of 45 [rad/s] to 330 [rad/s]. Also, the response was analysed on 5 internal points of the cube, $z = 0, 10$, $z = 2, 55$, $z = 5, 00$, $z = 7, 45$, e $z = 9, 90$.

Figure 2 shows the comparison between the frequency response obtained by BB and Bempp, using the open-closed boundary conditions. The vertical lines are the expected resonance frequencies found trough the analytical analysis. Only the real part of the response was considered. The location inside the cube is $z = 5, 00$.

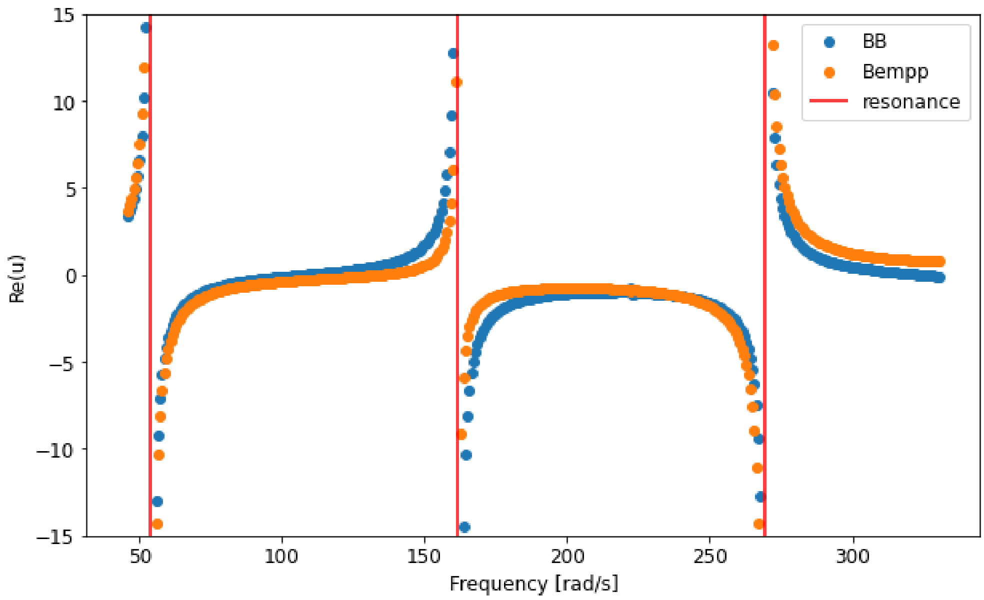

Figure 2. Real part of the frequency response for the open-closed conditions

The results obtained confirmed the capability of both programs in identifying the resonance frequency with good precision. Some differences in the obtained response were spotted in points between the resonance frequencies. Comparisons as the ones showed in Figure 2 were conducted for all three boundary conditions, closed-closed, open-closed and open-open. The method used to obtain the numerical resonance frequency was to identify the frequency whose response had the greatest magnitude of absolute value. The percentage errors were obtained using $error = (\omega_t - \omega_a)/\omega_a$, where ω_t is the frequency obtained numerically and ω_a is the analytical frequency, and are available at Table 2.

Table 2. Resonance frequency errors obtained by BB and Bempp

		n=1	n=2	n=3
BB	closed-	0,137%	0,087%	0,071%
Bempp	closed	0,137%	0,087%	0,071%
BB	open-	0,425%	0,325%	0,21%
Bempp	closed	0,425%	0,117%	0,055%
BB	open-	0,083%	0,308%	0,144%
Bempp	open	0,137%	0,087%	0,15%

3.3 Processing time analysis in a closed cubic cavity

After assessing the precision of both programs, a study was conducted to compare the time consumed by them to solve problems. The problem with closed-closed boundary conditions was solved using 5 different meshes, with 410, 1476, 6226, 10378 and 13526 elements.

One thing that requires attention is the fact that the comparison of both programs is not straightforward. As described in Section 1, both programs use different methodologies of the BEM, with different inputs. In view of all these differences, the solution to proceed with the comparison of processing times was to keep the settings that were used previously for the closed-closed case. This choice safeguards the particularities of each of the programs, using the formulation that is best suited to each one of them. As the implementation is the same used previously, it is coherent to state that the comparison of processing time was performed for a certain level of precision of the results, previously presented in the form of an error in relation to the analytical solution.

The processing time study was conducted in 3 different computers, in order to reduce hardware related errors. Machine 1 is Google Colaboratory, a hosted Jupyter notebook service that provides free access to computing resources including GPUs. The processors are 2 Intel(R) Xeon(R) with 2,3 GHz clock speed, with 2 siblings each. It has a RAM memory of 12 GB. Machine 2 is a PC with AMD FX(tm)-8300 motherboard, a 8 core processor and 16 GB RAM memory. Machine 3 is a notebook with Intel(R) Core(TM) i7-7500U CPU@2,70 GHz and 8 GB RAM memory. The results of this comparison, for Machine 2, can be seen in Figure 3.

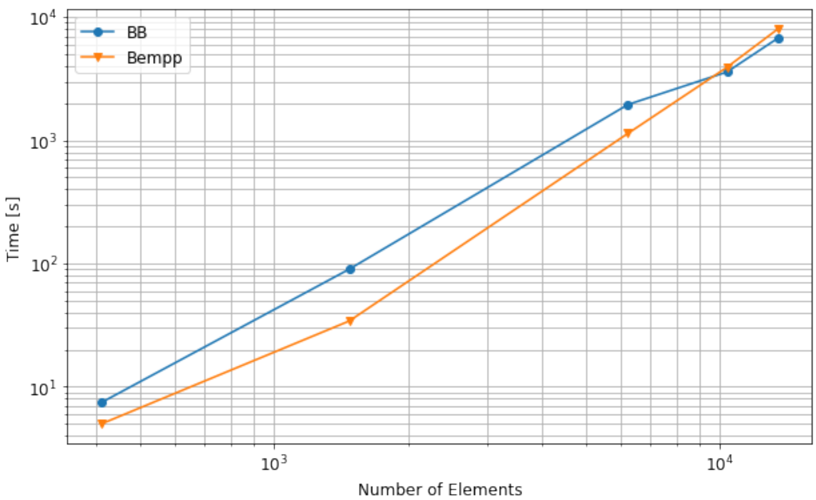

Figure 3. Processing time results for Machine 2

A summary of the results for the processing time study, obtained by the 3 machines, can be seen in Table 3. The results on Table 3 show that, in general, Bempp had an inferior processing time. However, these results must be taken with care. The reason is that, for the solving of the numerical problems using BB, 12 Gauss point were

Table 3. Time consumed for BB and Bempp

Num. of elements	Program	Machine 1	Machine 2	Machine 3
410	Bempp	7,10	4,98	8,73
	BB	9,92	7,43	5,79
1476	Bempp	24,24	34,16	8,40
	BB	83,39	90,02	63,91
6226	Bempp	795,54	1139,47	380,07
	BB	1963,36	1953,34	1076,65
10378	Bempp	2652,93	3919,44	1366,21
	BB	5553,90	3590,34	2848,80
13526	Bempp	5222,41	8065,73	2927,81
	BB	8908,38	6788,67	-

used for the numerical integrations. On the other hand, Bempp does not offer the possibility to define the number of Gauss Point, nether informs how many are used. If less points of Gauss were used for BB, the time consumed would be considerably reduced, while precision would also decrease. In a fast analysis, using 6 Gauss points for BB, with Machine 2, reduced the time consumed by 33,2%, while the error increased by 39,3%.

4 Conclusions

The results in this paper showed that both open-source BEM programs studied, BB and Bempp, were able to obtain solutions with good precision and time consumption. In average, BB predicted the behavior of modes with a RMS error of 0.0061816 and resonant frequencies with an error of 0.199%, while Bempp presented an RMS error of 0.0157416 for modes and 0.141% for the expected resonant frequencies.

Besides the precision analysis, a comparison was also conducted to assess the time consumed by both programs for meshes with different levels of refinement and for different computers. The results showed that, in general, Bempp needed less processing time than BB, although this result should be further explored by changing the number of Gauss points used by BB.

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References

[1] A. Marzo, A. Barnes, and B. W. Drinkwater. Tinylev: A multi-emitter single-axis acoustic levitator. *Review of Scientific Instruments*, vol. 88, 2017.

[2] A. Brancati. *Boundary Element Method for Fast Solution of Acoustic Problems: Active and Passive Noise Control*. PhD thesis, Imperial College London, London, UK, 2010.

[3] C. A. Brebbia and J. Dominguez. *Boundary Elements An Introductory Course*. WIT Press/Computational Mechanics Publications, Southampton, 2 edition, 1992.

[4] S. Kirkup. *The Bounday Element Method in Acoustics*. Integrated Sound Software, 2007.

[5] Y. Liu. *Fast Multipole Boundary Element Method*. Cambridge University Press, New York, 2009.

[6] W. Smigaj, T. Betcke, S. Arridge, J. Phillips, and M. Schweiger. Solving boundary integral problems with bem++. *ACM Trans. Math. Softw.*, vol. 41, n. 2, pp. 40, 2015.

[7] R. J. Gilbert. *Vibrations des structures - Interactions avec les fluides - Sources d'excitation aleatoires*. Ed. Eyrolles, Paris, França, 1988.

[8] M. V. G. Morais. Modelagens numéricas pelo método dos elementos finitos em problemas de iteração fluido-estrutura. Master's thesis, Universidade de Brasília, Brasília, Brasil, 2000.

[9] A. C. Ferreira. Análise harmônica de cavidades acústicas pelo método dos elementos de contorno direto. Master's thesis, Universidade de Brasília, Brasília, Brasil, 2004.