# Vibratory response of a wind tower considering soil-structure interaction

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**Abstract.** This work presents a model of the vibratory response of a wind tower structure supported by an embedded pile group, considering the soil-pile-structure interaction. The model for the pile group is obtained using the impedance matrix method. The soil is considered as an isotropic, viscoelastic, three-dimensional half-space, the response of which is obtained through a boundary element discretization of the pile-soil contact tractions. Piles are modeled as one-dimensional finite beam elements. Coupling between the systems is achieved by establishing equilibrium and continuity conditions at the soil-pile and pile-structure interfaces. The results consider arbitrary harmonic loads applied to the structure in terms of nodal equivalents. This analysis show that disregarding the influence of the pile group in the model of the tower may incur in considerable misrepresentation of the tower's dynamic response.

**Keywords:** Wind turbine, Dynamic soil-structure interaction, Piled structures

## 1 Introduction

Wind turbines are a well-established, fast-growing source of renewable energy. These are structures that, due to their geometry, can present high levels of vibration caused by both the operation of the turbine and by wind loads. Their industrial success has fostered thriving fields of study on their vibratory behavior, recent literature reviews of which can be seen in Amano [1] and Honrubia-Escribano et al. [2]. The importance of taking into account the foundation behavior in wind turbine tower models is also widely recognized [3]. However, many of such models resort to considerably limiting approximations with regards to the foundation behavior. Models such as that of Wang and Ishihara [4] and Banerjee et al. [5] approximate the foundation behavior in the sense of Winkler-Pasternak [6], which fail to represent wave propagation to and from the soil, and between different parts of the foundation. Models such as that of Kementzetzidis et al. [7] consider full finite element discretizations for the soil-foundation-structure system, an approach that is famously know for its domain truncation problems and its failure to comply with Sommerfeld's radiation condition [8].

This article proposes a model of the vibratory response of wind turbine towers considering soil-structure interaction. The tower is modeled with classical finite elements, which enables the consideration of arbitrarily-shaped structures. The piles are modeled using the impedance matrix method proposed by Kaynia and Kausel [9]. Coupling between the pile and the structure is obtained the same way, except that in this case these are imposed between the pile head and the prescribed node of the structure with which it interacts. Arbitrary harmonic loads can be applied anywhere in the structure in terms of nodal equivalents.

## 2 Problem statement

This study considers a wind tower with geometry and dimensions illustrated in Fig. 1. The tower is made of steel and has a height of 78 m. The foundation of the tower is composed of a conical concrete base with 14 m in diameter. The ensemble is supported by a group of 18 concrete piles of 0.7 m in diameter and 10.8 m in length, spaced radially at a distance of 6.25 m from the center. The material properties considered for each part are given in Table 1. The material damping $\beta$ is incorporated into the soil's elasticity modulus according to Christensen's elastic-viscoelastic correspondence principle [10].

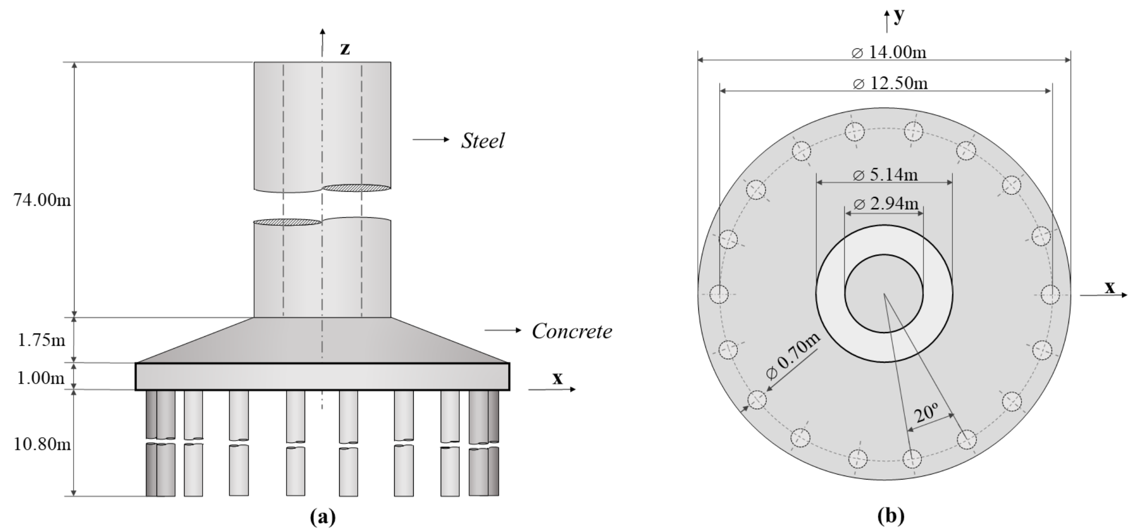


Figure 1. Wind tower geometry: (a) front view; (b) top view.

Table 1. Material properties

| | $E\,(GPa)$ | $\nu$ | $\rho\,(kg/m^3)$ | $\beta\,(m/s)$ |
|---|---|---|---|---|
| Steel | 200.0 | 0.3 | 7850 | - |
| Concrete | 21.5 | 0.2 | 2500 | - |
| Soil | 21.5 $10^{-3}$ | 0.4 | 1250 | 0.05 |

# 3 Formulation

## 3.1 Formulation of the pile group

The pile group model in this paper is obtained through the impedance matrix method [9]. This method provides the dynamic response of the group of piles, connected or not, due to external or seismic loads, and can consider both homogeneous or layered soils, which can be layers over rigid base or unbounded half-spaces. The piles in the group can have arbitrary geometrical and constitutive properties, and these properties can vary along the length of each pile. Only a small, yet representative subset of this method's capabilities will be used in this paper.

In the impedance matrix method, piles are modeled as one-dimensional, two-noded finite element beams in bonded contact with the soil. The soil is modeled as a homogeneous, viscoelastic, isotropic half-space. Contact tractions transferred at the pile-soil interface are modeled as piece-wise constant boundary elements, which yields an accurate description of wave propagation in the soil, and compliance with Sommerfeld's radiation condition. Coupling between the pile and soil elements is obtained by establishing continuity and equilibrium conditions at their interface. After laborious mathematical manipulation, the dynamic stiffness matrix $K_s$ of the pile group system can be written as:

$$K_s = K_p + \Psi^T (F_s + F_p)^{-1} \Psi, \tag{1}$$

in which $K_p$ is the dynamic stiffness matrix of the finite-body piles, $\Psi$ is the dynamic flexibility matrix of the pile elements, and $F_p$ and $F_s$ are pile and soil flexibility matrices, respectively. A full description of these terms is outside the scope of this article, but can be found in Kaynia and Kausel [9].

## 3.2 Formulation of the structure

In this paper, the tower is modeled with classical eight-noded, linear-elastic, hexahedral finite elements with three degrees-of-freedom per node. The use of full finite element discretization for the tower part enables the consideration of arbitrarily-shaped structures, as well as arbitrary loads to be applied in terms of nodal equivalents. The elemental stiffness and mass matrices $K_e$ and $M_e$ are given by

$$K_e = \int_{V_e} B^T DB\, dV = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} B^T DB det(J)\, d\xi d\eta d\zeta \tag{2}$$

and

$$M_e = \int_{V_e} \rho N^T N\, dV = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \rho N^T N det(J)\, dxid\eta d\zeta, \tag{3}$$

in which $V_e$ is the volume of the element, $\rho$ is its mass density, $D$ is its constitutive matrix, $J$ is the Jacobian matrix that relates the physical and natural domains, $N$ the vector of shape functions, and B the matrix of its derivatives. The structure's dynamic stiffness matrix is given by:

$$\overline{K} = K_G - \omega^2 M_G, \tag{4}$$

in which $K_G$ and $M_G$ are the global stiffness and mass matrices, and $\omega$ is the frequency of excitation. The assembly of elementary matrices $K_e$ and $M_e$ into global matrices $K_G$ and $M_G$ follows the classical procedure of the finite element method [11].

### 3.3 Coupling between the structure and the foundation

In order to obtain the coupled response of the structure and its underlying pile group foundation, direct equilibrium and continuity conditions are imposed at their interface. In this case of 1D piles, the interface is the connection between the pile head and specific nodes of the structure. In order for the coupling to be imposed, it is necessary that there is one node of the structure at the position of each pile head. Most commercial finite element software feature dimensionless points within their geometric entities, and these points can be placed so that they result in nodes at the pile head locations once the mesh is generated.

Imposing equilibrium and continuity conditions between pile heads and their corresponding nodes of the structure results in a modification of the equation of motion of the structure (Eq. 4). The modification is that the stiffness of the degrees-of-freedom in $\overline{K}$ corresponding to the nodes that are connected to pile heads will be summed to the stiffness of the pile that is connected to that node (Eq. 1). For a detailed deduction of this coupling scheme, refer to Vasconcelos [12].

## 4 Numerical results

This section considers the response of the tower described in Section 2 [13]. Uniformly distributed vertical ($z-$direction) and horizontal ($x-$direction) harmonic loads or circular frequency $\omega$ and magnitude $p_0$ are applied to the top surface of the tower. Results are presented in terms of the normalized displacements $U_{ij} = u_{ij}/p_0$, in which $u_{ij}$ $(i, j = x, z)$ is the displacement in the $i-$ direction due to loads in the $j-$direction, and of the normalized frequency $a_0 = \omega d/c_s$, in which $c_s$ is the shear wave speed in the soil medium, and $d$ is the pile diameter. Displacements are measured of point (x=2.57 m, z=76.75 m), on top of the tower (see Fig. 1).

In order to understand the effect of considering different assumptions for soil-structure interaction in a wind turbine tower problem, three cases were considered. Case 1 considers that all nodes of the base of the tower are fixed. In this assumption, the tower essentially behaves as a clamped-free beam. In Case 2, only the 18 nodes where the piles would be located (see Fig. 1) are fixed. And Case 3 considers the proper interaction of the tower with its flexible pile group foundation, attached to the 18 points described.

Figures 2 to 5 show the amplitude and phase components of the dynamic displacement of the tower in different directions. These results show that the dynamic response of the tower is strongly influenced of the coupling condition. Figures 2 to 5 show that the resonant frequencies in Case 2 are slightly smaller than in Case 1 in the horizontal direction, and much smaller than Case 1 in the vertical direction. This is physically consistent: clamping a few nodes of the base of the tower instead of clampling the entire base makes the model more flexible, which results in lower natural frequencies. The difference between Cases 1 and 2 is more pronounced in the vertical response because of the distribution of the 18 clamped nodes in this particular case.

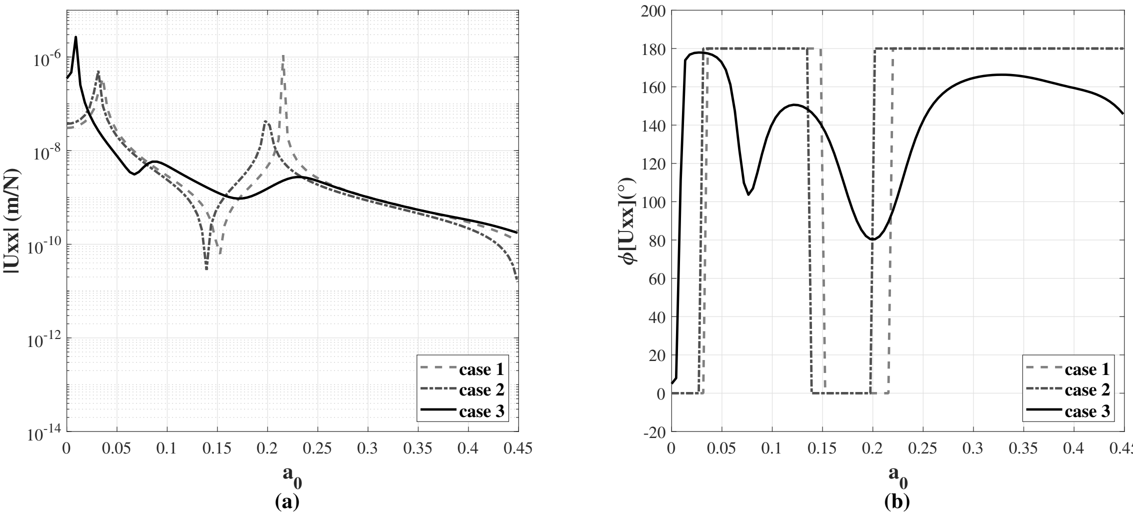


Figure 2. Horizontal displacement due to horizontal load: (a) absolute and (b) phase

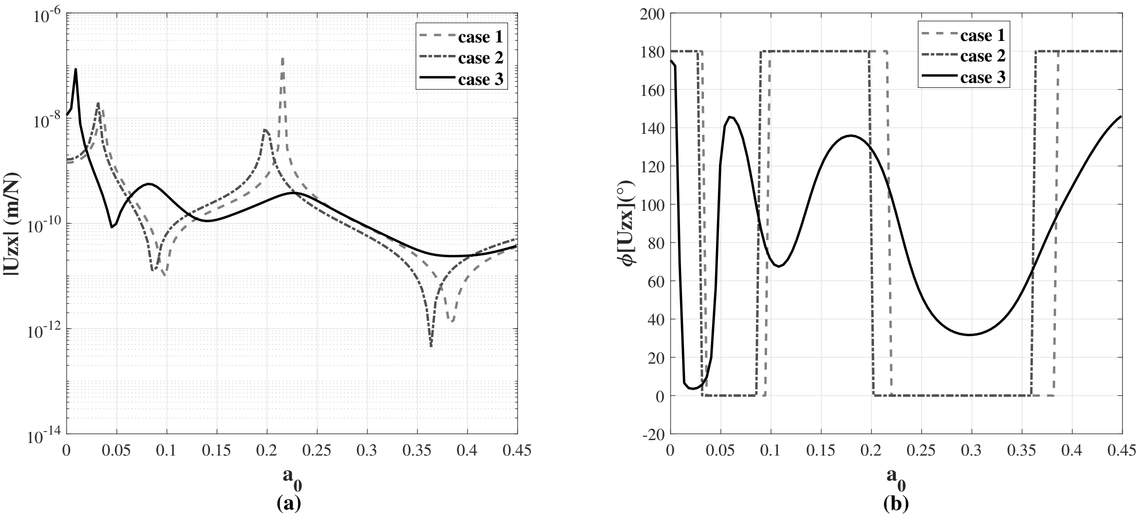


Figure 3. Vertical displacement due to horizontal load: (a) absolute and (b) phase

However, the most striking differences are observed between the clamped cases 1 and 2 and the piled Case 3. In Case 3, not only the natural frequencies are significantly changed, but the amplitude of the response is severely attenuated throughout the frequency spectrum. Due to the energy-absorption characteristics of soils, resulting from their geometric damping behavior as an unbounded, wave-propagating medium, the inclusion of foundations in the tower model corresponds to the inclusion of a damping component in the response of the system. This attenuation is manifested in these results as the strong attenuation of the amplitude component, and as the smooth transitions between phase angles, in contrast with cases 1 and 2. The static ($a_0 = 0$) horizontal displacement due to the horizontal load $U_{xx}$ in Case 3 is about 10 times larger than in Case 1, while the static vertical displacement due to the vertical load $U_{zz}$ is about 60 times larger. The increase in the overall attenuation of the tower's response for increasing frequencies of excitation is also physically consistent.

# 5 Conclusions

This article presented a model and case study of the vibratory response of wind turbine towers. The presented model considers a boundary element-based model of pile group foundations, and a classical finite element model for the tower. The results showed that both the static and dynamic responses of the tower can be severely

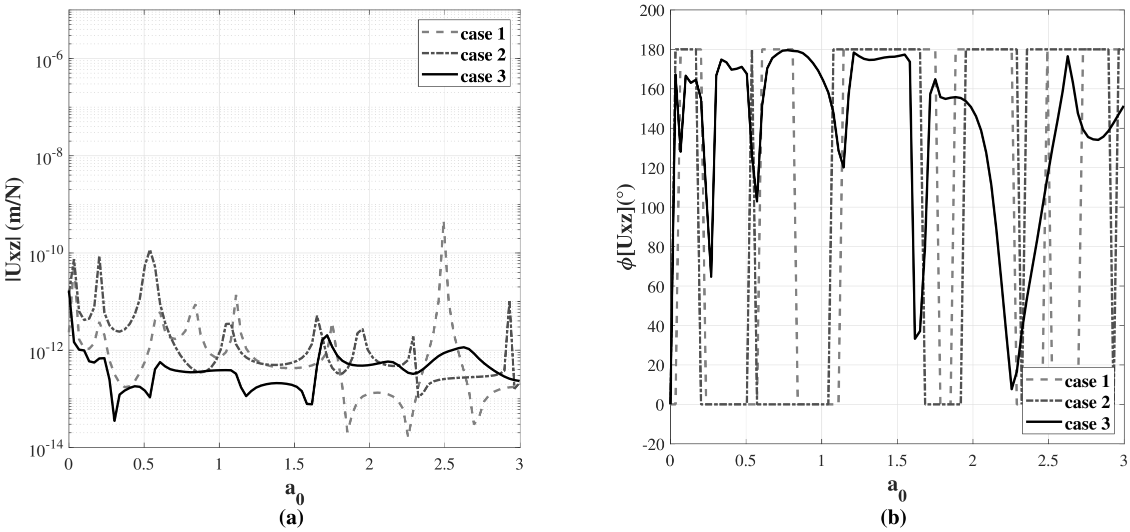


Figure 4. Horizontal displacement due to vertical load: (a) absolute and (b) phase

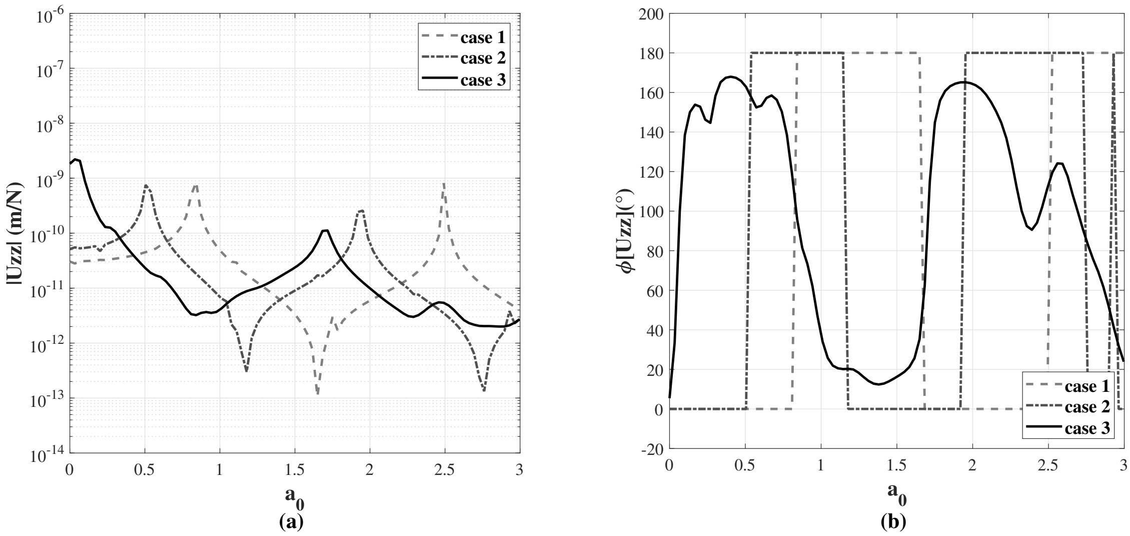


Figure 5. Vertical displacement due to vertical load: (a) absolute and (b) phase

misrepresented by considering rigid-support approximations for the tower.

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