



A Particle Swarm Optimizer for the Optimal Allocation and Sizing of Distributed Generators in Distribution Grids

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Abstract. Recently, there is a strong worldwide trend for renewable energy sources to reach increasingly higher levels of penetration, mainly connected at the level of distribution grids. In this paper, a Particle swarm optimization (PSO) have been applied to allocate and size the Distributed Generator (DG) in distribution grids. The proposed algorithm proved to be an excellent tool to identify the places and size of DG in electric power distribution systems. Tests are carried out on real large scale 141-bus system of AES- Venezuela. A detailed performance analysis demonstrate the effectiveness and robustness of the proposed method to find good solutions with reduce power losses and improvement the voltage profile. For three DG, the proposed algorithm was able to find a solution that reduces network losses by more than 95%.

Keywords: Allocation, Distributed Generation, Particle Swarm Optimization, Distribution Grids, Optimization.

1 Introduction

In recent years, the electric power systems have been changed [1]. The concept of Distributed Generation (DG) has arise to complement or replace the conventional model of energy production by integrating small generation units into the electrical grids, gaining special attention due to its several technical, economic and environmental advantage [2, 3]. According to [4], technical benefits involve reducing power losses in lines, improve the voltage profile, and increase the overall power system performance. In economic view, can reduce the on-peak operation cost [5]. Above all, DG contribute to decrease greenhouse gas emissions [6]. However, the inappropriate placement or size of DG can lead to elevation of system losses [7, 8]. The problem of Optimal Placement of Distributed Generation (OPDG) is a renowned topic usually addressed through Mixed-integer Nonlinear Programming (MINLP), involving discrete and continuous variables [9]. In the specialized literature, several techniques have been applied to DG placement divided in traditional and approximate methods. Among traditional methods, an analytical approach is used by [10] for DG placement. In [11], a mixed-integer linear programming approach is used for multiple DG types. Others numerical and analytical methods can be found in an interesting review of [8, 12]. As reported by [13], the drawback of traditional methods is low computational effort and accuracy of the solution, however, the simplification of the problem may threaten the accuracy of the solution when the problem incline to be complex such as multiple DG placement and constraints. Recently, evolutionary methods have been used for OPDG problem, i.e, approximate techniques. These algorithms are efficient of take into account several types of single or multiple objectives. However, the solutions are not be the optimal [14]. Methods based on Computational Intelligence (CI) same as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Coyote Optimization Algorithm (COA) are advocated in [15–17]. In [18], the authors proposes the Sine Cosine Algorithm (SCA) associated with Loss Sensitivity Factor (LSF) for DG placement. In [19], the authors suggested the Equilibrium Optimizer (EO) for different types of generators and tests are carried out on 69-bus and a real 94-bus from Portugal. Certainly, is an explosion of studies conducted by meta-heuristics in this relevant problem [20–23].

From analysis review, a few studies have been considered to place DG in a real systems [19, 24, 25]. In this sense, we propose a methodology based on PSO for DG placement in a practical large scale system. The proposed method is tested on real 141-bus radial distribution from Caracas Venezuela. The results shows the PSO is an

excellent tool for DG placement offering good solutions in relation of power losses.

2 Distributed Generation and Problem Formulation

DG has changed the paradigm of energy generation around the world, the use made us have in practice new approaches and new purposes for the installation of distributed generation, such as having a new or larger coverage area to control how variations in power transmission, such as minimizing losses, improving the voltage profile level and maintaining the stability of the system [26]. DG can be modeled in different ways, either with Active and Reactive Power can be subdivided [27] into:

- Active energy producers, such as photovoltaic arrays, micro-turbines;
- Reactive energy producers, such as synchronous compensators;
- Active and reactive energy producers as synchronous generators;
- Active energy producers and reactive energy consumers such as fixed speed wind turbines.

In addition, DGs are also classified according to their power and technology [28], as can be classified as Micro DG between 1W up to 5kW, Small DG are between 5kW up to 5MW, Medium DG are between 5MW up to 50MW, Large DG are between 50MW up to 300MW.

According to [29], the most studies focus on DG placement have been done for reduce active power losses. In this respect, the objective function has been designed for minimize power losses in lines and improve the voltage profile as in [25], as in eq. (1):

$$\min \sum_{l=1}^{N_L} P_{losses,l} \quad (1)$$

where $P_{losses,l} = g_{km}(V_k^2 + V_m^2 - 2V_k V_m \cos \theta_{km})$ are the active power losses in line l . The voltage magnitudes in buses k and m are given by V_k and V_m ; g_{km} is the conductance between buses k and m ; θ_{km} is the corresponding angular difference; and N_L is the number of lines of the system.

The objective function eq. (1) is subject to the constraints given as follows:

Load flow constraints

Equations (2) and (3) represent the active and reactive power balance, respectively.

$$P_{gk} + P_{DGk} - P_{lk} = V_k \sum_{m \in \kappa} V_m (G_{km} \cos \theta_{km} + B_{km} \sin \theta_{km}) \quad (2)$$

$$Q_{gk} + Q_{DGk} - Q_{lk} = V_k \sum_{m \in \kappa} V_m (G_{km} \sin \theta_{km} - B_{km} \cos \theta_{km}) \quad (3)$$

where P_{gk} and Q_{gk} , respectively, represent the active and reactive power injections existing in the network. P_{DGk} and Q_{DGk} are, respectively, the active and reactive power generated by allocated DG at bus k ; κ represents the set of buses m connected to bus k ; P_{lk} and Q_{lk} are the active and reactive power load in the same bus; and G_{km} and B_{km} represent the real and imaginary parts of the k - m element of the network admittance matrix ($Y = G_{km} + jB_{km}$).

Equality equations (2) and (3) are evaluated in solving the LF, using Matpower [30].

Bus voltage constraints

Inequality (4) represents the minimum and maximum limits imposed on voltage V_k . The voltage limits established by Brazilian Agency Regulator (ANEEL) were considered, in which the limits are 0.93 p.u. up to 1.05 p.u.

$$V_k^{min} \leq V_k \leq V_k^{max} \quad (4)$$

Constraint (4) is evaluated based on the LF solution, if not satisfied, the objective function is penalized by adding a value to the objective function, so that the function is not attractive.

DG constraints

Inequalities (5) express the minimum and maximum limits of the active and reactive power injected at location k by DGs. The Power of DG inserted in the distribution system varies from 0 to 4 MW/MVAR.

$$P_{DG}^{min} \leq P_{DGk} \leq P_{DG}^{max} \quad \text{and} \quad Q_{DG}^{min} \leq Q_{DGk} \leq Q_{DG}^{max} \quad (5)$$

Constraints in inequalities (5) are evaluated while updating the variables k , Equation (7). If a limit is violated, the variable returns the value of the boundary (upper or lower limit).

3 Solution Approach

3.1 Particle Swarm Optimization

The PSO is a stochastic algorithm, whose optimization technique is based on swarm, which was originally proposed by Kennedy and Eberhart [31]. Each particle $X_i = (X_{i1}, X_{i2}, \dots, X_{in})$ in the PSO is a candidate solution evolving in the search space with velocity (or rate of change) $V_i = (V_{i1}, V_{i2}, \dots, V_{in})$. During the iterative process, the position and velocity of a particle i are given by (6) and (7), respectively:

$$V_i^{t+1} = wV_i^t + rand_1 C_1 (P_i - X_i^t) + rand_2 C_2 (P_g - X_i^t) \quad (6)$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \quad (7)$$

$$w = W_{max} - \frac{(W_{max} - W_{min}) * It}{I_{max}} \quad (8)$$

where $P_i = (P_{i1}, P_{i2}, \dots, P_{in})$ is the best position of particle i as of the current iteration, and the current best solution of the swarm is denoted by P_g . In this work, an inertia weight w was used to control the velocity (6), according to [32] and [17]. Equation (8) presents the expression used for w , which is assigned to the particle's previous velocity (V_i); $rand_1$ and $rand_2$ are two random vectors in the range $[0;1]$; C_1 is the constant acceleration of cognitive learning and C_2 is the constant acceleration of the social learning.

The flowchart of the proposed algorithm is presented in Fig. 1. The inputs are the network data, the number of DGs (N_{DG}) and the configuration parameters of PSO (p , population size and maximum iteration, C_1 and C_2 , w , V_{max} and V_{min}). An initial topology preprocessing is recommended to improve the LF efficiency. In the optimization process the PSO optimizes the vector (X), composed by the place and sizes of the DGs. The process ends when a maximum number of iterations is achieved.

The vector X (dimension of the problem) is made up of the elements $X_i^A, X_j^{SP}, X_j^{SQ}$. If reactive power is not considered in DG, the X_j^{SQ} element is neglected.

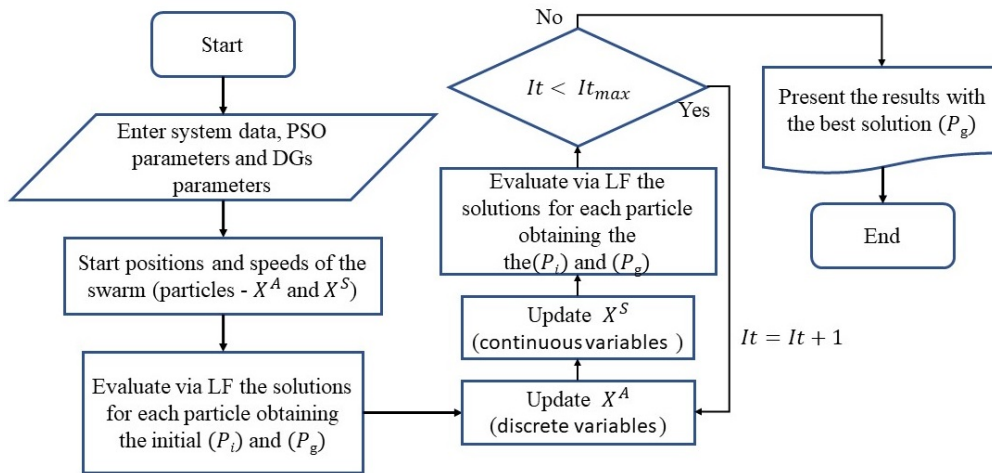


Figure 1. PSO Flowchart

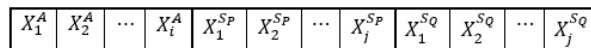


Figure 2. PSO particle

$i = 1, 2, \dots, NB$ and $j = 1, 2, \dots, NDG$

Where: X_i^A integer value representing the bus i to DG allocation.

In this work, the MATLAB function "ceil" was utilized. The ceil function rounds the element X_i^A to the nearest whole number greater than or equal to that element.

X_j^{SP} represents the active power of DG j ;

X_j^{SQ} represents the reactive power of DG j .

Because the elements of X have different characteristics, the PSO search process is separated by variables. The integer variables corresponding to the allocation buses are updated considering the limits and their discrete

characteristic. The other variables only power limits. Thus, the PSO search procedure was changed, favoring the search of the optimal solution. This being one of the contributions of the work.

4 Test results and discussion

The proposed PSO algorithm was developed in Matlab® and the LF calculated by Matpower[30] to be tested on three-phase 141-bus distribution system of AES-Venezuela in the metropolitan area of Caracas. The rated voltage is 12.47 kV, 141-bus, and 140 branches whereas the total peak load active power are 12.19 MW and reactive power 6.2897 MVar, the initial power loss of the system is 629.06 kW, the loads, lines and length data and all these system data can be seen originally from [33]. The PSO parameters were obtained after a series of test performance. And the best results are simulated with: number of particles = number of buses; maximum number of iterations = number of buses; $C_1 = 1.2$; $C_2 = 0.8$; $w_{max} = 0.9$; $w_{min} = 0.4$; $V_{max} = 5$; and $V_{min} = 0.5$.

4.1 Simulation with fixed generation capacity

We have considered 4 cases, with different power insertion in distribution network. The maximum generation power is fixed and can DGs range from 1 MW and 1 MVar up to 4 MW and 4 MVar in all tested cases.

In Figure 3, is presented a statistical analysis using the box plot of active losses after 30 runs. It is possible to observe that, the decrease of losses when DG are capable to provide both active and reactive power. In all simulations, the dispersion of results was small, proving that the algorithm is suitable for the allocation of DGs.

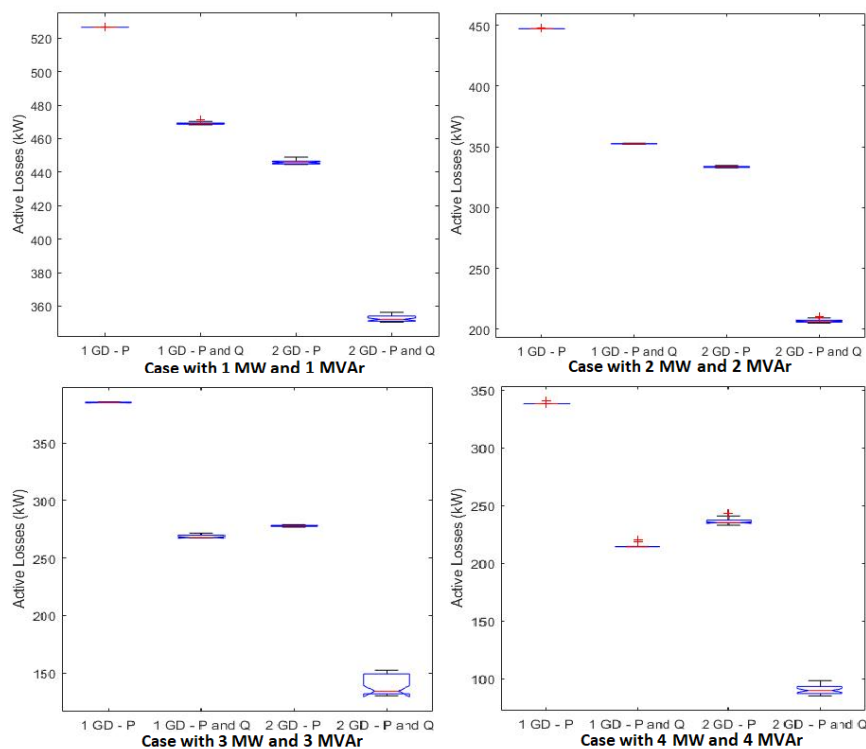


Figure 3. Box plot of active losses after 30 runs - 141-bus, case with active and reactive power fixed

The voltage profile has increased in all cases simulated and after the DG insertion, the voltage profile kept between limits that is 0.93 p.u. and 1.05 p.u. It is demonstrated at figure 4, the cases with a power insertion of 4MW in the circuit.

Another important approach is through table 1. In which, it has the best results for the case. In this table, we see that the insertion of DG at low powers cannot reduce significantly the power losses. At moment that we increase the power insertion, the power losses decrease as demonstrated in table 1. For this circuit the power insertion that has returned the best results was with 4MW and the best scenario was with 2 DG allocated with power active and reactive, that had 85.20 kW of power losses, but, in these scenarios, the power factor of (PF) 0.71, which can be an impediment for some generators.

Another important point is that DG power insertion with active and reactive power has better results compared

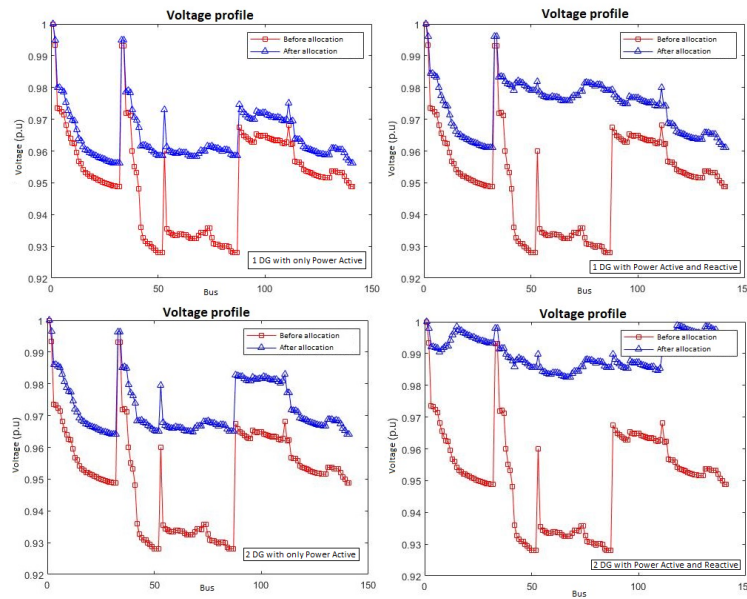


Figure 4. Voltage profile for the case with power generation up to 4 MW and 4 MVar

with the cases where the DG has inserted only active power.

Table 1. Simulation with fixed maximum generation capacity

Nominal Power	Qty DG	Best Buses	DG 1 (MW;MVar)	DG 2 (MW;MVar)	Losses kW	Best of Runs
Without DG	0	-	-	-	629,06	> 30
1 MW	1	86	(1;0)	-	526.58	> 30
1 MW	1	50	(1;1)	-	468.24	> 30
1 MW	2	(50;80)	(1;0)	(1;0)	444.62	> 30
1 MW	2	(50;80)	(1;1)	(1;1)	350.10	> 30
2 MW	1	49	(2;0)	-	447.12	> 30
2 MW	1	47	(2;2)	-	352.14	> 30
2 MW	2	(48;63)	(2;0)	(2;0)	332.70	> 30
2 MW	2	(48;55)	(2;2)	(2;1.8)	205.02	> 30
3 MW	1	44	(3;0)	-	384.94	> 30
3 MW	1	44	(3;3)	-	267.28	> 30
3 MW	2	(44;55)	(3;0)	(3;0)	276.77	> 30
3 MW	2	(44;15)	(3;2.9)	(3;2.56)	130.06	> 30
4 MW	1	44	(4;0)	-	338.09	> 30
4 MW	1	44	(4;4)	-	214.29	> 30
4 MW	2	(13;44)	(4;0)	(4;0)	232.96	> 30
4 MW	2	(14;44)	(4;3.45)	(4;2.5)	85.20	> 30

4.2 Simulation with free generation capacity

In order to find the best solution for circuit with 141 bus, we simulated the OPDG without a limit for the power generation, so the algorithm was able to find the best solution for the system. The algorithm has stipulated the best scenario for each case and at all cases the power factor was around 0.85 capacity. Also in this case, we simulated with insertion up to 3 DG in the system. In table 2, we can see the best results obtained for these cases.

As seen in Table 2, the best scenario was with the insertion of 3 DGs in the system, even the voltage profile improved compared to the other analyses.

As demonstrated in figure 5, the voltage profile has increased in all 141 buses, which is one of the main

Table 2. System simulation results 141-bus system with free generation capacity

Qty. DG	Best Buses	DG 1 (MW;MVar)	DG 2 (MW;MVar)	DG 3 (MW;MVar)	Losses kW	Best of Runs
Without DG	0	-	-	-	629,06	> 30
1	42	(7.3;0)	-	-	276.74	> 30
1	42	(7.3;4.5)	-	-	151.75	> 30
2	(15;42)	(3.46;0)	(6.35;0)	-	208.6	> 30
2	(15;42)	(3.43;2.13)	(6.3;3.9)	-	61.56	> 30
3	(15;42;92)	(2.53;0)	(5.7;0)	(3.35;0)	184.86	> 30
3	(15;44;91)	(2.52;1.56)	(5.06;3.14)	(3.73;2.31)	30.95	> 30

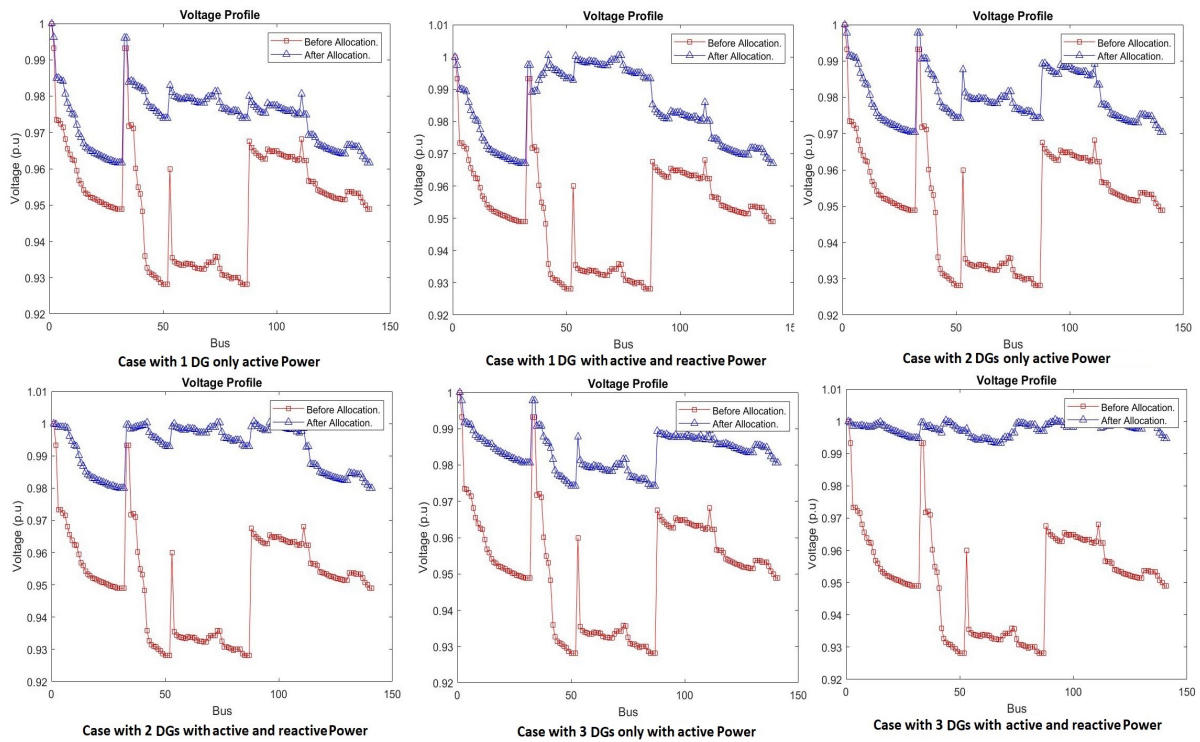


Figure 5. Voltage profile

importance for the system as a whole and for future application studies. In figure 6, we prove the effectiveness of the algorithm because its results are close each other.

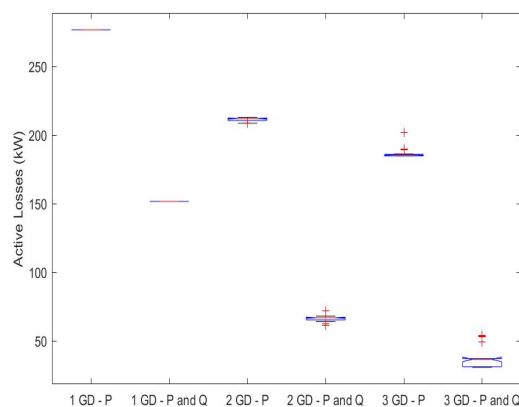


Figure 6. Box plot of active losses after 30 runs - 141-bus, case with free generation capacity

Analyzing the results, it is noted that for the cases without a fixed power generation had better results at the voltage profile, power factor approximately 0.85 capacity and reduce the losses on the system. The OPDG for the case with 3 DG with active and reactive power and without limit for the power generation, was the best scenario for this circuit which demonstrates an 95.08% reduction in system losses compared to the case without DG installed and had the best the voltage profile, therefore the system will have future capacity to increase loads and avoid expansion costs.

5 Conclusion

In this work, the impact of DG units on a practical distribution system was studied with main aspects that are losses reduction and voltage profile improvement. The PSO algorithm was implemented to find the best size and optimal place for DG and applied to a practical large-scale 141-bus system of AES-Venezuela in the metropolitan area of Caracas. The maximum benefits was obtained using DGs with active and reactive power, therefore the size and place require a adequate methodology for optimization and the proposed approach demonstrated effectiveness and accurate solutions. The PSO developed proved to be qualified to solve the problem for allocate and size the DG in distribution grid, showing excellent performance in solving the sizing and OPDG. Results have shown significant enhancement of the distribution system efficiency while using the proposed PSO algorithm. Finally, improvements can be made to the algorithm in addition to considering more realistic situations, such as load variations and DG with intermittent characteristics, such as photovoltaic and wind power. The authors firmly believe that this article can be useful to researchers and engineers in the related field.

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Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property of the authors.

References

- [1] A. Rezaee Jordehi. Allocation of distributed generation units in electric power systems: A review. *Renewable and Sustainable Energy Reviews*, vol. 56, n. C, pp. 893–905, 2016.
- [2] S. Nagaballi and V. S. Kale. Pareto optimality and game theory approach for optimal deployment of dg in radial distribution system to improve techno-economic benefits. *Applied Soft Computing*, vol. 92, pp. 106234, 2020.
- [3] L. S. Göran Andersson. Thomas Ackermann. Distributed generation: a definition. *Department of Electric Power Engineering, Royal Institute of Technology, Electric Power Systems, Teknikringen 33, 10044 Stockholm, Sweden and Electric Power Systems Group, Swiss Federal Institute of Technology, ETL 626, 8092 Zurich, Switzerland*, 2000.
- [4] H. M. H. Farh, A. M. Al-Shaalan, A. M. Eltamaly, and A. A. Al-Shamma'A. A novel crow search algorithm auto-drive pso for optimal allocation and sizing of renewable distributed generation. *IEEE Access*, vol. 8, pp. 27807–27820, 2020.
- [5] J. A. Martín García and A. J. Gil Mena. Optimal distributed generation location and size using a modified teaching–learning based optimization algorithm. *International Journal of Electrical Power & Energy Systems*, vol. 50, pp. 65–75, 2013.
- [6] A. A. Abo El-Ela, S. M. Allam, A. M. Shaheen, and N. A. Nagem. Optimal allocation of biomass distributed generation in distribution systems using equilibrium algorithm. *International Transactions on Electrical Energy Systems*, vol. 31, n. 2, pp. e12727, 2021.
- [7] K. L. Anaya and M. G. Pollitt. Integrating distributed generation: Regulation and trends in three leading countries. *Energy Policy*, vol. 85, pp. 475–486, 2015.
- [8] E. Karunarathne, J. Pasupuleti, J. Ekanayake, and D. Almeida. Optimal placement and sizing of dgs in distribution networks using mlps algorithm. *Energies*, vol. 13, n. 23, 2020a.
- [9] F. C. Coelho, da I. C. Silva Junior, B. H. Dias, and W. B. Peres. Optimal distributed generation allocation using a new metaheuristic. *J Control Autom Electr Syst*, vol. 29, n. 1, pp. 91–98, 2018.
- [10] T. Gözel and M. H. Hocaoglu. An analytical method for the sizing and siting of distributed generators in radial systems. *Electric Power Systems Research*, vol. 79, n. 6, pp. 912 – 918, 2009.
- [11] A. C. Rueda-Medina, J. F. Franco, M. J. Rider, A. Padilha-Feltrin, and R. Romero. A mixed-integer linear programming approach for optimal type, size and allocation of distributed generation in radial distribution systems. *Electric Power Systems Research*, vol. 97, pp. 133–143, 2013.

- [12] W. L. Theo, J. S. Lim, W. S. Ho, H. Hashim, and C. T. Lee. Review of distributed generation (dg) system planning and optimisation techniques: Comparison of numerical and mathematical modelling methods. *Renewable and Sustainable Energy Reviews*, vol. 67, pp. 531–573, 2017.
- [13] Z. Abdmouleh, A. Gastli, L. Ben-Brahim, M. Haouari, and N. A. Al-Emadi. Review of optimization techniques applied for the integration of distributed generation from renewable energy sources. *Renewable Energy*, vol. 113, pp. 266–280, 2017.
- [14] A. Ehsan and Q. Yang. Optimal integration and planning of renewable distributed generation in the power distribution networks: A review of analytical techniques. *Applied Energy*, vol. 210, pp. 44–59, 2018.
- [15] Z. Zhao and J. Mutale. Optimal conductor size selection in distribution networks with high penetration of distributed generation using adaptive genetic algorithm. *Energies*, vol. 12, n. 11, 2019.
- [16] G. W. Chang and N. Cong Chinh. Coyote optimization algorithm-based approach for strategic planning of photovoltaic distributed generation. *IEEE Access*, vol. 8, pp. 36180–36190, 2020.
- [17] I. L. Silva, E. A. B. , and D. José. Silva. Sensitivity analysis associated with particle swarm optimization for distributed generation allocation. *Brazilian Journal of Development*, vol. 6, n. 8, pp. 63479–63494, 2020.
- [18] A. Selim, S. Kamel, A. A. Mohamed, and E. E. Elattar. Optimal allocation of multiple types of distributed generations in radial distribution systems using a hybrid technique. *Sustainability*, vol. 13, n. 12, 2021.
- [19] A. Ramadan, M. Ebeed, S. Kamel, A. Y. Abdelaziz, and H. Haes Alhelou. Scenario-based stochastic framework for optimal planning of distribution systems including renewable-based dg units. *Sustainability*, vol. 13, n. 6, 2021.
- [20] H. M. H. Farh, A. M. Al-Shaalán, A. M. Eltamaly, and A. A. Al-Shamma’A. A novel crow search algorithm auto-drive pso for optimal allocation and sizing of renewable distributed generation. *IEEE Access*, vol. 8, pp. 27807–27820, 2020.
- [21] M. Suresh and J. B. Edward. A hybrid algorithm based optimal placement of dg units for loss reduction in the distribution system. *Applied Soft Computing*, vol. 91, pp. 106191, 2020.
- [22] E. S. Oda, A. A. Abdelsalam, M. N. Abdel-Wahab, and M. M. El-Saadawi. Distributed generations planning using flower pollination algorithm for enhancing distribution system voltage stability. *Ain Shams Engineering Journal*, vol. 8, n. 4, pp. 593–603, 2017.
- [23] K. Muthukumar and S. Jayalalitha. Optimal placement and sizing of distributed generators and shunt capacitors for power loss minimization in radial distribution networks using hybrid heuristic search optimization technique. *International Journal of Electrical Power & Energy Systems*, vol. 78, pp. 299–319, 2016.
- [24] A. Viet Truong, T. Ngoc Ton, T. Thanh Nguyen, and T. L. Duong. Two states for optimal position and capacity of distributed generators considering network reconfiguration for power loss minimization based on runner root algorithm. *Energies*, vol. 12, n. 1, 2019.
- [25] E. Karunarathne, J. Pasupuleti, J. Ekanayake, and D. Almeida. Optimal placement and sizing of dgs in distribution networks using mlpso algorithm. *Energies*, vol. 13, n. 23, 2020b.
- [26] M. Ghosh, S. Kumar, S. Mandal, and K. Mandal. Optimal sizing and placement of dg units in radial distribution system using cuckoo search algorithm. *International Journal of Applied Engineering Research*, vol. 12, pp. 362–369, 2019.
- [27] D. Hung, M. Nadarajah, and R. Bansal. Analytical expressions for dg allocation in primary distribution networks. *Energy Conversion, IEEE Transactions on*, vol. 25, pp. 814 – 820, 2010.
- [28] R. Viral and D. Khatod. Optimal planning of distributed generation systems in distribution system: A review. *Renewable and Sustainable Energy Reviews*, vol. 16, n. 7, pp. 5146–5165, 2012.
- [29] M. Pesaran H.A, P. D. Huy, and V. K. Ramchandaramurthy. A review of the optimal allocation of distributed generation: Objectives, constraints, methods, and algorithms. *Renewable and Sustainable Energy Reviews*, vol. 75, pp. 293–312, 2017.
- [30] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas. Matpower: Steady-state operations, planning, and analysis tools for power systems research and education. *IEEE Transactions on Power Systems*, vol. 26, n. 1, pp. 12–19, 2011.
- [31] R. Eberhart and J. Kennedy. A new optimizer using particle swarm theory. vol. , pp. 39–43, 1995.
- [32] J. Xin, G. Chen, and Y. Hai. A particle swarm optimizer with multi-stage linearly-decreasing inertia weight. In *2009 International Joint Conference on Computational Sciences and Optimization*, volume 1, pp. 505–508, 2009.
- [33] H. Khodr, F. Olsina, P. D. O.-D. Jesus, and J. Yusta. Maximum savings approach for location and sizing of capacitors in distribution systems. *Electric Power Systems Research*, vol. 78, n. 7, pp. 1192–1203, 2008.