Mixed-integer Optimization under Uncertainty in Reservoir Development and Management

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Abstract. Reservoir geoengineering is usually faced with large-scale optimization problems under uncertainty arising as part of development planning of smart wells locations, performing separated, jointly, or simultaneously optimization of well locations and control rates of water injection and hydrocarbon productions. This paper performs a simultaneous optimization of well locations and production rates under geological uncertainty using Monte Carlo samples over geostatistical realizations. Those optimization problems are of mixed-integer type. Traditionally, they have been solved by performing projections between real and integer variables using different strategies, Whitney and Hill [7]. This work investigates the performance of DSPSA, a discrete version of SPSA, recently described in Wang and Spall [13], and proposes a discrete variant to be applied in mixed-integer problems where all control variables are ceiling round, taking advantage of practical field implementations. One-sided deterministic constraints are imposed to reduce search space. For more general one-sided stochastic non-linear constraints, see Fonseca [3] and Fonseca et al. [9]. In the class of reservoir problem solved in this paper, functional and constraints derivatives a never available, mainly because industry solves the reservoir simulation problem using commercial software as a black box. Additional metaheuristics are used to construct the discrete version of DSPSA. This work makes a preliminary comparison between the new discrete version, DSPSA-R, with SPSA-Z, the mixed-integer version of SPSA in Fonseca [3].

Keywords: Mixed-integer optimization, Stochastic optimization, SPSA, DSPSA, well location, control rate.

1 Introduction

The research to be described proposes an optimization methodology for optimal reservoir development management and planning of smart wells locations and production optimization under geostatistical uncertainty, performing a simultaneous optimization of well locations and production rates in a water injection petroleum reservoir exploitation. The simultaneous strategy, first proposed in Fonseca [3], efficiently opposes to industry and academic conventional joint strategy that uses cycles of wells locations optimization followed by rate optimization, Shirangi [4].

Simulation-optimization techniques are used together with gradient-free optimization algorithms that execute high fidelity reservoir physics from any commercial reservoir simulator. Traditional surrogate-based objective functions techniques suffer from the well-known problems of accuracy, efficiency, and robustness whenever trying to estimate approximate derivatives of surrogate models. Instead, direct function measurements are used to make direct stochastic approximations of gradient functions.

SPSA - simultaneous approximation stochastic algorithm - is especially efficient in high-dimensional problems in terms of providing a good solution for a relatively small number of objective function evaluations, He et al. [1] and Spall [2]. This is confirmed by numerical experiments in Fonseca [3]. SPSA is an appropriate algorithm for use in random noise environments, as are reservoir geostatistical descriptions combined with stochastic direct measurements. These environments are typical in reservoir closed-loop control.

Shirangi [4] remarked a tendency to perform joint optimization of well locations and control. Because studies have reported that a sequential approach, i.e., optimizing first well locations then well rates, often yields suboptimal solutions compared to joint optimization. To perform the simultaneous or joint optimization, usually, production rates are treated as continuous variables and well locations as integer variables. In other words, this problem belongs to the mixed-integer optimization class problem. For such problems, neither combinatory methods nor classic methods of optimization with gradients and Hessians are suitable.

Whereas SPSA was developed for continuous optimization problems, to solve discrete optimization problems, several adapted versions have been proposed. Fu and Hill [5] applied SPSA over discrete systems, where integer variables were projected in the continuous domain. Gerencser and Hill [6] show a discrete SPSA version defining the gain parameter, a, as a fixed gain parameter instead of the original sequence a_k.

Whitney and Hill [7] solved constrained optimization problems over discrete sets via SPSA. Within a domain of interest, treated as a grid of points with discrete-value coordinates, they proposed three discrete methods that differed among themselves by the way control variables are updated, as shown further up. Other forms of projection techniques can be seen in Brooks [8], where the author tested six SPSA versions to solve discrete resource allocation problems. These versions were different from each other in aspects related to how to obtain stochastic gradient $\hat{g}(\theta_k)$, update control variable, θ_{k+1} and parameters c_k and a_k.

We revisit the simultaneous well placement and production rates optimization problem solved in Fonseca [9], where integer part of control variables are wells positions, supposed to be block-cantered, and the continuous part of control variables are injection and production rates. Applications in that work were made with a unique cycle after projections, Whitney and Hill [7], of integer variables in the continuous field, immediately before calling the black-oil model simulator, commercial software IMEX [10].

This work makes a preliminary comparison between the new discrete version, DSPSA-R, with SPSA-Z, the mixed-integer version of SPSA in Fonseca [3], taking advantage of practical constraints on the field implementation of optimization output. The methodology is fully coded in MATLAB [11].

2 Methodology

Algorithms that make stochastic approximations (SA) are suitable for problems that consider the inherent uncertainties of real issues. SPSA is an algorithm of stochastic nature that uses only two objective function measurements by iteration to make a stochastic gradient approximation of the objective function. Additionally, its performance is independent of the control variable's number. This characteristic allows a significant reduction in the optimization cost, mainly in problems with a high number of variables. Applications of this algorithm with hundreds or even thousands of control variables are shown in Spall [12], Fonseca [3]. The implementation of SPSA for unconstrained optimization, in its basic format, is given in Spall [2] and Spall [12]. We are aware of the relative efficiency of solvers that uses derivatives and adjoints of a smooth and convex objective function. However, the general set of problems solved here are neither smooth nor convex.

2.1 SPSA basic format for unconstrained and continuous optimization, Spall [2]

Step 1 Initialization and coefficient selection. Set counter index $k = 0$. Pick initial guess $\theta_0 \in G$ and nonnegative coefficients $a, c, A, \alpha,$ and γ. Usually, $\alpha = 0.602$ and $\gamma = 0.101$; $a, c,$ and A may be determined based on the practical guidelines given in Spall (2003).

Step 2 Generation of simultaneous perturbation vector. Generate by Monte Carlo method a p-dimensional random perturbation vector using a Bernoulli ± 1 distribution with probability 1/2 for each ± 1 outcome.

Step 3 Objective function evaluations. Obtain two objective function measurements $y(\theta_k + c_k\Delta_k)$ and $y(\theta_k - c_k\Delta_k)$.

Step 4 Gradient Approximation. Generate the SP approximation to the unknown gradient according to eq. (1)

$$\hat{g}_{k,l}(\theta_k) = \frac{y(\theta_k + c_k\Delta_k) - y(\theta_k - c_k\Delta_k)}{c_k\Delta_{k,l}} . \tag{1}$$

Average several gradients estimations at θ_k, if asked by the user. Benefits will be especially apparent if noise effects are relatively large.

Step 5 Update $\boldsymbol{\theta}$ estimate. Update θ_k through stochastic approximation equation,

$$\theta_{k+1} = \theta_k - a_k \hat{g}(\theta_k). \tag{2}$$

Then check if θ_{k+1} is in the feasible domain.

Step 6 Iteration or termination. Return to step 1 with $k+1$ replacing k. Terminate the algorithm if there is little change in several successive iterates or if the maximum allowable number of iterations has been reached.

2.2 SPSA versions with projection techniques.

Whitney and Hill [7] proposed three discrete methods to estimate θ_{k+1}, to deal with continuous variables in SPSA over discrete sets.

Method 1: $\theta_{k+1} = \theta_k - a \cdot round\big(\hat{g}_k(\theta_k)\big)$. Upon convergence, the final values of θ should be the next integer, given by $\theta_{final} = round(\theta_k)$.

Method 2: θ_{k+1} is obtained in a similar manner of the first method, except that the constant gain, a, is included in the rounding operation: $\theta_{k+1} = \theta_k - round\big(a \cdot \hat{g}_k(\theta_k)\big)$. In this case, upon convergence, θ values are already discrete.

Method 3: at each iteration, the entire parameter estimate is rounded either up or down to the nearest discrete value, i.e., $\theta_{k+1} = round\big(\theta_k - a \cdot \hat{g}_k(\theta_k)\big)$. According to the authors, this method had achieved better results in terms of efficiency and convergence.

2.3 DSPSA, the discrete SPSA.

Wang and Spall [13] described the DSPSA algorithm, for a θ p-dimensional vector, with $p = 1,2,3,\ldots$. They considered objective function noise measurements, $y = L + \varepsilon$, where $L: \mathbb{Z}^p \to \mathbb{R}$ and ε is noise. With the following steps:

Step 1 Initialization and coefficient selection. Set counter index $k = 0$. Pick initial guess θ_0 .

Step 2 Generation of simultaneous perturbation vector. Generate by Monte Carlo a p-dimensional random perturbation vector using a Bernoulli ± 1 distribution with probability 1/2 for each ± 1 outcome. $\Delta_{\boldsymbol{k}} = \big[\Delta_{k1}, \Delta_{k2}, \ldots, \Delta_{k1p}\big]^T$.

Step 3 Domain perturbation Consider $\boldsymbol{\pi}(\theta_k)$ is the middle point of a unitary hypercube, and $\lfloor\theta_k\rfloor = \big[\lfloor\theta_{k1}\rfloor, \ldots \lfloor\theta_{kp}\rfloor\big]^T$, then calculate

$$\boldsymbol{\pi}(\theta_k) = \lfloor\theta_k\rfloor + 1_p/2\ , \tag{3}$$

where 1_p is a p-dimensional vector with all components being unity, and $\lfloor\ \ \rfloor$ is a floor function operator.

Step 4 Objective function measurements and gradient Approximation. Evaluate y at $\boldsymbol{\pi}(\theta_k) = \lfloor\theta_k\rfloor + \Delta_k/2$ and $\boldsymbol{\pi}(\theta_k) = \lfloor\theta_k\rfloor - \Delta_k/2$, and estimate $\hat{g}(\theta_k)$.

$$\hat{g}_{k,l}(\theta_k) = \Big[y(\boldsymbol{\pi}(\theta_k) + \tfrac{1}{2}\Delta_k) - y(\boldsymbol{\pi}(\theta_k) - \tfrac{1}{2}\Delta_k)\Big]\Delta_k^{-1}. \tag{4}$$

Step 5 Update $\boldsymbol{\theta}$ estimate. Update θ_k through stochastic approximation equation, eq. (5)

$$\theta_{k+1} = \theta_k - a_k \hat{g}(\theta_k). \tag{5}$$

Step 6 Iteration or termination. Return to step 1 with $k+1$ replacing k. After the maximum number of allowed iterations, N, set the approximated optimal solution.

One should observe in this algorithm that control variables are intrinsically reals. Because of that, in step 3, they are truncated. But at the updating step, θ_{k+1} is a continuous variable. It should observe that DPSA does not use c_k parameter to perturb θ_k in the domain, instead, perturbations are centered in $\lfloor\theta_k\rfloor$ by $\pm 1/2$.

In this work, we introduce a DSPSA version called DPSA-R, where the letter "R" refers to rounding the updated estimate. Because both types of variables rates and well locations were treated as integers, only the product $a_k \hat{g}(\theta_k)$ is in the real domain. This has been solved by rounding the estimate updated in step 5, using eq. (6)

$$\theta_{k+1} = ceil[\theta_k - a_k \hat{g}(\theta_k)]. \tag{6}$$

3 Discussion and Results

3.1 Reservoir under geostatistical uncertainties

The methodology is applied on a stochastic version of a 2D-synthetic and deterministic oil reservoir model, Case 1 in Oliveira [14]. The reservoir was exploited with two producers, P1 and P2, and a water injector, I-1, for 16 years.

Figure 1 gives a view of the stochastic version, named Case 1S, for which a set of 1000 reservoir geostatistical realizations in the software Builder [15]. Permeability realizations were generated conditioned to hard data of the three wells. Both well producers can produce individually up to 12 m³/d of liquids. However, to explore the optimization algorithm to its full potential, the capacity of processing liquids in the production station was limited to 16 m³/d. The reservoir was modeled with a 51×51×1 grid with cells of 10×10×4 m comprising 2601 cells. All other reservoir properties are assumed to be deterministic. See Oliveira [14] and Fonseca [3] for a complete reservoir description.

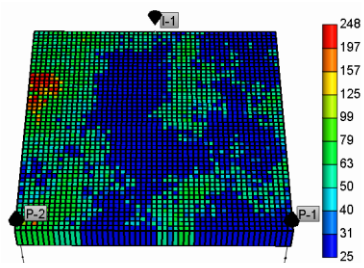


Figure 1. Realization number 10 of horizontal permeability generated.

3.2 Dynamic allocation of the production rates

Under the production and economics aspects, the objective function for optimization problems of exploitation in oil reservoirs is the Net Present Value (NPV). An economical package developed at Oliveira [14] was used to compute the NPV. The control variables are the dimensionless ratios given by

$$x_{p,t} = \frac{q_{p,t}}{\sum_{p \in P} q_{p,t}}, t = 1 \ldots n_t, \tag{7}$$

where $q_{p,t}$ is the rate (m³/d) of the well p in the interval of time t, n_t is the number of total time intervals of the production operation, and $\sum_p q_{p,t}$ is the total rates of the wells in the interval t, the same as the capacity of the production station. The control variables are the rates of well P1 during exploitation time, as the P2 well rates are secondary variables obtained through subtraction of the P1 well rate from the total rate production station. Which is put to produce at its full capacity. Upper $x_{p,t}^u$ and lower $x_{p,t}^l$ bounds constraints are imposed to production wells,

$$x_{p,t}^{\ell} \leq x_{p,t} \leq x_{p,t}^{u}, \tag{8}$$

where $x_{p,t}^{\ell} = 0.25$ and $x_{p,t}^{u} = 0.75$ in the current example. As the optimization process takes place, the set consisting of the NPV of each simulated realization is used to compute the Expected Net Present Value (ENPV) and other desired summary statistics using the Monte Carlo method. Simulations of each reservoir realizations can be distributed across cluster processors to reduce the computational cost. The standard problem of stochastic optimization is established as in April et al. [16]:

$$\max_{x_{p,t} \in R^n} ENPV(x_{p,t}, u). \tag{9}$$

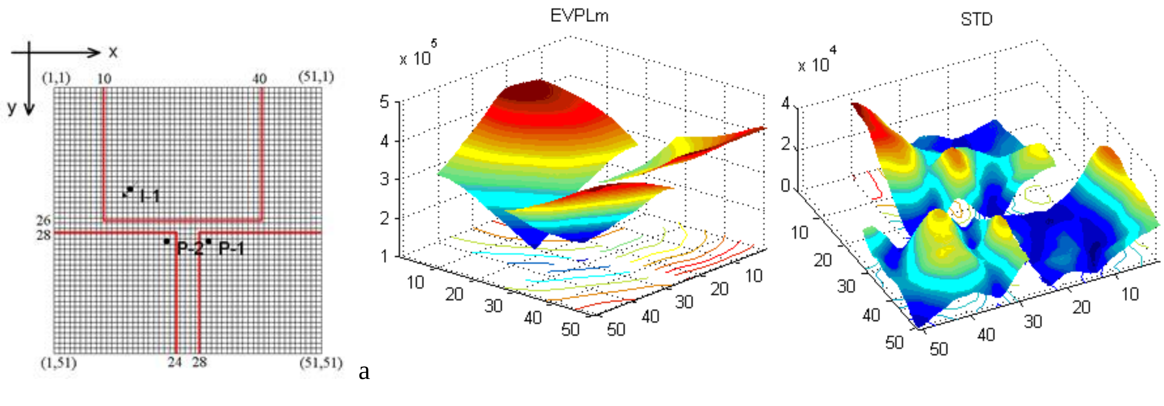
3.3 Simultaneous optimization of well placement and production rate

In this section, it is performed a joint optimization of well locations and production rates. Figure 2 shows the reservoir grid, described by Cartesian coordinates, X and Y. During the optimization process, wells can only be located at a block center. Usually, a well should be located at preferential regions, selected by geoengineers.

Boundaries of the feasible regions for each well can be seen in Fig. 2; also, it shows the arbitrary selected initial position for the injector and producers.

To obtain the maximum NPV which can occur during the time horizon of production, Bangerth et al. [17] divided the total exploitation time into sub-periods $T_1 < T_2 < T_3 < \ldots < T_f$. Where T_f is the final time of exploitation and performed a set of optimization problems equal to the number of sub-periods. In this study, the chosen objective function is the maximum ENPV during the time of exploitation. This maximum is obtained by solving a unique optimization problem in which the series of Bangerth's sub-problems was eliminated. Also, time was implicitly included as a control variable. Details of this technique were shown in Fonseca [3].

Because the locations of the three wells were simultaneously optimized, it was not possible to draw the multidimensional ENPV surface. Figure 2(a) gives us an idea of this problem's complexity, where two wells were fixed at their original positions, and the third one was moved to all feasible locations. Results of these simulations are depicted in Fig. 2(b), which shows the ENPV surface and the standard deviation, STD illustrated in Fig. 2(c). One should observe that possible feasible regions (low risk) are disconnected.



(a) (b) (c)

Figure 2. Initial positions of the wells and their limit borders of locations, (a). Response surfaces with three geostatistical realizations of ENPV (b) and (c) of the STD, obtained by changing the positions of one well at a time along with coordinates X and Y.

Table 1 shows some results for this application, where T1 and T2 are P1 rates for two control steps. Run (1) considers all wells fixed on their original positions to have a ENPV reference value. Except for runs (4) which started with rates T1 and T2 equal to 50%, all tests started with 25% of production station rate for well P1, for both steps of control. Moreover, Tab. 1 shows the final well positions and final producer P1 rates, at maximum ENPV, achieved at k-th iteration. Table 2 shows us the gain parameters corresponding to runs in Tab. 1.

Table 1. Results of mixed-integer optimization using three geostatistical realizations. The first run optimized only production rates with wells in original locations.

Run #	Algorithm version	N	ENPV$_0$ (E4)	ENPV (maximum) (E4)	k	P1		P2		I		Rate (%)	
						X	Y	X	Y	X	Y	T1	T2
(1)	SPSA	200	37.28	40.87	65	51	51	1	51	26	1	54.0	25.0
(2)	SPSA-Z	390	7.13	42.32	148	51	51	24	51	20	2	56.8	67.6
(3)	SPSA-Z	712	6.38	41.74	58	51	51	24	51	22	3	54.1	72.8
(4)	SPSA- Z	1000	5.52	42.06	128	49	51	24	51	18	6	75.0	42.1
(5)	DSPSA-R	300	7.13	42.59	52	46	51	24	51	19	3	65.0	61.0
(6)	DSPSA-R	270	7.13	42.62	112	48	51	24	51	20	1	75.0	75.0
(7)	DSPSA-R	230	7.13	42.39	33	46	51	22	51	22	3	55.0	46.0

Table 2. Gain parameters of algorithms SPSA and DSPSA used in the runs.

Run	Version	Gain Parameters N	c	a	A
(1)	SPSA	200	4	0.02	10
(2)	SPSA-Z	1000	14	0.05	30
(3)	SPSA-Z	712	14	0.05	30
(4)	SPSA-Z	390	14	0.0844	30
(5)	DSPSA-R	300	-	0.005	30
(6)	DSPSA-R	270	-	0.01	30
(7)	DSPSA-R	230	-	0.003	30

Figure 3 compares ENPV curves along with iterations for the best runs of SPSA-Z and DPSA-R. In Tab. 1, they correspond to runs #2 and #6, respectively. Again, we can see that DPSA-R has achieved the best value of ENPV at iteration 112 and is faster than SPSA-Z.

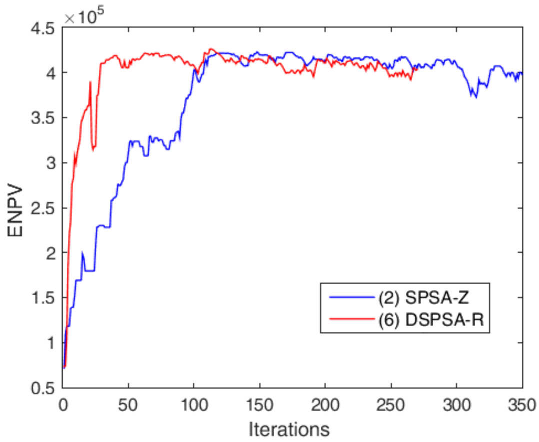


Figure 3. ENPV versus iterations for SPSA-Z and DPSA-R best runs of simultaneous optimization of production rates and well locations. Curves correspond to runs #2 and #6 in Tab. 1, respectively.

Figure 4 shows well positions at the highest ENPV values during optimization. Figures were numbered according to the cases in Tab.1, which offers a summary for the simultaneous optimization with three realizations. Well settings in (2) and (3) of Fig. 5 show that simultaneous optimization of well placement and rates production has modified the original positions of wells producers.

Additionally, the optimized well configurations show a tendency of producers P1 and P2 be located on the right-bottom of their feasible region. Fonseca et al. [9] had studied this case with 3, 10, 30, 50 realizations, and the results showed the same tendency for any number of realizations.

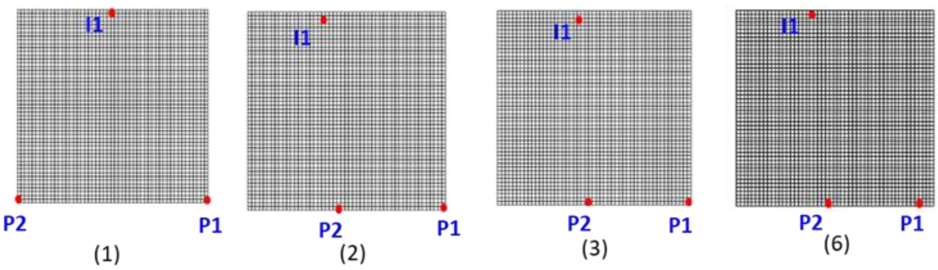


Figure 4. Wells arrangement after optimization. (1) original positions, (2) and (3) optimization with SPSA-Z version, and initial rates 25% and 50%, respectively. (6) the best optimization with DPSA-R version and initial rates at 25%.

4 Conclusions

For the integer-mixed type of optimization, the literature used to perform two sequential cycles: one external cycle with respect to the integer variables and an internal cycle with respect to the continuous ones, or the reverse. This work shows a simultaneous optimization of well placement and production rates in a bidimensional petroleum reservoir.

Comparing performances of SPSA-Z and DSPSA-R versions to solve the simultaneous optimization, results showed that both algorithms achieved similar well positions and ENPV values, but with DSPSA-R running faster.

Relating to production rates, it was not possible to observe a tendency in their values for optimal values of ENPV, regardless of the kind of set, continuous or integer. We should investigate this fact more.

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