An elastic-viscoplastic load-transfer method for a single pile and pile groups

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Abstract. A nonlinear load-transfer method for determining the load-settlement curve for a single pile and pile groups under axial loads is presented. The model allows slippage, or plastic soil deformation, and introduces a new exponential model for load-transfer modeling either strain hardening or softening. Long term deformations of soil layers and pile material are accounted for by a viscoplastic model. Elastic soil deformation and simultaneous soil hardening induced by neighboring pile shafts are considered. Validation of the model is done by comparing numerical and field static load tests from literature. This research is mainly motivated by the need of estimate piles and pile caps loads when periodic vertical settlement measures are done during building construction in sites where there exist layers of very soft clay. Present work describes basic formulation and initial applications.

Keywords: Axial loaded piles, pile groups, load-transfer method, soil ageing, strain hardening, strain softening.

1 Introduction

A nonlinear load-transfer method for determining the load-settlement curve for a single pile and pile groups under axial loads is presented. The model allows slippage, or plastic soil deformation, that can easily be tracked by introducing a new exponential model for load-transfer that is able to capture either strain hardening or softening. A total base displacement control is used to trace the load-settlement response curve, where the slippage part is extract from total displacement by solving a nonlinear equation using Newton-Raphson method. Long term soil ageing deformations of soil layers and pile material are accounted for. Elastic soil deformation induced by neighboring pile shafts and by simultaneous soil hardening are considered. Applied moments at top of caps by eccentric vertical loads are estimated in a postprocessing step. Validation of the model is done by comparing numerical and field static load tests from literature. The present simplified model is showed to be accurate enough to be used in the simultaneous design of the whole building foundation, considering the mutual interactions between piles of all pile groups inside an elastic influence radius. This integration has been proved to be important when soft soil layers are present. Despite of their possible general use, present model was mainly developed to be used in desired situations when the construction is accompanied by periodic estimation of incremental loads acting along time at foundation level and periodic measures of absolute settlements of pile caps. In this situation, the structural engineering can analyze the structure by more realistic incremental construction methods using an integrated model for soil-structure interaction or, alternatively, to compare obtained foundation loads from their structural analysis with the geotechnical solutions proposed in this paper. In this last scheme, the load at interface between structure and their foundations determined by the two approaches (structural and geotechnical) should match in all incremental construction steps. This research is mainly motivated by the need of estimate piles and pile caps loads when periodic vertical settlement measures are done during building construction in sites where there exist layers of very soft clay.

Nonlinear load-transfer approach that consider neighbors piles was developed in Zhang at al [1] using a hyperbolic load-transfer function. The three parameters of hyperbolic laws are not flexible enough to be adjusted to specific strain softening or hardening, motivating the development of an adaptive exponential model with variable, from two to four, number of parameters, introduced in this paper. More flexible, kinked, or smooth, exponential models are described in Araújo and Costa [2]. There are numerous schemas for trace load-settlement curve of piles, see references in Boonyatee and Lai [3]. Some of them model the incremental increases of soil strength and stiffness acquired during primary consolidation, secondary compression, and creep, but no one appears to model ageing settlement. Modeling deformation under constant loads is essential for evaluating temporal series of settlement field measures when building rests on layers of soft clays; if soil ageing displacement are not considered, estimated caps loads will be in large error. This work model ageing settlement occurring during secondary compression and creep based on the elastic-viscoplastic theory developed in Kelln at al [4].

2 Proposed load-transfer method

2.1 Model formulation

A displacement based finite element approximation for pile shaft is used in which the pile is divided into a set of reticulated structural elements that are laterally loaded by uniform longitudinal soil shear stresses (skin friction) reacting to pile top axial load and pile tip soil reaction. The nonlinear formulation is cast as a two-point boundary value problem, allowing a sequential iterative solution, without the needing of a structural matrix assemble and solution. The settlement, z_i, of each structural element $i \epsilon \{1\colon n\}$, measured at their middle, is composed of a soil immediate nonlinear elastic part, z_i^e, and a slippage, plastic, or irrecoverable part, z_i^s. Slippage occurs at a narrow band near to structural element. Clayey soils suffer from additional soil ageing deformation along time under constant load that is modeled as viscoplastic displacement, z_i^{vp}. The total settlement of a structural pile element becomes,

$$z_i = z_i^e + z_i^s + z_i^{vp}. \tag{1}$$

Displacement due to concrete pile creep, z_i^{cr}, under constant load can be added to above relation.

2.2 Exponential models for soil hardening and softening

For modeling the soil shear stress acting in the pile structural element, this work introduces a new exponential relation between the slippage, z_i^s, and the mobilized soil shear stress, τ_i, giving by

$$\tau_i = a_i\left(1 - e^{-b_i z_i^s}\right), if\ z_i^s \le z_i^{su}, \tag{2a}$$

$$\tau_i = \tau_i^{su} - \tau_i^r\left(1 - e^{-r_i(z_i^s - z_i^{su})}\right), if\ z_i^s > z_i^{su}, \tag{2b}$$

where the pair (z_i^{su}, τ_i^{su}) specify the yield stress after which the soil strain softening if the conventional curvature $r_i > 0$, hardening, if $r_i < 0$, or the stress remains uniform if $r_i = 0$. As z_i^s goes to infinite, the shear stress approaches the specified residual stress $\tau_i = \tau_i^r$. The rate at which the residual stress is attained is in inverse proportion to the conventional curvature, r_i. This model contains one more parameter than well-known hyperbolic and Zhang models. The conventional curvature can be determined by specifying an additional observed value $(z_i^D > z_i^{su}, \tau_i^D)$. Conventional exponential model is recovered if $r_i = (a_i b_i / \tau_i^r) e^{-b_i z_i^{su}}$, or for z_i^{su} great enough. For other values of the conventional curvature, the model is not smooth at the yield point. The yield stress is defined by, $\tau_i^{su} = R_i^{su} a_i$, where $R_i^{su} < 1$, from where $z_i^{su} = -\ln\left(1 - R_i^{su}\right) / b_i$. New similar exponential smooth models can be finding in Araújo and Costa [2].

Parameter a_i can be estimated from the maximum skin friction stress and a conventional reduction factor, $R \epsilon [0.8, 0.95]$, as $a_i = \tau_i^f / R$. Although, it is more accurate to determine the yield skin friction from field tests, such information is not available at time of design, it can be estimated from instrumented pile test, or by empirical relations such as

$$\tau_i^f = \alpha_i s_{u,i},\ for\ clay, \tag{3a}$$

$$\tau_i^f = \beta_i \sigma_{v,i}^f,\ for\ sand. \tag{3b}$$

In above equations, $s_{u,i}$ is the undrained shear strength, $\sigma_{v,i}^f$ is the total overburden stress at middle of the structural element, and α_i and β_i are related to the undrained shear strength and soil internal friction angle, ϕ_i. The value of b_i can be determined from the conventional expression,

$$b_i = \frac{G_{s,i}}{a_i r_0 ln\left(\frac{r_{m,i}}{r_{0,i}}\right)}, \tag{4}$$

in which $G_{s,i}$ is the shear modulus of the soil layer, $r_{0,i}$ is the radius of the pile shaft, and $r_{m,i} = 2.5L_p(1 - \upsilon_{s,i})$, where L_p is the pile length, and $\nu_{s,i}$ is the Poisson's ratio of the soil layer. The radius $r_{m,i}$ is an estimatimative of the elastic influence of the pile, that is, the distance from the center of pile shaft where the elastic soil vertical deformations induced by the pile are negligible. The elastic deformation of the soil is estimated from

$$z_i^e = \frac{r_{0i}}{G_{s,i}} ln\left(\frac{r_{m,i}}{r_{0,i}}\right)\tau_i = C_i \tau_i, \tag{5}$$

where C_i is the elastic stress compliance of the soil layer. The shear modulus can be determined from overconsolidation ratio or SPT correlations like

$$G_{s,i} = \Omega(N_{SPT})^{\omega}; \tag{6}$$

factors $\Omega = 14.12$ and $\omega = 0.68$ has been used for various soil types and for the shear modulus expressed in MPa. Settlement z_b at pile tip is related to the mobilized tip force by the exponential model,

$$P_b = a_b(1 - e^{-b_b z_b^s}); \tag{7}$$

$z_b^s = z_b - z_b^{vp}$, z_b^{vp} being the current viscoplastic base ground displacement. For soft clayed soil the mobilized tip force is too small to justify better model than pure exponential one dictated by Equation (7). Parameter a_b is obtained by a similar relation used for unit skin friction, that is, $a_b = P_b^f / R$, where the tip force reaction can be expressed by,

$$P_b^f = A_p\left(9s_{u,b} + \sigma_{v,b}^f\right),\ for\ clay, \tag{8a}$$

$$P_b^f = A_p \sigma'_{v,b} N_q^*,\ for\ sand; \tag{8b}$$

$s_{u,b}$ is the undrained shear strength of the base layer, $\sigma_{v,b}^f$ and $\sigma'_{v,b}$ are the overburden stress and effective overburden stress, respectively, at the base of the pile, A_p is the area of the pile tip, and N_q^* is a bearing capacity coefficient. The base soil parameter, b_b, is determined using,

$$b_b = \frac{4G_{s,b} r_{o,b}}{a_b(1-\nu_{s,b})}. \tag{9}$$

For some stiff clay base layer, the strain soft model described by Equations (2) can be useful.

2.3 Elastoplastic models

Elastoplastic models described in this subsection are not time dependent. Settlement z_i^b of the bottom of each structural element, $i \epsilon \{1{:}\,n\}$, are related to the settlement parts at the middle of the element by

$$z_i^b = z_i^e + z_i^s - z_i^{vp} - z_i^c, \tag{10}$$

where z_i^c is the elastoplastic displacement of the compressed lower half of the structural element. Under a finite element quadratic displacement field approximation, z_i^c can be written as

$$z_i^c = \frac{1}{2}\left(P_i^b + 0.5\left(P_i^b + P_i^t\right)\right)\frac{0.5 l_i}{E_{p.i}A_{p,i}} = \left(4P_i^b + A_{s,i}\tau_i\right)\frac{l_i}{8E_{p,i}A_{p,i}}; \tag{11}$$

$P_i^b$and P_i^t are compression forces at bottom and top of the structural element, respectively, and l_i is its length. $E_{p,i}$ and $A_{p.i}$ are the secant Young's modulus and cross section area of the element, respectively, while $A_{s,i}$ is its perimetrical area. Structural steel and reinforced concrete piles are modeled, both with elastoplastic models. Hognestad's nonlinear softening model for concrete is used, and their reinforcements are modeled by a elastic perfectly plastic model. Equation (11) comes from linear finite element approximation for pile normal forces, meaning assumed axial quadratic displacement field, linear axials deformations and internal forces. Time dependent viscoplastic creep deformation through a creep function for concrete can be added to z_i^c in Equation (11). Substituting Equations (2), (5) and (11), into Equation (10), the element base displacement is obtained as a function of the plastic slippage,

$$z_i^b(z_i^s,t) = z_i^s + C_i\tau_i(z_i^s) - z_i^c(z_i^s) - z_i^{vp}(t). \tag{12}$$

Above nonlinear Equation (12) should be solved for z_i^s for each displacement increment at each layer in the incremental solution of the nonlinear two-point boundary problem by an iterative Newton-Raphson method. The first derivative of the function $f(z_i^s) = z_i^b(z_i^s,t)$ sought at each NR-iteration for, a fixed time t, is

$$f'(z_i^s) = f_1'(z_i^s) = 1 + a_i b_i D_i e^{-b_i z_i^{su}},\ \ for\ z_i^s < z_i^{su} \tag{13a}$$

$$f'(z_i^s) = f_2'(z_i^s) = 1 + a_i r_i D_i e^{-r_i(z_i^s - z_i^{su})},\ for\ z_i^s > z_i^{su} \tag{13b}$$

$$where\ \ D_i = C_i - \frac{A_{s,i}l_i}{8E_{p,i}A_{p,i}}. \tag{13c}$$

Computing above derivatives, it could be happening that $z_i^s = z_i^{su}$, a zero-probability event. In this case, a local subgradient, $g(z_i^s)$, of $f(z_i^s)$ should be used. It appears that the canonical subgradient, $g(z_i^s) = \frac{1}{2}\left(f_1'(z_i^s) + f_2'(z_i^s)\right)$, works well. However, it is noteworthy that the modified NR-method, in which the derivative is kept constant during some or all the iterations, works nearly equally well. For this last case, $f'(0) = 1 + D_i a_i b_i$ has been successfully used.

2.4 Pile groups

Present formulation can be generalized to model interaction between piles in a pile group through elastic settlement field in two ways: displacement induced by neighboring loaded piles, and the deformation caused by the reinforcement effect of adjacent piles, which can reduce the elastic settlement around the pile. Considered these two more elastic effects, the elastic compliance of Equation (5) can be rewritten as,

$$C_i = \frac{r_{0.i}}{G_{s,i}} ln\left(\frac{r_m}{r_{0.i}}\right) + \sum_{j=1,j\neq i}^{n} \frac{r_{0,i}}{G_{s,i}} ln\left(\frac{r_m}{r_{ij}}\right) - \sum_{j=1,j\neq i}^{n} \frac{r_{0,i}^2}{G_{s,i} r_{ij}} ln\left(\frac{r_m}{r_{ij}}\right), \tag{14}$$

where r_{ij} is the distance between axis of pile i and axis of n other pile j. Equation (14) assumes that piles have same depth and same equivalent diameter. Equation (14) can be used too for piles out current group.

2.5 Viscoplastic model

Strength and settlement of long-term load application in bored or driven piles can be predicted when drained properties are used instead of undrained properties. Better results for clayed soils are obtained if primary consolidation, secondary compression, and creep are considered. A parallel work is being done that take the effects of both, primary consolidation, and soil ageing (secondary compression and creep) to determine the long-term soil strength and load carrying capacity of pile and pile groups, using a well-accepted elastic-viscoplastic model. It uses an expression for time dependent quasi consolidation ratio to determine the relationship between soil strength and stiffness with time. This work is mainly interested in the ageing effect part, $z^{vp}(t)$, of the observed (measured) caps settlements along time because this creep deformations are not followed by increases in loads. Direct use of observed total displacement in the load-transfer method to infer caps load would be in serious error. Viscoplastic displacement plots vertically in the $p' - V$ plane state, where p' is the mean effective stress and V is the specific volume. The isotropic normal compression states $p_m' - V_m$, beginning after the primary consolidation occurring after a time t_0, when the excess of pore-water pressure is insignificant, forms the isotropic normal compression line (*iso-ncl*). This line has the functional equation in the semi-log plane, $V_{ncl}(p'_m)$= $V_{ncl}(p'_m = 1) - \lambda ln p'_m = N - \lambda ln p'_m$, where λ is the slope of the *iso-ncl* line. Secondary compression, initiating at time t_0 corresponding to the end of primary consolidation, and developing under constant stress, is a purely viscoplastic phenomenon called creep by the civil engineering community. The development of creep in time follows a vertical line called secondary consolidation line (*scl*) that plots vertically in the compression plane (deformation without stress increases). Adopting a logarithm creep function, $\Phi(t_0, t)$, the points in the vertical plane are moving along time over the vertical secondary consolidation line (*scl*), starting with $V_m = V_{ncl}$ at $t = 0$. The *scl* functional equation becomes

$$V_m(p'_m) = V_{ncl}(p'_m) - \psi\Phi(t_0, t) = V_{ncl}(p'_m) - \psi ln \frac{t+t_0}{t_0}, \tag{15}$$

where the creep parameter ψ is a material constant that is independent of stress state or overconsolidation ratio. Equation (14) can be cast in incremental form as,

$$\delta V_m^{vp} = \frac{\partial V_m^{vp}}{\partial t}\delta t = -\psi \frac{\partial\Phi(t_0,t)}{\partial t}\delta t = -\frac{\psi}{t+t_0}\delta t. \tag{16}$$

The incremental viscoplastic volumetric strain can then be obtained as

$$\delta\varepsilon_p^{vp} = \frac{\delta V_m}{V_m} = \frac{\psi}{V_m(t+t_0)}\delta t, \tag{17}$$

from which the viscoplastic volumetric strain rate takes the form

$$\dot{\varepsilon}_p^{vp} = \frac{\delta\varepsilon_p^{vp}}{\delta t} = \frac{\psi}{V_m(t+t_0)}. \tag{18}$$

This equation shows that viscoplastic volumetric strain rate slowly converges to zero at infinite, as is the viscoplastic increments given by Equation (17), because of the reduction of specific volume in time becomes negligible. Equations (17) and (18) form the basis for the determination of the viscoplastic displacements of each soil layer. Structural element embedded in clayed soil layer suffer an incremental rigid body displacement at a given δt by

$$\delta z_i^{vp} = l_i\delta\varepsilon_p^{vp} = \frac{\psi}{(t+t_0)}\frac{l_i}{V_m}\delta t. \tag{19}$$

The viscoplastic displacement is obtained by integration of above equation in time.

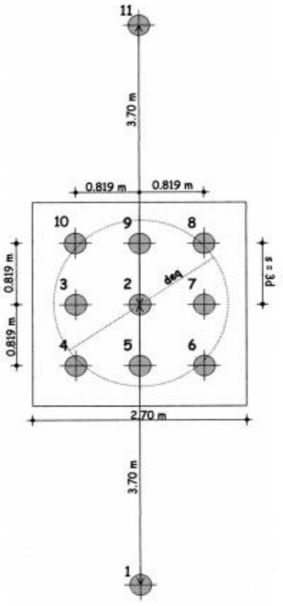

Figure 1. Configuration of 9 pile group and the 2 auxiliar piles

3 Applications

In this section, the simulation of a single pile load and pile group test reported by O'Neill, Hawkes and Mahar [5] is done for validation of the proposed method. Figure 1 shows 9 caped piles and 2 reference piles. Table 1 shows soil layers properties, original parameters and modified a parameter by Boonyatee and Lai [3] based on final observed load test. All piles are closed end steel pipe in stiff over-consolidated clays, with an external diameter of 274 mm, a wall thickness of 9.3 mm, and were driven to a penetration of 13.1 m. The 9 piles connected by concrete cap were installed at a spacing of $6r_0$, while each one of the two remaining piles was located at 3.7 m from the center of the group in opposed sides. The Young's modulus of the steel was taken as $210\ GPa$. The shaft stress capacity of the soil was taken as $\tau^f = 19\ kPa$ at the surface, increasing linearly to $93\ kPa$ at the pile base, as recommended by the authors of the test. The average value of the shear modulus was taken as $99.45\ MPa$ and the Poisson's ratio assumed to be 0.35. The stress reduction factor of R=0.90 was used. Table 1 shows other required soil parameters. The isolated pile 1 was chosen for simulation.

Table 1 – Soil profile and analysis parameters

Strata	**Depth(m)**	$\tau^f\ (kPa)$	**Original parameters**		**Modified parameters**	
			$a\ (kPa)$	$b\ (1/m)$	$a\ (kPa)$	$b\ (1/m)$
Silty clay	0-9.15	35.1	39	197.2	42.9	197.2
Silty clay	9.15-12.45	53.1	59	208.8	64.9	208.8
Silty	12.45-17.25	45	50	166.9	55	166.9
Fine sand	17.25-27.46	57.6	64	353.8	70.4	353.8
Silty	27.46-35.50	57.6	64	347.1	70.4	347.1
Silty	35.50-47.60	62.1	69	608.7	75.9	608.7
			1344.0*	150.0**	1344.0*	150**

* a_b for base layer; ** b_b for base layer

Figure 2 shows simulations done in this work for a single pile and the 9-pile group in Fig. 1, using the classical exponential model and experimental results extracted from reference [5], and from a partial tabulated data in Castelli and Maugeri [6]. Simulating the 9-pile group accounts for mutual influence of all 11 installed piles. Measurements are from [5], as reported by [6].

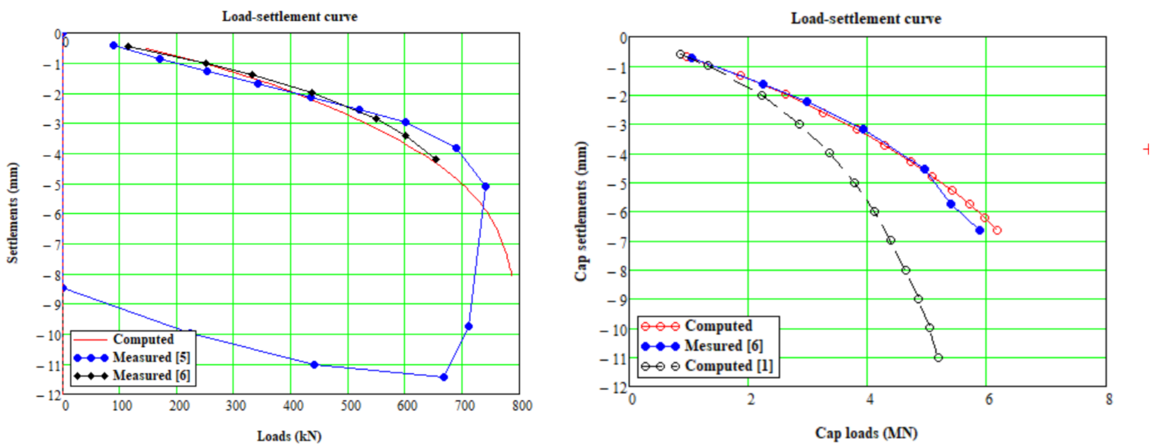

Figure 2. Experimental and simulation results using conventional exponential model

Careful observation of the simulation of a single pile indicates that the yielding of some layers being certainly the main responsible for the suddenly lost of capacity showed in Fig. 2. Conventional exponential model cannot model soft behavior. Results using the new proposed exponential model is showed in Fig. 3, after some minor adjustment of the free parameters: conventional curvature radius, r_i , and residual stress, τ_i^r.

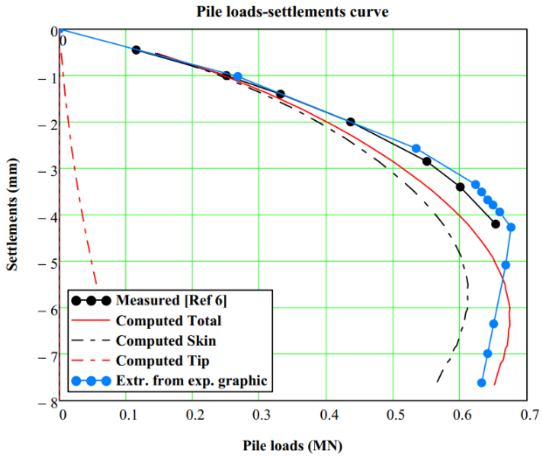

Figure 3. Experimental and simulation results using the new proposed exponential model

4 Conclusions

A load-transfer method for determining the load-settlement curve of a single pile and pile group under axial loading is presented. The proposed method was validated by comparing with an isolated single pile and a 9 piles group tested and analyzed in literature. Computed results in this work show good agreement with tests and are at least as accurate as literature simulation results. A new exponential model for capturing strain hardening and softening that is more flexible than well-known exponential and hyperbolic models are introduced. Formulation for incremental settlement with clayed soil ageing during secondary compression and creep under constant loading are present too.

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References

[1] Q. Q. Zhang et al, “Simplified non-linear approaches for response of a single pile and pile groups considering progressive deformation of pile-soil system”. *Soil and Foundations”*, vol. 56, n. 3, pp. 472-484, 2016.
[2] E. da R. Araújo, Victória F. R. da Costa, “Generalized and flexible nonlinear exponential models for strain softening and hardening description of skin friction for axially loaded pile”, submitted, 2021.
[3] T. Boonyatee, Q. V. Lai, “A non-linear load transfer method for determining the settlement of piles under vertical loading”, *Intern. J. of Geotechnical Engineering,* vol. 14, n. 2, pp. 206-327, 2020.
[4] C. Kelln et al, “An improved elastic-viscoplastic soil model”, *Can. Geotech. Journal,* vol. 45, pp. 1356-1376, 2008.
[5] M. W. O’Neill, R. A. Hawkins, L. J. Mahar, “Load-transfer mechanism in piles and pile groups”, *Journal of Geotech. Engineering,* vol. 108, n. 12, pp. 1605-1623, 1982.
[6] F. Castelli, M. Maugeri, “Simplified nonlinear analysis for settlement prediction of pile groups”, *Journal of Geotech. Geoenviron. Eng.,* vol. 128, n. 1, pp. 76-84, 2002.