Comparison of the response of reinforced concrete structures modeled in mesoscale for different damage models

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Abstract. The continuous damage mechanics allows the construction of constitutive models that represent well the behavior of concrete structures. In this work, a computational code is developed, using the positional approach of the Finite Element Method (FEM), which allows to represent the concrete degradation using two different damage models. In the first, the degradation is distributed in the elements that discretize the concrete. In the second, the damage model is applied to interface elements and makes it possible to represent discrete cracks in the structure. The objective is to compare the performance of these models by applying them to the same structure to verify the proximity of the force-displacement curves and cracking patterns. Where do we seek to define the best for each type of use. The concrete is represented in mesoscale, where the finite element meshes that represent the coarse aggregates and the mortar are generated independently and are superimposed to form the composite material. The damage models were applied to a reinforced concrete beam and it was found that the model applied to interface elements represents the cracks better, but the computational cost is higher.

Keywords: Damage models, Mesoscale concrete, Embedded particles

1 Introduction

Concrete structures after a certain load intensity present a microcracking process, no longer having a linear elastic mechanical behavior. Furthermore, after reaching the maximum stress, the response curve shows softening. The continuous damage mechanics allows the construction of constitutive models that represent this behavior well. When using a damage model, the mechanical properties of the material are penalized based on the evolution of the degradation process. Thus, it is possible to represent the appearance of microcracks and even macrocracks as regions where localized deformations occur. Thus, the hypothesis of continuity of the material remains valid, even when very degraded.

Over time, different damage models have emerged to treat concrete degradation. Different damage criteria, different evolution laws and scalar or tensor damage variables are used. Models with a scalar damage variable can reproduce the behavior of concrete well, as can be seen in the models presented by Mazars [1] and Manzoli et al. [2]. The Mazars [1] model is suitable to degrade the mechanical properties of bulk elements subjected to any state of stress, uses a damage criterion based on maximum elongation strain and allows it to obtain response curves of the structure close to the experimental ones. But it does not allow to model the cracks in a discrete way, since the degradation is distributed in the most stressed elements. The Manzoli et al. [2] model was developed to be used in high aspect ratio interface elements, which according to the continuum strong discontinuity approach presented in Oliver et al. [3] makes them suitable for representing discrete cracks. The damage criterion is based on normal tensile stress at the base of the interface elements, so it is not able to represent the degradation of compressed elements.

The cracking of concrete is explained by the heterogeneity of the material that can be seen at the smaller scales. The mesoscale is the scale where the matrix, coarse aggregates and the interfacial transition zone (ITZ) are observed, which is the weakest link and where degradation begins. At this scale, concrete can be understood as a composite of particles and it is possible to relate its heterogeneity to the shape of the crack, giving it physical meaning. Thus, numerical modeling at this scale leads to complex answers even with the use of simple constitutive models for each component of the material [4]. At mesoscale, using FEM, Ramos et al. [5] modeled reinforced concrete beams using the Mazars [1] model and Rodrigues et al. [6] and Vieira et al. [7] modeled simple and reinforced concrete beams, respectively, using the Manzoli et al. [2] model. The results obtained in these works show that the different damage models allowed to obtain response curves close to the experimental ones. Furthermore, the representation of the heterogeneity of concrete brought improvements in the identification of degraded regions.

Amaral et al. [8] made a comparative study of the applicability of different damage models for ductile materials. The focus was to identify the advantages and disadvantages of each for their more efficient use in the automobile industry. Focused on the construction industry, looking for a more efficient concrete modeling, in this work a comparative study of the performance of the damage models presented by Mazars [1] and Manzoli et al. [2] is developed. A computational code is developed for the analysis by FEM of the phenomenon of concrete degradation in which the two damage models were implemented and concrete is represented in mesoscale. An alternative form is used to model the mesoscale in which the meshes representing the mortar and coarse aggregate are generated independently and superimposed to form the composite. The goal is to make the pre-processing step simpler. Furthermore, the finite elements that represent the aggregates do not add degrees of freedom to the problem, so it also aims to reduce the computational cost of the analysis. To improve the quality of the answers, the formulation used in the implementation is geometrically exact. It is a positional approach to FEM developed by Coda and Greco [9]. The implemented code is used to model a $2D$ reinforced concrete sample. Steel is modeled embedded in concrete and does not add degrees of freedom to the problem.

2 Numerical Modeling

2.1 The Finite Element Method based on positions

The positional approach is a naturally non-linear geometric version of the FEM in which the unknowns are the element node coordinates rather than the displacements used in the classical method. It emerged in the works of Bonet et al. [10], who used positions such as unknowns, and Coda and Greco [9] which they proposed the method and applied to $2D$ frame elements. In the method, as it is geometrically exact, the internal force vector is not obtained from a linear relationship between the stiffness matrix and the nodal positions. Then the Newton-Raphson method is used for the iterative solution of the problem. To use the technique, it is necessary to calculate the internal force vector and the tangent stiffness matrix for each trial. These terms are obtained as a function of the specific deformation energy and depend on the constitutive model and the type of element used. The elements used in this work are trusses and $2D$ solids with linear approximation. The way to calculate the internal force vector and the tangent stiffness matrix of these elements is detailed in Coda [11]. The constitutive model used is the Saint-Venant-Kirchhoff (SVK) model, suitable to describe the behavior of materials that present large displacements and moderate strains. It uses the second Piola-Kirchhoff stress tensor and the Green-Lagrange strain tensor.

The SVK specific deformation energy is given by

$$\Psi(E) = \frac{1}{2} E_{kl} \mathfrak{C}_{klij} E_{ij}, \tag{1}$$

where E is the Green deformation and $\mathfrak{C}$ is the elastic constitutive tensor.

2.2 Coupling matrix and reinforcement

The strategy used to model the coarse aggregates and steel allows the representation of these reinforcements with perfect adherence, without adding degrees of freedom to the problem and without the need for nodes to coincide with those of the matrix. The idea appeared in Vanalli et al. [12] to couple fiber and matrix elements using traditional FEM. Sampaio et al. [13] extended the idea to the FEM positional formulation and used it for fiber elements with any order of polynomial approximation. Using the same strategy, Paccola and Coda [14] were successful in coupling particles and matrix to model composites with elastic behavior. In this work, to connect the matrix with fibers and particles, the formulations presented in Sampaio et al. [13] and Paccola and Coda [14] are used respectively. The idea is to represent the nodal positions of the reinforcement elements using the shape functions of the matrix elements. The deductions presented by Sampaio et al. [13] and Paccola and Coda [14] allow us to conclude that the internal force vector and the tangent stiffness matrix of the reinforcements can be calculated as a function of their own nodes and then distributed over the nodes of the elements of the matrix. Thus, these values are added to those obtained for the matrix itself and the problem can be solved.

3 Damage models

3.1 Mazars damage model

The damage model of Mazars [1] is defined by a scalar variable and can represent the degradation for any stress state. The damage criterion can be defined as

$$f\left(E_{eq}, D\right) = E_{eq} - E_{lim}(D) \leq 0, \tag{2}$$

where variable E_{lim} is the maximum elongation strain in the strain history and E_{eq} is a is an equivalent strain given by

$$E_{eq} = \sqrt{(E_1)_+^2 + (E_2)_+^2 + (E_3)_+^2}. \tag{3}$$

The term $(E_i)_+$ corresponds to the positive components of the principal values of the strain tensor. At the beginning of the process E_{lim} is the strain corresponding to the concrete tensile strength E_{d0}. The formulation used here employs Green-Lagrange strain.

The damage variable is given by

$$D = \alpha_T D_T + \alpha_C D_C. \tag{4}$$

The terms D_T and D_C are damage variables related to tensile and compressive behavior of the concrete. These variables are given by:

$$D_T = 1 - \frac{E_{d0}\left(1 - A_T\right)}{E_{eq}} - \frac{A_T}{e^{B_T\left(E_{eq} - E_{d0}\right)}} \quad \text{and} \quad D_C = 1 - \frac{E_{d0}\left(1 - A_C\right)}{E_{eq}} - \frac{A_C}{e^{B_c\left(E_{eq} - E_{d0}\right)}}, \tag{5}$$

where A_T, A_C, B_T and B_C are parameters of the Mazar's damage model to be identified based on results of uniaxial tensile and compression tests.

The coefficients α_T and α_C are calculated by

$$\alpha_T = \sum_i^3 \left(E_i^T\right)_+ / \left(E_V\right)_+ \quad \text{and} \quad \alpha_C = \sum_i^3 \left(E_i^C\right)_+ / \left(E_V\right)_+. \tag{6}$$

The terms $\left(E_i^T\right)_+$ and $\left(E_i^C\right)_+$ are

$$\left(E_i^T\right)_+ = \frac{1+\nu}{\mathbb{E}} \left(S_i^T\right)_+ - \frac{\nu}{\mathbb{E}} \sum_{j=1}^3 \left(S_j^T\right)_+ \tag{7}$$

and

$$\left(E_i^C\right)_+ = \frac{1+\nu}{\mathbb{E}} \left(S_i^C\right)_- - \frac{\nu}{\mathbb{E}} \sum_{j=1}^3 \left(S_j^C\right)_-. \tag{8}$$

However

$$\left(E_i^T\right)_+ = 0 \quad \text{if} \quad \left(E_i^T\right)_+ < 0 \quad \text{and} \quad \left(E_i^C\right)_+ = 0 \quad \text{if} \quad \left(E_i^C\right)_+ < 0. \tag{9}$$

The term $\left(S_i^T\right)_+$ is given by the value of the i-th principal stress if it is positive and zero if it is not. The term $\left(S_i^C\right)_-$ is given by the value of the i-th principal stress if it is negative and zero if it is not. The variable $(E_V)_+$ represents the total state of elongation, given by the principal strains as

$$\left(E_V\right)_+ = \sum_i \left(E_i^T\right)_+ + \sum_i \left(E_i^C\right)_+. \tag{10}$$

Now all terms in eq. (4) can be calculated. Then the damage variable can be used to penalize material stiffness and determine nominal stress tensor. This is done as

$$\tilde{S}_{ij} = (1 - D)\mathbb{C}_{ijkl}E_{kl} \tag{11}$$

where $\tilde{S}$ is second Piola-Kirchhoff stress tensor , $\mathbb{C}$ is the fourth-order elastic constitutive tensor and E is the Green-Lagrange strain tensor.

3.2 Manzoli damage model

The damage model used by Manzoli et al. [2] is designed to be used on high aspect ratio interface elements. Therefore, when using it, only the interface elements are degraded and the other elements present linear elastic behavior. As in this model only the tension component of the stress tensor is used, it is limited to cases where the mode I fracture predominates. The criteria for the existence of damage ϕ is given by

$$\phi = \sigma_{nn} - q(r) \leq 0, \tag{12}$$

where σ_{nn} is the Cauchy stress normal to the major base of the interface element, q and r are the stress and strain-like internal variables, respectively. The function $q(r)$ defines the softening law. Dividing all terms of ϕ by $(1-d)$, the damage criterion for the effective stress is given by

$$\overline{\phi} = \overline{\sigma}_{nn} - r \leq 0, \tag{13}$$

where was made $(r = q/(1-d))$ to control the elastic domain in the space of effective deformations. Isolating d, the damage variable is given by

$$d = 1 - \frac{q(r)}{r}. \tag{14}$$

Considering the loading-unloading conditions and the consistency condition, for a pseudo-time t associated with the loading process, the variable r is always given by the highest value between $\overline{\sigma}_{nn}$ up to that moment and the initial value of the process r_0 given by the tensile strength f_t of the material. So it can be written as

$$r = max(\overline{\sigma}_{nn}(s), r_0) \;\mid\; s \in [0, t]. \tag{15}$$

To represent the softening, the same exponential law already used successfully by Rodrigues et al. [6] for mesoscale concrete was chosen. It is given by

$$q(r) = f_t \exp\left(\frac{f_t^2}{G_f \mathbb{E}} h(1 - r/f_t)\right), \tag{16}$$

where h is the smallest height of the interface element, G_f is the fracture energy for mode I, and $\mathbb{E}$ is the Young's modulus of the material.

The damage implementation algorithm is adapted from the version used by Manzoli et al. [2] and it is the implicit-explicit integration scheme (IMPL-EX) developed in Oliver et al. [15]. As in the implemented constitutive model the second Piola-Kirchhoff stress tensor is used, it is necessary to calculate the Cauchy stress tensor through it.

To create the interface elements, the mesh fragmentation technique presented in Manzoli et al. [2] is used, which is divided into 3 main steps. Step 1 consists of generating the regular mesh for the entire sample. Step 2 consists of reducing the dimensions of all elements, leaving an empty space between them for the positioning of the interface elements. Finally, in Step 3 the interface elements are inserted into these spaces.

4 Numerical example

4.1 Four-point bending test

This example simulates a four-point bending test on a reinforced concrete beam. This beam was experimentally tested by Álvares [16]. The geometry and boundary conditions of the structure are shown in Fig. 1 (a). The beam length is 2600 mm, the height 300 mm and the thickness 120 mm. To represent a structure with the dimensions described in mesoscale, it is necessary to use a very refined mesh. Therefore, the computational cost is high. So, trying to get around this problem, as there is an axis of symmetry in the beam, it was decided to simulate only half of it as shown in Fig. 1 (b). The load is applied as a prescribed displacement at the point where the reaction force F is observed. The Fig. 1 (c) shows the cross-section of the beam.

The structure was analyzed 3 times with the developed code. In the first analysis, concrete is represented as a composite formed by mortar and coarse aggregate. Among the elements that represent the mortar, interface elements are inserted. Their constitutive model is the damage model presented by Manzoli et al. [2]. This analysis is referred to as Mesoscale-interface damage. In the second analysis, concrete is represented as a homogeneous material, interface elements are not created and all elements have a mechanical behavior defined by the damage model of Mazars [1]. This analysis is referred to as Macroscale-Mazars. In the third analysis, concrete is represented as a composite formed by mortar and coarse aggregate, interface elements are not created and all elements have a mechanical behavior defined by the damage model of Mazars [1]. This analysis is referred to as Mesoscale-Mazars. The mesh that represents the concrete is formed by linear approximation triangular elements and in the three cases analyzed were created starting from a mesh with 87012 elements and 43907 nodes. This mesh is already suitable for use in Macroscale-Mazars and Mesoscale-Mazars analysis. But in Mesoscale-interface damage analysis, another 260236 interface elements were created, totaling 347248 elements and 261036 nodes. In all analyses, the same mesh was used to represent the steel. Linear approximation fiber elements totaling 400 elements and 800 nodes were used. In the analyses with the presence of coarse aggregates, they were randomly positioned, the particle size distribution used the Fuller curve, occupied 40% of the sample volume and the dimensions ranged from 4.80 mm to 19 mm. The mesh of aggregates is formed by linear approximation triangular elements and totals 34852 elements and 33632 nodes.

The known mechanical properties of the study by Álvares [16] for concrete are the Young's modulus, tensile strength and Poisson's ratio given respectively by $E = 29200$ MPa, $f_t = 2.04$ MPa and $\upsilon = 0.2$. It was assumed that the aggregate used is granite and has $E = 55300$ MPa and $\upsilon = 0.16$, values determined by Lee et al. [17]. The properties of the mortar were obtained using the parallel model of Counto [18] from the information of concrete and coarse aggregate. The values obtained are $E = 19150$ MPa and $\upsilon = 0.23$. The interface elements were adopted with the same Young's modulus of the mortar and $\upsilon = 0.0$. The parameters $A_T = 0.995$, $A_C = 0.85$, $B_T = 8000$ and $B_C = 1620$, determined for concrete by Álvares [16], were used in the damage model for the Macroscale-Mazars and Mesoscale-Mazars analysis. The term E_{d0} was used for concrete in Macroscale-Mazars analysis as $E_{d0} = 7 \cdot 10^{-5}$, as was done by Álvares [16]. In the Mesoscale-Mazars analysis, it was calculated as a function of the Young's modulus adopted for the mortar, also assuming that its tensile strength is the same as that of concrete. So we got $E_{d0} = 1 \cdot 10^{-4}$. For the Mesoscale-interface damage analysis, the damage model parameters are the fracture energy for mode I $G_f = 0.10$ MPa/mm, defined by Vieira et al. [7], and the concrete tensile strength, already known from the experimental result. For the reinforcement, the perfect elastic behavior with $E = 196000$ MPa was considered. The analyses were carried out considering a plane stress state with the application of a vertical displacement of 10.0 mm. The prescribed displacement was divided into 100 increments when applying the damage model of Mazars [1] and 800 increments when using the damage model presented by Manzoli et al. [2].

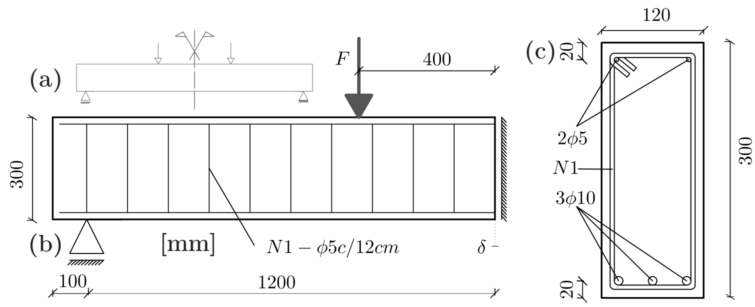

Figure 1. Geometry and boundary conditions

In Fig. 2 are presented the curves that relate the reaction force F with the displacement δ in the middle of the beam span, obtained for each analysis. In general, it was possible to observe a good agreement between the results obtained in this work and the experimental results of Álvares [16]. The curve obtained for the Mesoscale-Mazars analysis was the most different from the experimental results, which is attributed to the simplified form used to adopt the damage model parameters, since the response of the Macroscale-Mazars analysis, which uses the same damage model, was adequate. In Fig. 3 (a) the discrete cracking obtained in the Mesoscale-interface damage analysis is presented. It shows multiple approximately spaced cracks and their inclination change when approaching the support, as observed experimentally. In Fig. 3 (b) and (c) we can see the damage distribution in the structure for the Macroscale-Mazars and Mesoscale-Mazars analysis respectively. The blue color represents that the region is completely degraded and the red color that there is no damage. Comparing the damage distribution in the Macroscale-Mazars and Mesoscale-Mazars analysis, it is observed that the representation of the particles contributed to the greater location of the damage, with more parts of the matrix remaining intact, as it actually happens. However, it is still not possible to visually represent the cracking of concrete with the quality of the Mesoscale-interface damage analysis. Comparing the computational cost, the analyses that use the damage model of Mazars [1] are more economical, as they have less degrees of freedom and need less loading steps for the results to converge. The higher computational cost of the Mesoscale-interface damage analysis is due to the presence of

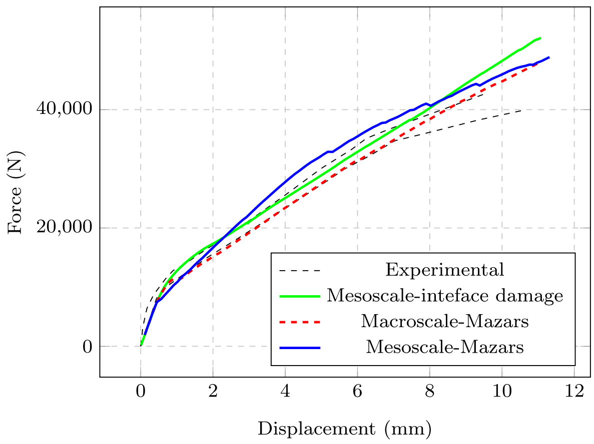

Figure 2. Force-displacement curve for each analysis

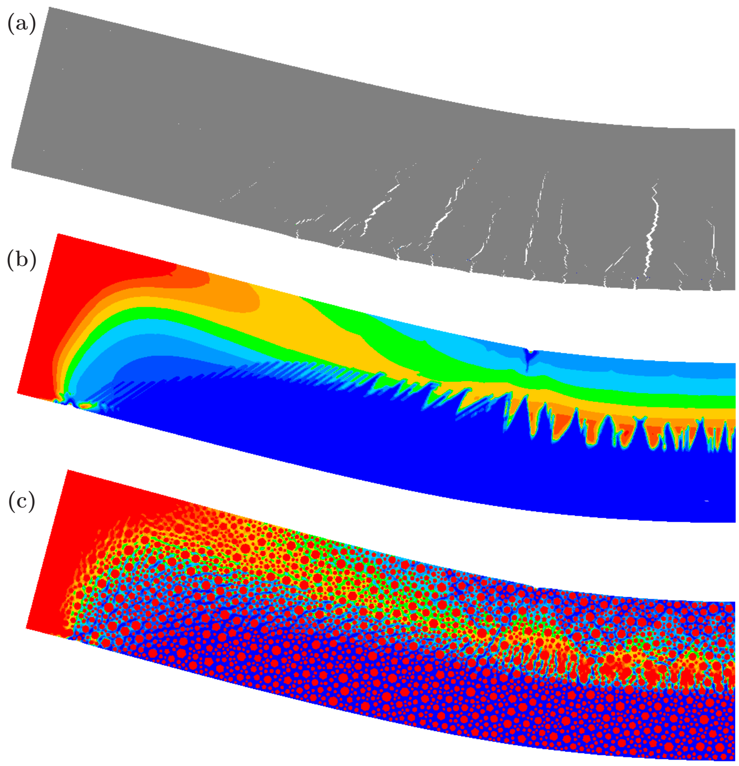

Figure 3. Discrete crack obtained in Mesoscale-interface damage analysis (a) and structure damage distribution for Macroscale-Mazars (b) and Mesoscale-Mazars (c) analysis

the interface elements and the need for more loading steps to use the IMPL-EX, which improves the convergence of the analysis but has this cost.

5 Conclusions

In this work a computational code was developed using FEM and the performance of two different damage models applied to a reinforced concrete beam was compared. The model presented by Mazars [1], as implemented here, is more efficient than the model presented by Manzoli et al. [2] in relation to the computational cost of the analysis, but it does not allow the representation of discrete cracks. Therefore, with the modeling strategies presented, it is concluded that, for a beam similar to the one analyzed, the Mazars [1] model is better when the objective is to determine only the response curve. Nor is the representation of particles necessary. But if the objective is to analyze the cracking, the model presented by Manzoli et al. [2] is most suitable.

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