An algorithm for distribution of short fibers in multiscale numerical analysis

André Peres da Silva[1], Luís Antônio Guimarães Bitencourt Jr.[1]

[1] *Department of Structural and Geotechnical Engineering, Polytechnic School at the University of Sao Paulo*
Av. Prof. Almeida Prado, trav.2 nº. 83, 05508-070, Sao Paulo/SP, Brazil
andre.peres.silva@usp.br, luis.bitencourt@usp.br

Abstract. Nowadays, it is widely known that the behavior of Steel Fiber Reinforced Concrete (SFRC), especially the failure process is highly influenced by the distribution of the fibers. For this reason, recently, many numerical models with a discrete and explicit representation of steel fiber have been proposed in order to consider the effect of the fiber distribution. In this context, this work presents an algorithm for the distribution of short fibers in multiscale numerical analyses. Some features and capabilities of the algorithm are assessed through the numerical analyses of some beams under distinct degrees of fiber segregations.

Keywords: finite element method, multiscale model, SFRC, fiber distribution, segregation

1 Introduction

The use of steel fiber reinforced concrete (SFRC) in structural members has grown in recent years in order to improve their performance in terms of serviceability and ultimate limit states. Steel fibers act as stress transfer bridges across the crack, obtaining a cracking frame with a greater number of cracks and smaller openings. In this way, a material with high ductility and toughness is obtained ([1]) .

Several researchers have investigated the influence of the addition of steel fibers to concrete, especially when subjected to tensile, bending and shear stress [2]. It is noted that, for an appropriate steel fiber content, the use of SFRC in structural members subjected to shear stress is very promising ([3]; [4]).

The behavior of SFRC elements strongly depends on the distribution and orientation of fibers ([5]; [6]). Fibers improve the performance of the structural elements if they are perpendicular to the crack plane, losing efficiency when they are parallel to this plane. Therefore, the efficiency of fibers depends, among other factors, on the number of fibers and their inclination angles in relation to the crack plane ([7]). Distribution and orientation of fibers in a structural members are influenced, among other aspects, by: formwork geometry, concrete vibration, fresh mix properties, wall effect, etc. ([7]; [8]). fib Model Code 2010 (2013) provides equations for the design of SFRC structural members and allows passive reinforcements to be partially or fully replaced by fibers when certain residual strength conditions are attended. The residual strengths are obtained according to EN14651 (2005).

In this work, numerical simulations of the SFRC beams are performed using a mesoscale model with a discrete and explicit representation of steel fibers [9]. A non-uniform distribution model is used to generate clouds of fibers with different segregation levels. This model is used to simulate 4-point-bending beams without shear reinforcements. The results obtained with different distributions are used to analyze the influence of the segregation of fibers on the shear resistance of the beams.

2 Numerical modeling

2.1 Concrete

The numerical model used in this work is based on coupling finite elements developed by Bitencourt Jr. et al. [10]. In this model, the mechanical behaviors of concrete, fibers and the fiber/concrete interaction are independently represented. The behavior of concrete is represented by a continuous isotropic damage model with two scalar damage variables, to represent the tensile and compression behavior of the material [11]. Concrete is discretized in three-noded triangular finite elements (constant strain triangle - CST), while the fiber/matrix interaction is described by four-noded triangular finite element coupling (EFA) (Figure 1), whose behavior is described by a constitutive model based in the mechanics of continuous damage, adjusted according to the law proposed by Cunha [12] for steel fibers with a hook.

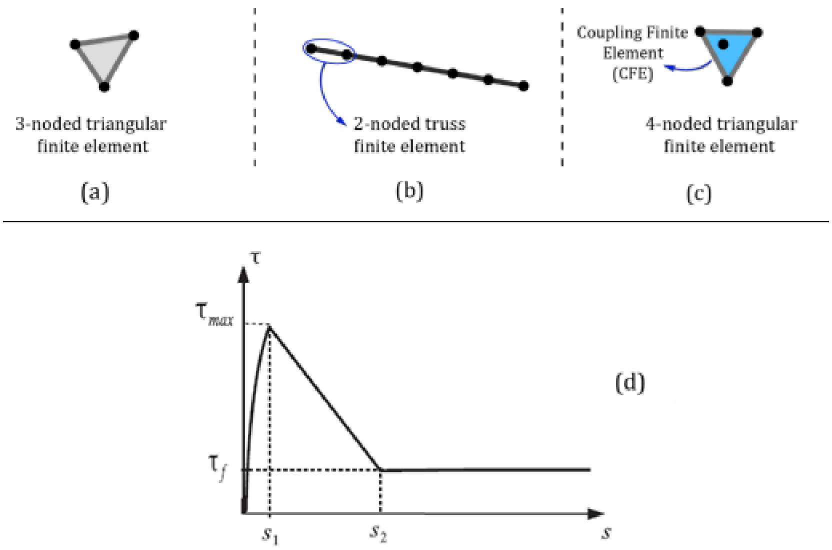

Figure 1. Coupling finite elements method: (a) CST element (concrete); (b) truss element (fiber); (c) CFE (fiber/matrix interaction); (d) bond-slip law adopted. [13]

2.2 Steel Fiber distribution

Steel fibers were generated using a non-uniform random distribution considering the presence of vertical segregation of the fibers. Such model uses a Exponential Random Distribution $Y = e^{-\lambda x}$ with parameter $\lambda > 0$ and $x \in [0, 1]$ (Figure 2). In this situation, the amount of fibers in each third of the rectangular cross section $(\theta_1, \theta_2, \theta_3)$ is related to the area under the curve of the function $y = e^{-\lambda x}$ as follows:

$$A_1 = \int_0^{1/3} e^{-\lambda x} dx \tag{1}$$

$$A_2 = \int_{1/3}^{2/3} e^{-\lambda x} dx \tag{2}$$

$$A_3 = \int_{2/3}^{1} e^{-\lambda x} dx \tag{3}$$

$$A = A_1 + A_2 + A_3 \tag{4}$$

$$\theta_i = \frac{A_i}{A} \tag{5}$$

Table 1 shows some examples of theoretical values of fibers for different values of λ.

3 Numerical simulations

In this paper, SFRC beams with $2000mm$ length and $125x250mm$ cross-section were simulated using the parameters of Trindade et. al (2019) work [3] (Figure 3). Concrete was modeled using the following parameters: modulus of elasticity $E = 34960MPa$; Poisson's coefficient $\nu = 0.2$; compressive strength $f_c = 50MPa$; tensile strength $f_t = 2.35MPa$; fracture energy $G_f = 0.1N/mm$ and compression parameters $A^- = 1.0$

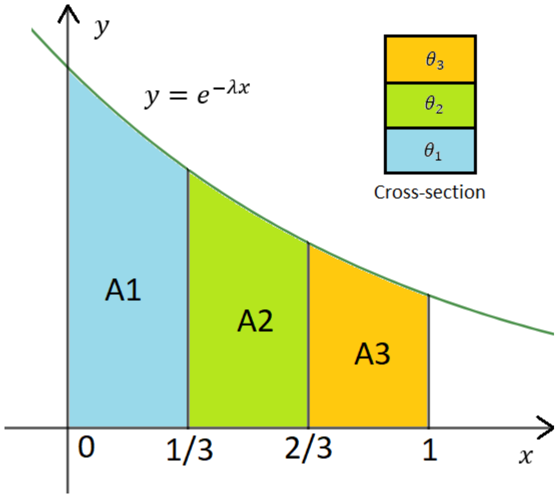

Figure 2. Scheme of the exponential random distribution used in the vertical distribution of fibers on beams.

λ	A_1	A_2	A_3	A	$\theta_1(\%)$	$\theta_2(\%)$	$\theta_3(\%)$
1.5	0.26230	0.15910	0.09650	0.51790	50.7	30.7	18.3
1	0.28347	0.20311	0.14554	0.63212	44.9	32.1	23.0
0.75	0.29490	0.22969	0.17889	0.70348	41.9	32.7	25.4

Table 1. Examples of percentage of fibers in each third of the cross-section for different values of λ.

and $B^- = 0.89$. The parameters used in fiber modeling were: modulus of elasticity $E = 210000MPa$ and yield stress $f_y = 1250MPa$. The fiber/matrix interaction parameters adopted were: $\tau_{max} = 8.5MPa$, $\tau_f = 4.5MPa$, $s_1 = 0.01mm$; $s_2 = 6.5mm$, $\alpha = 0.4$, $k_x = 10^3MPa/mm$ and $k_y = 10^6MPa/mm$. The fiber used is a hook-ended fiber with the following geometry parameters: $60mm$ of length and $0,75mm$ of diameter. Conventional bending bars ($2\Phi16mm^2$) were used in all beams tested, with $2.5mm$ of cover. The bars/matrix interaction parameters adopted were: $\tau_{max} = 14.8MPa$, $\tau_f = 5.9MPa$, $s_1 = 1mm$; $s_2 = 2mm$, $s_3 = 10mm$ $\alpha = 0.4$, $k_x = 10^3MPa/mm$ and $k_y = 10^6MPa/mm$. Three beams with 45 kg/m³ (Figures 4 and 5) of fibers were simulated with different segregation levels (Table 2) and the results are shown in Figure 6. The elevated value of the compressive strength of the concrete was used to consider only the effect of the tensile stresses on beams.

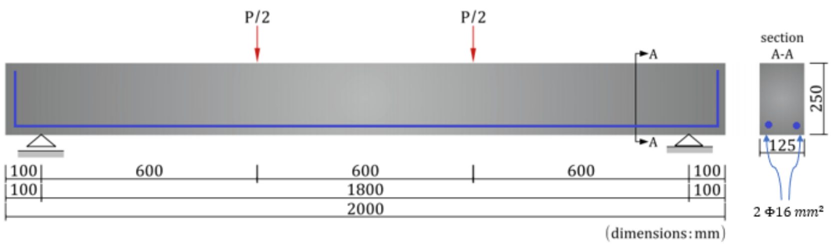

Figure 3. Characterization of the beams: dimensions, load and boundary conditions ([3]).

4 Conclusions

In this study, the following conclusion were obtained:

- It's important to consider the distribution of fibers along the height of the cross-section in the designs of SFRC beams under shear stresses.
- Segregation of fibers induced a bigger number of flexural cracks at the bottom of the beams.
- Segregation of fibers increased the ductility of beams in comparison to the beam with uniform distribution.

Beam	Dosage (kg/m³)	Number of fibers	$\theta_1(\%)$	$\theta_2(\%)$	$\theta_3(\%)$
V0	0	0	-	-	-
V45-0	45	13499	33.1	33.6	33,3
V45-1	45	13499	55.7	30.0	14,3
V45-2	45	13499	75.1	19,4	5,5

Table 2. Setup of beams studied in this work. Percentages of fibers in each region of the cross-section considering all the fibers of each beam.

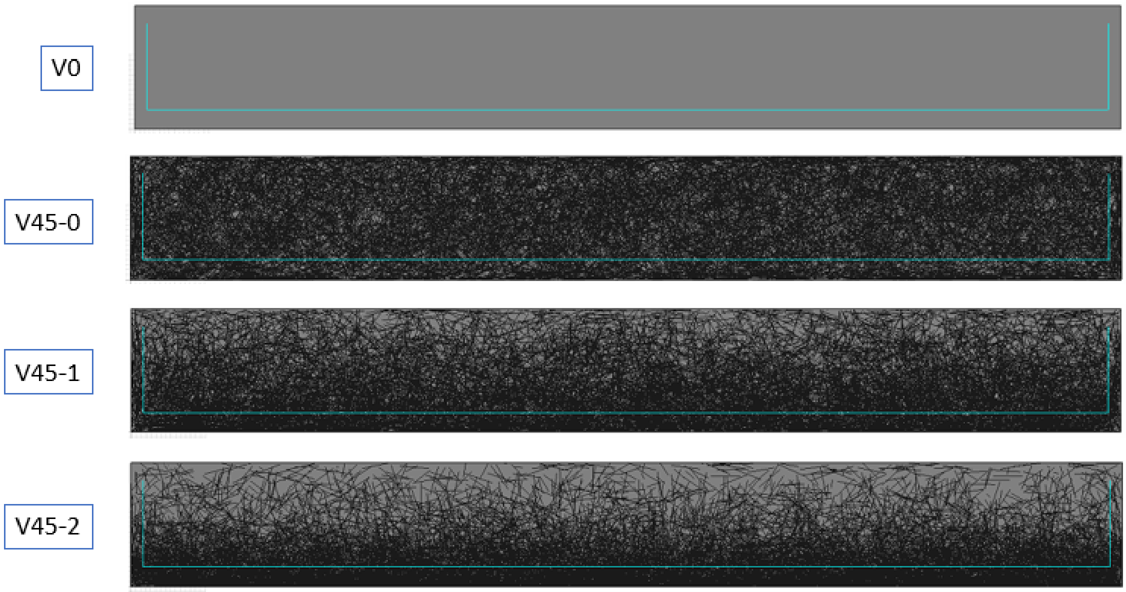

Figure 4. Fiber distributions on tested beams.

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Authorship statement

The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

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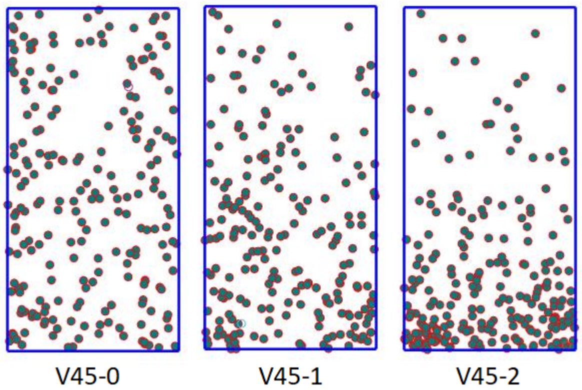

Figure 5. Cuts on central cross-sections of tested beams.

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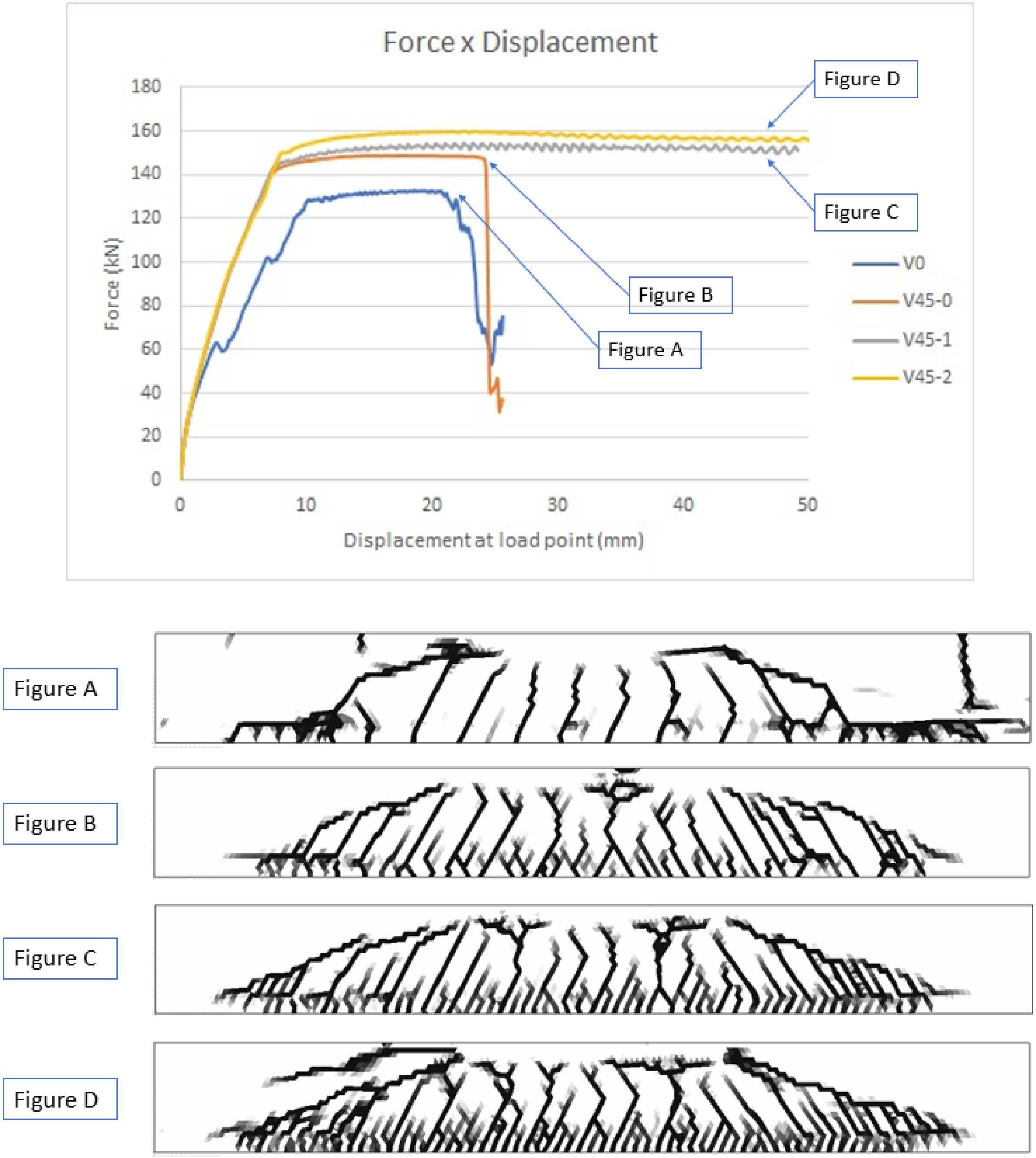

Figure 6. Load x Displacement of 4-point bending tests and failure mode of beams..