Direct Transient Response of Coupled Structures-Soil Systems Modelled by Matrix Methods and Green's Functions Approach

Amauri C. Ferraz[1], Euclides Mesquita[1], Luis F. V. Lima[1], Josué Labaki[2]

[1]*Dept. of Computational Mechanics – FEM, University of Campinas*
200 Mendeleyev St, and CCES – Center for Computational Engineering and Science (http://www.escience.org.br), 13083-860, São Paulo, Brazil
amauriferrazz@gmail.com, euclides@fem.unicamp.br, lfvl_@hotmail.com
[2]*Dept. of Integrated Systems – FEM, University of Campinas*
200 Mendeleyev St, 13083-860, São Paulo, Brazil
labaki@fem.unicamp.br

Abstract. Even today it is still a big computational challenge to obtain the dynamic transient response of complex systems constituted of a structure interacting with a foundation supported by the soil. The most common numerical solution schemes for the two sub-systems, the structure and the soil-foundation support are distinct. Usually, the foundation dynamics is modelled by the Finite Element Method (FEM) or related matrix methods. On the other hand, the soil dynamics has been best solved using either the direct or indirect version of the Boundary Element Method (BEM). A coupling of both numerical schemes is the most versatile scheme to solve Dynamic Soil-Structure Interaction (DSSI) problems. The coupling of the methods to obtain transient solutions is still computationally very demanding. I this work we propose an alternative approach. For subsystem 1, the structure, modal quantities are obtained in coordinate relative to the structure base displacement. For sub-system 2, the soil-foundation scheme, the BEM is applied to obtain frequency domain solution of foundations resting on soils or on piles. A coupling of the structure with foundations in the frequency domain delivers modified Frequency Response Functions (FRFs). New or modified modal quantities are now extracted from the FRFs that already include the influence of the soil-foundation dynamics on the structure. These extracted modal quantities may be used to integrate directly the equations of motion in the time domain. All degrees of freedom of the soil-foundation system are incorporated in the modified modal quantities, which retain the number of degrees of freedom of the original structure. The numerical examples will address the transient response of structures resting foundation scheme composed of a homogeneous half-space and a pile embedded in the half-space.

Keywords: Dynamic Soil-Structure Interaction, Transient Response, Modal Analysis.

1 Introduction

This article presents a methodology to obtain transient response of structures interacting with soils or pile foundations embedded in the soil. The structures are considered to be modelled by matrix methods such as the Finite Element Method (FEM). The soil and foundation dynamic responses are synthesized by a version of the Boundary Element Method (BEM) based on a set of specially developed Green's functions. A typical set of problems being addressed in this article is shown in Figure 1. In this figure three systems are depicted. System I represent a classical model of a Mass-Damping-Stiffness (MCK) structure, with a fixed base. System II represents the same structure supported by a homogeneous half-space, whereas System III represents the structure supported on a pile, which, in turn is embedded in soil modelled as a half-space.

There scientific community has dedicated a lot of efforts to develop methodologies to solve transient soil-structure problems as those indicated in Figure 1. Some significant contributions can be found in the articles Coda [1], Soares *et al* [2], Soares and Araújo [3], Soares *et al* [4] and von Estorff [5], among others. One important aspect of these previous works is the high computational cost of the solution procedures. In this article an alternative procedure to obtain the transient response of structures interacting with soil and foundation arrangements is presented, which is based on two key aspects.

First, the response of the structure is developed in terms of modal superposition obtained for displacements relative to the movement of the foundation arrangement. The structural decoupled equations of motion in time domain are, in the sequence, transformed to the frequency domain, as in Wu and Smith [6] and in Louzada [7]. The dynamic response of the soil or the soil-pile system is obtained directly in the frequency domain, according to Labaki *et al* [8] and Barros *et al* [9]. A coupling of the structural equations of motion with the soil-foundation system is performed by classical equilibrium and kinematic compatibility requirements. The frequency domain response of the resulting soil-foundation-structure system may be expressed in terms of Frequency Response Functions (FRFs).

Classically the transient response of the coupled soil-foundation-structure systems may be obtained by applying the inverse Fourier Transform, or its fast algorithm IFFT, to the synthesized FRFs defined by Wu and Smith [6] and adapted by Louzada [7]. In this work, nevertheless, another approach is chosen. In the second step of this procedure, modal quantities are extracted from the modified FRFs, which include the structure response considering the soil influence. A new set of uncoupled equations of motion are created based on this modified modal basis. This new set of equations of motion are directly integrated in the time domain, rendering the transient response of the soil-foundation-structure system as seen in Ferraz [10]. The main steps of the methodology are described in the sequence.

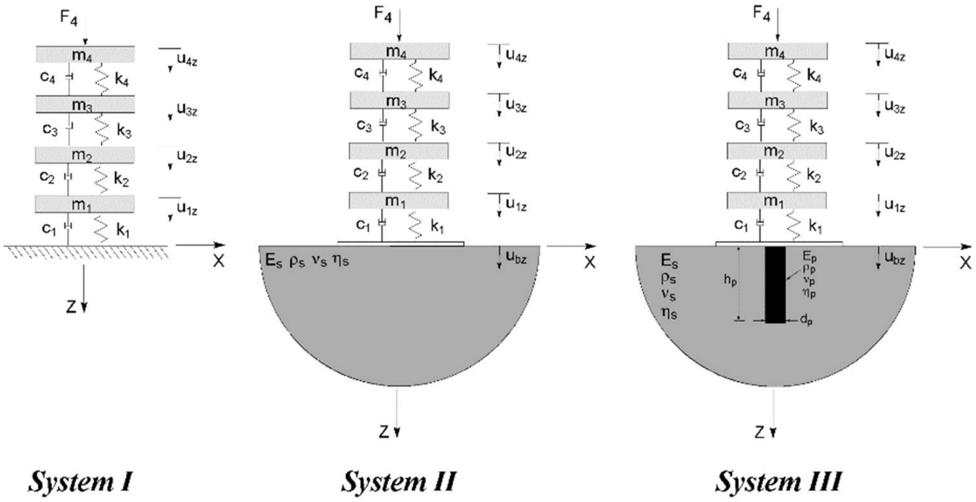

Figure 1. Structure and Foundation Arrangements Systems

2 Structural Response by Modal Superposition

Consider systems II and III in Figure 1. The foundation displacement is given by $u_{bz}(t)$. The total displacement of the i-th structural degree of freedom is given by $u_{iz}(t)$, The structural displacements relative to the foundation movement can be written as $u_{iz\text{-}rel}(t) = u_{iz}(t) - u_{bz}(t)$. According to Louzada [7], the equations of motion for the structure with mass [M], damping [C] and stiffness [K] can be written in terms of the relative displacements $u_{iz\text{-}rel}(t)$ as:

$$[M]\{\ddot{u}_{z-rel}(t)\}+[C]\{\dot{u}_{z-rel}(t)\}+[K]\{u_{z-rel}(t)\}=\{F(t)\}-[M]\{1\}\ddot{u}_{bz}(t).$$

In Equation (1) the vector {F(t)} represent the external forces directly applied to the foundation and $\ddot{u}_{bz}(t)$ represents the acceleration of the foundation basis. Considering a proportional damping matrix [C], the equations of motion (Eq. (1)) may be decoupled by a classical modal analysis procedure and can be transformed to the frequency domain, as defined in Fu and He [11] and applied by Louzada [7]. If the foundation displacement is added, the total displacement of the structure degrees of freedom $\{U_z(\omega)\}$ may be expressed as:

$$\left\{U_z(\omega)\right\} = \left[\Phi\right]\left[H(\omega)\right]\left(\left[\Phi\right]^T\left\{F(\omega)\right\} + \omega^2\left[\Phi\right]^T\left[M\right]\left\{1\right\}U_{bz}(\omega)\right) + \left\{1\right\}U_{bz}(\omega). \tag{2}$$

In Equation (2) the term [H(ω)] represents the transfer function matrix of the structural system without considering the soil-foundation influence and [Φ] is the modal matrix.

3 Soil and Soil-pile Responses

In this article the structure will be supported by a homogeneous half-space, as in Labaki *et al* [8], or by a pile embedded in the half-space, as in Barros *et al* [9]. These arrangements are shown in Figures 2 and 3 respectively. The soil response in the frequency domains can be expressed in terms of a complex dynamic flexibility $S_Z(\omega)$, relating the external excitation force $F_I^2(\omega)$ to the soil displacement $U_{bS}(\omega)$:

$$U_{bz}\left(\omega\right) = S_Z\left(\omega\right) F_I^2\left(\omega\right). \tag{3}$$

Figure 2a shows a half-space, characterized by an elasticity modulus E_S, a Poisson coefficient ν_S, a density ρ_S and an internal damping coefficient η_S. The real and imaginary parts of a typical frequency dependent half-space flexibility as synthesized by Labaki *et al* [8] is given in Figure 2b.

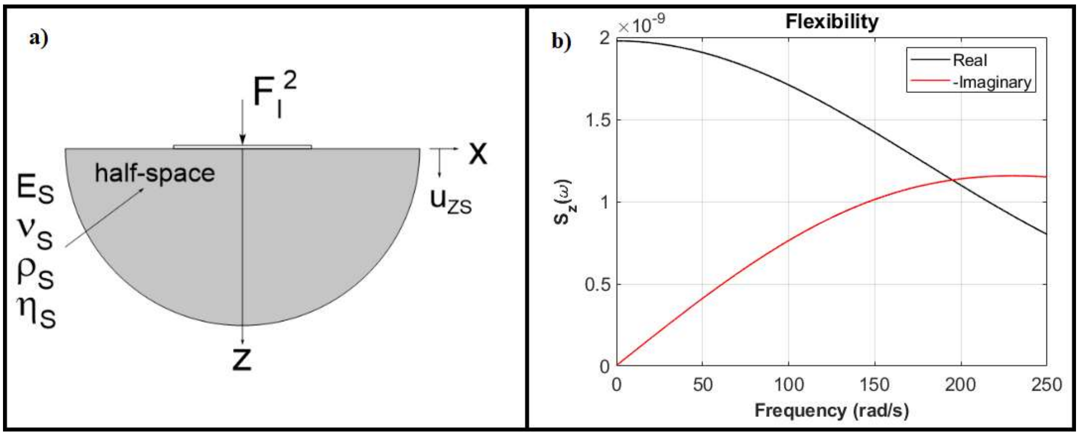

Figure 2. a) Half-space properties and b) dynamic flexibility

Figure 3a shows a pile embedded in a half-space, characterized by an elasticity modulus E_P, a density ρ_P a length h_P and a diameter d_P. The real and imaginary parts of a typical frequency dependent flexibility at the pile head (z=0) as synthesized by Barros *et al* [9] is given um Figure 3b.

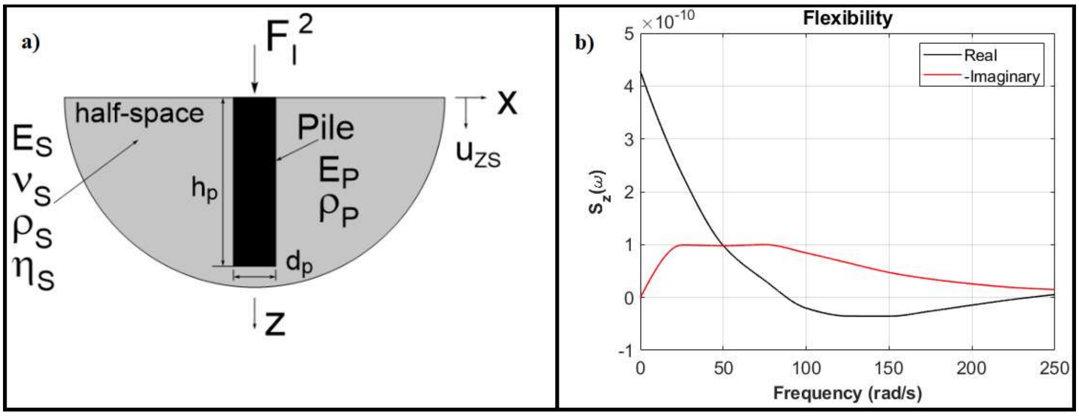

Figure 3. a) Pile embedded in a half-space properties and b) dynamic flexibility

4 Frequency Response of the Coupled Soil-Foundation-Structure System

According to Louzada [7], the structure response given in Equation (2) and the soil response furnished in Equation (3) may be coupled by considering force equilibrium and kinematic compatibility at the soil-structure interface leading to the modified frequency dependent structural response $\{U_Z^{mod}(\omega)\}$ due to an external excitation vector $\{F(\omega)\}$, which already incorporates the influence of the soil-foundation arrangement:

$$\left\{U_Z^{\mathrm{mod}}\left(\omega\right)\right\} = \left(\left[\Phi\right]\left[H(\omega)\right]\left(\left[\Phi\right]^T + \omega^2\left\{\Gamma\right\}\left\{H_S\left(\omega\right)\right\}^T\right) + \left\{1\right\}\left\{H_S\left(\omega\right)\right\}^T\right)\left\{F(\omega)\right\}. \quad (4)$$

In Equation (4) the vector of the generalized modal load coefficients is $\{\Gamma\}=[\Phi]^T[M]\{1\}$ and the vector the structural system transfer function without the soil effect is designated by $\{H_S(\omega)\}$. Equation (4) can be used to obtain modified frequency response functions for the structure, in which the soil or soil-pile responses are already incorporated. In the next step a method to extract modal quantities from the modified FRFs will be discussed.

5 Extraction of Modal Parameters from Modified FRFs

Modal parameter extraction methods are applied to the FRFs of a system to obtain natural frequencies, damping factors and modal shapes. In Ewins [12], the methods are classified according to the number of modes analyzed simultaneously, which can be only one mode (SDOF) or multiple modes (MDOF).

In this study, the Rational Fraction Polynomial method (RFPM) implemented by Richardson and Formenti [13] is applied, which is a MDOF method based on the determination of an approximate equation for the FRF under analysis, according to Equation (5), where c_k and d_k are the indeterminate coefficients, $\phi_{j,k}^+$ and $\theta_{j,k}^+$ are the complex orthogonal polynomials.

$$H(\omega_j) = \frac{\sum_{k=0}^{2N-1} c_k \phi_{j,k}^+}{\sum_{k=0}^{2N} d_k \theta_{j,k}^+} \quad j = 1,...,L. \quad (5)$$

6 Numerical Results

In this section, the influence of the inclusion of a pile in a homogeneous half-space is studied from the systems seen in Figure 1. First, the parameters of mass, stiffness and damping used in the structure with four degrees of freedom are defined, as noted in Table 1. The model of a stiffness-proportional damping was used as a reference to perform the following analyses. It should be noted that the Fourier Transform method and linear convolution defined in Cheng [14] were used to determine system I responses.

Table 1. Input data

Mass	Stiffness	Damping
$m_1 = 17.10 \times 10^4$ kg	$k_1 = 17.27 \times 10^8$ N/m	$c_1 = 5.31 \times 10^5$ kg/s
$m_2 = 14.40 \times 10^4$ kg	$k_2 = 14.54 \times 10^8$ N/m	$c_2 = 4.47 \times 10^5$ kg/s
$m_3 = 11.70 \times 10^4$ kg	$k_3 = 11.82 \times 10^8$ N/m	$c_3 = 3.63 \times 10^5$ kg/s
$m_4 = 9.00 \times 10^4$ kg	$k_4 = 9.09 \times 10^8$ N/m	$c_4 = 2.80 \times 10^5$ kg/s

Then, from the properties defined in Table 2 and the studies of Labaki *et al* [8] and Barros *et al* [9], it is possible to obtain the flexibilities for the homogeneous half-space and for the pile embedded in the half-space, seen in Figures 2 and Figure 3, respectively.

Table 2. Soil properties

Half-space properties					
$\rho_s = 2700$ kg/m^3	$E_s = 234$ MPa	$G_s = 90$ MPa	$\upsilon_s = 0.3$	$\eta_s = 0.01$	$c_s = 183$ m/s
Pile properties					
$\rho_p = 3000$ kg/m^3	$E_p = 2340$ MPa	$h_p = 15$ m	$d_p = 2$ m		

After defining the flexibilities, the modified FRFs for system II and III can be determined. Figure 4 compares the FRFs obtained for the three systems, in which it is noted that the inclusion of the pile results in smoother resonances. Furthermore, the natural frequencies for system III are quite similar to those for system I.

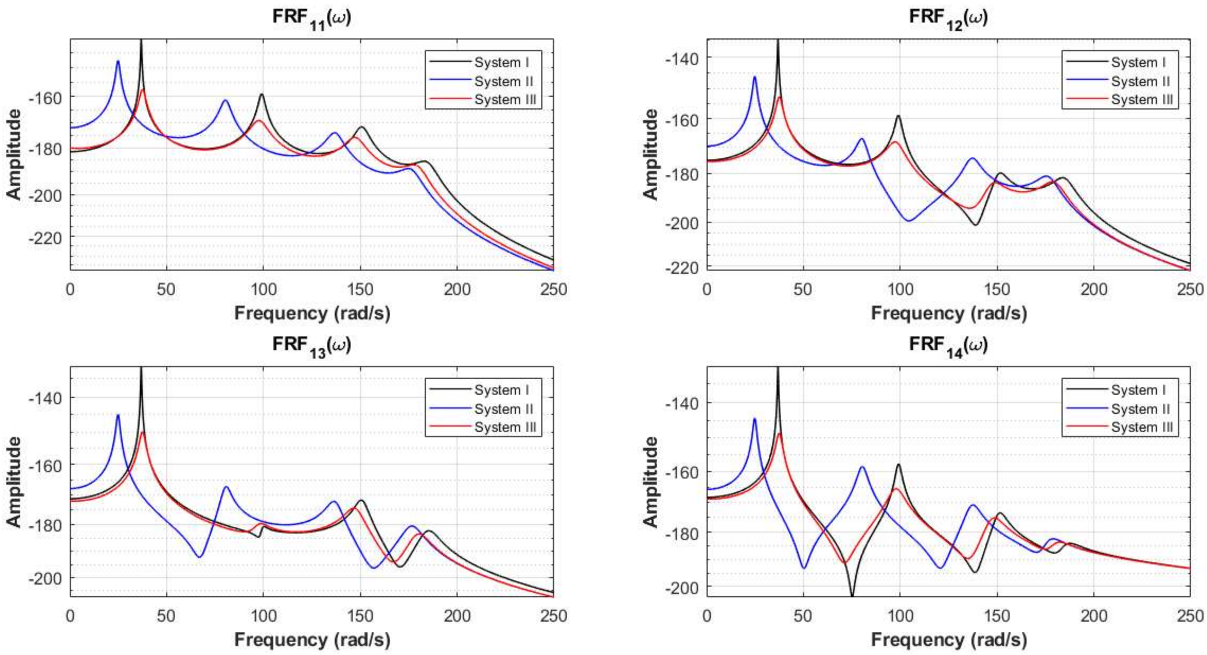

Figure 4. Comparison between the FRFs of the analyzed systems

From the application of the RFPM in the FRFs obtained above, it is possible to extract the modal quantities with influence from the soil in systems II and III. Table 3 compares the natural frequencies and damping factors extracted for the analyzed systems, which confirms the similarity between the natural frequencies of systems I and III. It is also confirmed the increase in the structure's damping factors due to the influence of the soil in system II, and an even greater increase occurs when the pile is included, according to the values obtained for system III.

Table 3. Extracted Modal Parameters

Modal Parameter	System I	System II	System III
Natural Frequencies	$\omega_1 = 36.86$ rad/s	$\omega_1 = 24.93$ rad/s	$\omega_1 = 37.56$ rad/s
	$\omega_2 = 99.13$ rad/s	$\omega_2 = 80.46$ rad/s	$\omega_2 = 97.65$ rad/s
	$\omega_3 = 150.94$ rad/s	$\omega_3 = 137.06$ rad/s	$\omega_3 = 147.51$ rad/s
	$\omega_4 = 184.89$ rad/s	$\omega_4 = 176.53$ rad/s	$\omega_4 = 180.03$ rad/s
Damping Factors	$\xi_1 = 0.0056$	$\xi_1 = 0.0352$	$\xi_1 = 0.0404$
	$\xi_2 = 0.0150$	$\xi_2 = 0.0306$	$\xi_2 = 0.0430$
	$\xi_3 = 0.0228$	$\xi_3 = 0.0283$	$\xi_3 = 0.0309$
	$\xi_4 = 0.0280$	$\xi_4 = 0.0291$	$\xi_4 = 0.0303$

In Figure 5 it is possible to observe the modal forms of the analyzed systems, in which there are no significant changes, either by the influence of the soil or the soil with the pile.

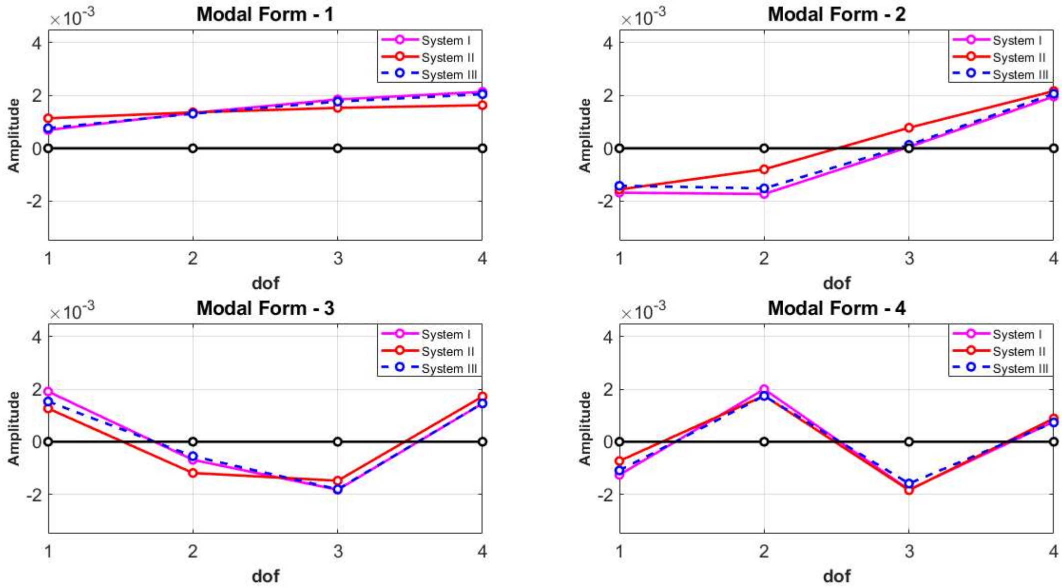

Figure 5. Modal forms

To obtain the transient responses, the excitation force, $F_4(t)$, in Figure 1, is defined by Equation (6).

$$F_4(t) = \begin{cases} 10N, & se \quad 0.1s < t < 0.2s \\ 0, & se \quad 0 < t < 0.1s \quad ou \quad t > 0.2s \end{cases}. \tag{6}$$

Thus, by the Modal Superposition method, the transient responses are calculated using the modal parameters in Table 3 and in Figure 5, as the excitation force, $F_4(t)$. Figure 6 compares the transient responses of systems I, II and III, in which shows the greatest amplitudes for system II and the greatest amplitude decay for system III.

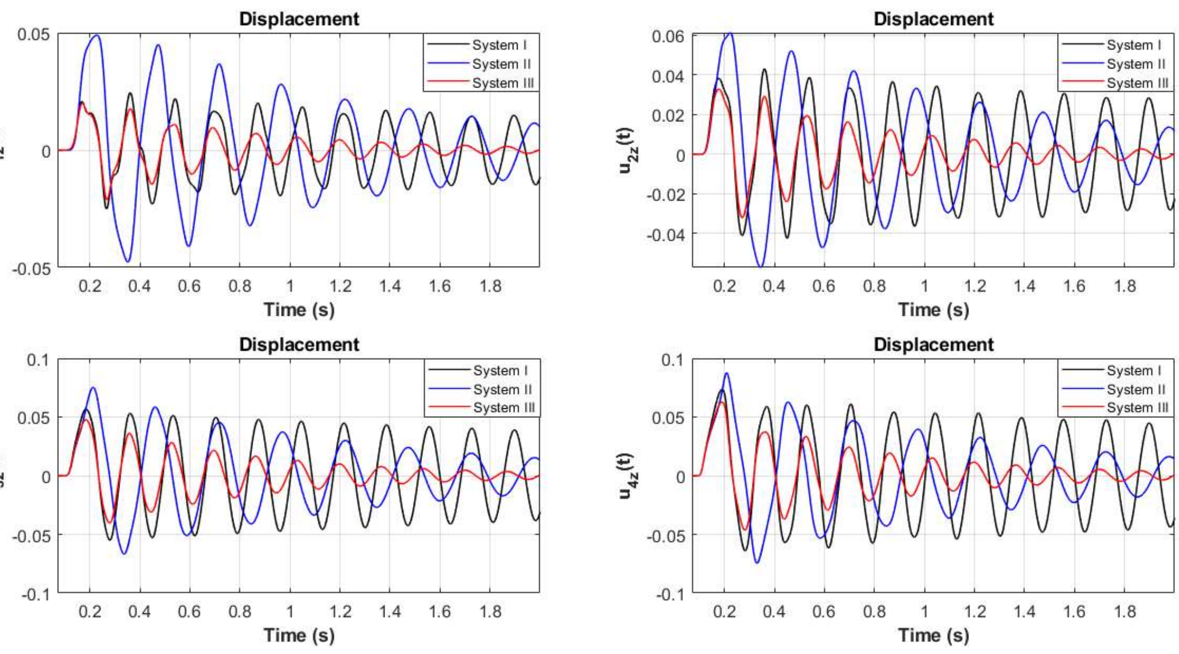

Figure 6. Transient responses

7 Conclusions

In this article, the influence of different soil-foundation arrangements on the structure responses was analyzed, as seen in Figure 1. From the comparison between systems I and II, it is noticed a great influence of the soil in the modal quantities related to the structure. When comparing systems II and III, it is noted that the inclusion of the pile to the half-space results in a reduction in the amplitudes of transient responses, as well as an increase in the natural frequencies and damping factors of the structure.

Acknowledgements. The research leading to this work was funded by the São Paulo Research Foundation (Fapesp) grant FAPESP CEPID Process 2013/08293-7 and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. The support of CNPq and Faepex/Unicamp is also gratefully acknowledged.

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References

[1] H. B. Coda, "Dynamic and static non-linear analysis of reinforced media: A BEM/FEM coupling approach". *Computers and Structures*, vol. 79, pp. 2751–2765, 2001.
[2] D. Soares, O. von Estorff and W. J. Mansur, "Iterative coupling of BEM and FEM for nonlinear dynamic analysis". *Computational Mechanics*, vol. 34, pp. 67–73, 2004.
[3] D. Soares and F. C. Araújo, "An explicit direct FEM–BEM coupling procedure for nonlinear dynamics". *Engineering Analysis with Boundary Elements,* vol. 103, pp. 94–100, 2019.
[4] D. Soares Jr., J. A. M. Carrer and W. J. Mansur, "Non-linear elastodynamic analysis by the BEM: an approach based on the iterative coupling of the D-BEM and TD-BEM formulations". *Engineering Analysis with Boundary Elements,* vol. 29, pp. 761–774, 2005.
[5] O. von Estorff, "Coupling of bem and fem in the time domain: Some remarks on its applicability and efficiency". *Computers and Structures,* vol. 44, pp. 325–337, 1992.
[6] W. H. Wu and H. A. Smith, "Efficient modal analysis for structures with soil-structure interaction". *Earthquake Engineering and Structural Dynamics*, v. 24, pp. 283–299, 1995.
[7] T. S. Louzada. *Resposta transiente de estruturas interagindo com o solo utilizando Análise Modal*. Master's thesis, University of Campinas, 2019.
[8] J. Labaki, E. Mesquita and R. K. N. D. Rajapakse, "Vertical Vibrations of an Elastic Foundation with Arbitrary Embedment within a Transversely Isotropic, Layered Soil". *Computer Modeling in Engineering & Sciences*, vol. 103, n. 5, pp. 281–313, 2014.
[9] P. L. A. Barros, J. Labaki and E. Mesquita, "IBEM-FEM model of a piled plate within a transversely isotropic half-space". *Engineering Analysis with Boundary Elements*, vol. 101, pp. 281–296, 2019.
[10] A. C. Ferraz. *Análise transiente de estruturas por superposição modal incluindo os efeitos da interação solo-fundação*. Master's thesis, University of Campinas, 2021.
[11] Z. F. Fu and J. He. *Modal Analysis*. Butterworth-Heinemann, 2001.
[12] D. J. Ewins. *Modal testing: theory, practice and application*. Wiley, 2000.
[13] M. H. Richardson and D. L. Formenti, "Parameter estimation from frequency response measurements using rational fraction polynomials". *In Proceedings of the 1st International Modal Analysis Conference*, pp. 1–15, 1982.
[14] D. K. Cheng. *Analysis of Linear Systems*. Addison-Wesley, 1972.