DEEP LEARNING FOR INTERFACIAL DAMAGE ESTIMATION IN AN INVERSE ULTRASOUND SCATTERING ANALYSIS

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Abstract. We formulate and solve a time-harmonic inverse scattering problem to estimate the interfacial defect distribution at an adhesion interface of a composite plate. We use the incident field that mostly interact with such defects and assume prior knowledge of the material properties of each layer of the laminate. We model the adhesion interfaces using the Quasi-Static-Approximation, where approximates it by a set of tangential and normal springs, and allow the interfacial stiffness to depend upon the position along the interface. To solve the direct problem, the interfacial defect distribution is generated by a prior of smooth stochastic field. In addition, we develop a deep learning neural network, using the reflected signal as input, to solve the formulated inverse problem. We validate our implementation and evaluate the presented method's performance for noisy input data and different defect distribution scenarios with the aid of numerical simulations. From the obtained numerical results, we may say that the proposed method is robust to the presence of noise and has the potential to detect and classify interfacial defects.

Keywords: Deep Learning, Acoustic Scattering, Quasi-Static-Approximation

1 Introduction

Techniques to estimate the damage field in composites are very desirable in structural health monitoring of laminated structures [1–3]. Over the last years, countless computational models have been developed to predict damage evolution and failure in composite materials [3–5]. In particular, deep learning (DL) has recently shown exceptional performance on object classification and segmentation tasks in computer vision. Motivated by these results, many works in literature are applying DL to several research fields including inverse problems [3, 6–8].

According to Mass [9], one of the hottest topics discussed over the last few years in literature is the use of neural networks for large data applications. Furthermore, deep learning is achieving outstanding results in large-scale nonlinear problems such as classification tasks in computer vision and speech recognition. This encouraged researchers to use deep learning networks to solve inverse problems. Theoretically, when comparing inverse problems defined by analytical models with data-driven approaches, such as deep learning, it is expected that data-driven approaches have some advantages, e.g., an incomplete model of the underlying physical or engineering system or if the search for a parameter x is actually restricted to a characteristic subset, which, however, escapes precise mathematical modeling. A simple deep learning inverse approach consists on considering a basic problem given by an operator $A : x \to y$ and noisy data $y^{(i)} = Ax^{(i)} + \eta^{(i)}$, which set $x^{(i)}$'s as inputs and $y^{(i)}$'s as outputs to train the forward operator, while the training of a neural network to solve the inverse problem is given using $y^{(i)}$'s as inputs and $x^{(i)}$'s as outputs.

Mass [9] indicates that training even a most simple neural network for a well-posed matrix-vector multiplication yields good results. However, the natural approach for training a network for an inverse problem by reversing inputs and outputs fails in cases where you have an unstructured matrix with high level of sparsity.

There are many works in literature that uses learning approaches to predict damage. In Liu *et al.* [10], the authors investigate machine learning methods to predict the length of the path across delamination area in composite materials. In Ritto and Rochinha [11], the authors tested different classifiers and parameters in order to build a fast digital twin (machine learning), by using a physics-based model, that will be connected to the physical twin to support real time engineering decisions. The computational model consists in approximate a bar structure with actuators and sensors to a 6-DOF lumped parameter, where some parameters are random in order to give the model a stochastic approach. Zobeiry *et al.* [3] used a continuum damage finite element model to train interconnected

Neural Networks (NN) in series, based on macroscopic load-displacement data in order to characterize damage in quasi-isotropic composite laminates. Then, they used experimental measurements obtained through cumbersome non-destructive testing to validate the predicted damage properties. Santos *et al.* [12] investigate and compared the classification performance of four kernel-based algorithms (one-class support vector machine, support vector data description, kernel principal component analysis and greedy kernel principal component analysis) by using measures of an acceleration time-series from an array of accelerometers obtained from a laboratory structure. And Pathirage *et al.* [13] propose an autoencoder deep network for structural damage identification in highly non-linear problems. They use the natural frequencies and mode shapes of vibration as input and the structural damage as output in order to train the network.

Chen *et al.* [6] mention that deep learning is becoming an increasingly important tool for solving inverse scattering problems (ISPs) in recent years. ISPs can be seen as a problem that consists in determining the nature of an unknown scattering distribution from the measure of the scattered fields. ISPs are challenging to solve because they are intrinsically ill-posed and nonlinear. ISPs can be tackled by either traditional objective-function approaches (as can be seen in Leiderman and Castello [14], Cakoni *et al.* [15], Gaikovich and Gaikovich [16], to cite a few) or learning approaches, that is the scope of the present work. To avoid using DL as a purely data-driven black-box solver, it is important to address the problem of how profitably is combining DL with the available knowledge on underlying physics as well as traditional objective-function approaches. Many research efforts have been made in this direction to achieve real-time quantitative results. The physical-insight perspective applies not only to ISPs, but also to many other physical regression problems. In fact, in many practical applications, data collected by sensors are automatically governed by physical laws. Some of these physical laws present well-known mathematical properties or analytical formulas, which do not need to be learnt by training with a lot of data as stated by Chen *et al.* [6].

In the present work, we develop an inverse formulations in order to identify and estimate damage fields in interfaces of composite laminates. The use of a deep learning regression technique is investigated aimed to recover the damage field at any interface and direction of a composite laminate immersed in an acoustic fluid from the scattered reflected field.

2 Mathematical Formulation

Here we formulate a generic multilayered laminate fully immersed in an acoustic fluid as shown in Fig. 1.

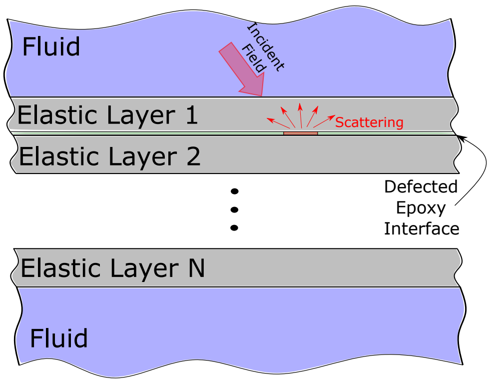

Figure 1. A representation of an acoustic scattering problem in a composite laminate immersed in a fluid medium.

In this technique, we divide the displacement $\mathbf{u}$ and traction $\mathbf{t}$ fields into up and downgoing fields, i.e., fields propagating (or being attenuated) in the positive vertical (z) direction and fields propagating (or being attenuated) in the negative vertical (z) direction.

The solid layers are treated as elastic, isotropic and homogeneous, where stress $\boldsymbol{\sigma}$ and displacement field $\mathbf{u}$ satisfy

$$\nabla \cdot \boldsymbol{\sigma} + \rho\, \omega^2 \mathbf{u} = \mathbf{0}, \tag{1}$$

and

$$\boldsymbol{\sigma} = \mathbf{C} : \nabla \mathbf{u}, \tag{2}$$

where ρ is the mass density, ω is the angular frequency and $\mathbf{C}$ is the elasticity tensor. However, we point out that this formulation could accommodates for anisotropic layers as well, as can be seen in [17].

The displacement field $\mathbf{u}$ and traction vector $\mathbf{t}$ are related by [18]:

$$\bar{\mathbf{u}}_a(z_2) = \mathbf{M}_a(z_2 - z_1)\bar{\mathbf{u}}_a(z_1), \quad a = 1, 2, \tag{3}$$

$$\bar{\mathbf{t}}_a = -i\,\omega\,\mathbf{Z}_a\bar{\mathbf{u}}_a, \quad a = 1, 2, \tag{4}$$

where $(\bullet)_1$ and $(\bullet)_2$ denote up and downgoing fields, respectively, z_1 and z_2 are the vertical coordinates of the bottom and the top of the layer under analysis, operators $\mathbf{M}_1(\bullet)$ and $\mathbf{M}_2(\bullet)$ propagate the up and downgoing displacement fields in each layer, respectively, and $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are the local impedance tensors, which relate up and downgoing traction vectors $\bar{\mathbf{t}}_\alpha$ to the corresponding displacement fields $\bar{\mathbf{u}}_a$ [17, 19]. $\mathbf{M}_a(\bullet)$ and $\mathbf{Z}_a(\bullet)$ are obtained by the exact solution of the elastodynamic equations of motion and its computation are presented in [17, 19] for isotropic and anisotropic media. The bar on top of the field variables stands for a single Fourier Transform over the x-direction, indicating the representation of these in the wavenumber vector x component domain.

As can be seen in Fig. 1 the damage field at interfaces is heterogeneous, leading to an acoustic scattering phenomenon. This changes the formulation of the acoustic direct problem into an optimization problem, which can be solved in one step [20], as follows:

1. Transform the given incident field $\mathbf{u}_2^+$.
2. Choose an initial guess for the transmitted field at the upper interface $\bar{\mathbf{u}}_2^-$ and assemble its real and imaginary parts into a minimization parameters vector $\mathbf{p}^{(0)}$:

$$\begin{aligned}\mathbf{p}^{(0)} = [[real\left\{\bar{\mathbf{u}}_{21}^-\right\}]^T, [imag\left\{\bar{\mathbf{u}}_{21}^-\right\}]^T, [real\left\{\bar{\mathbf{u}}_{22}^-\right\}]^T, [imag\left\{\bar{\mathbf{u}}_{22}^-\right\}]^T,\\ ..., [real\left\{\bar{\mathbf{u}}_{2N}^-\right\}]^T, [imag\left\{\bar{\mathbf{u}}_{2N}^-\right\}]^T]^T.\end{aligned} \tag{5}$$

   Notice that it can be any initial guess.
3. Calculate $\mathbf{u}_1^-$ through

$$\mathbf{u}_1^- = [\mathbf{Z}_1 - \mathbf{Z}_{f_2}]^{-1}[-z_{f_2}(2\bar{u}_{2z}^+ - \bar{u}_{2z}^-)\mathbf{n} - \mathbf{Z}_2\bar{\mathbf{u}}_2^-], \tag{6}$$

   where $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are the elastic layer local impedance tensors, $\mathbf{Z}_{f_2}$ and z_{f_2} are the local impedance of the fluid and $\mathbf{n}$ is the outward unit vector in the z direction.
4. Then we propagate $\bar{\mathbf{u}}_1^-$ and $\bar{\mathbf{u}}_2^-$ to the bottom of the layer, using eq. 3 to determine the up and downgoing fields ($\bar{\mathbf{u}}_1^+$ and $\bar{\mathbf{u}}_2^+$) at the top of the next interface.
5. Calculate $\bar{\mathbf{u}}^+ = \bar{\mathbf{u}}_1^+ + \bar{\mathbf{u}}_2^+$, calculate $\bar{\mathbf{t}}^+$ through

$$\bar{\mathbf{t}}^+ = -i\,\omega\,[\mathbf{Z}_1^+\bar{\mathbf{u}}_{\mathbf{1}}^+ + \mathbf{Z}_2^+\bar{\mathbf{u}}_{\mathbf{2}}^+], \tag{7}$$

   and perform the inverse Fourier transform to determine $\mathbf{u}^+$ and $\mathbf{t}^+$.
6. Calculate $\mathbf{u}^-$ and perform the Fourier transform to determine $\bar{\mathbf{u}}^-$.
7. By using the boundary condition of traction continuity, eq. 4, $\bar{\mathbf{t}}^- = \bar{\mathbf{t}}_1^- + \bar{\mathbf{t}}_2^-$ and $\bar{\mathbf{u}}^- = \bar{\mathbf{u}}_1^- + \bar{\mathbf{u}}_2^-$, compute $\bar{\mathbf{u}}_1^-$ through

$$\bar{\mathbf{u}}_{\mathbf{1}}^- = [\mathbf{I} - \mathbf{Z}_2^-]^{-1}[\mathbf{Z}_1^-]^{-1}(-1\,\omega)^{-1}\bar{\mathbf{t}}^- - \mathbf{Z}_2^-\bar{\mathbf{u}}^-, \tag{8}$$

   then use $\bar{\mathbf{u}}^- = \bar{\mathbf{u}}_1^- + \bar{\mathbf{u}}_2^-$ to calculate $\bar{\mathbf{u}}_2^-$.
8. Use steps 4-7 recursively to determine $\bar{\mathbf{u}}_1^+$ and $\bar{\mathbf{u}}_2^+$ at the lower interface of the structure.
9. At the lower interface, the solid-inviscid fluid boundary conditions must be satisfied. Furthermore, one can notice that the Sommerfeld condition takes place in the lower half-space, i.e., $\bar{\mathbf{u}}^- = \bar{\mathbf{u}}_2^-$ and $\bar{u}_{z_2}^-$ can be straightforwardly calculated.

10. Then the residual vector can be calculated by

$$\mathbf{r}_j = \mathbf{t}_j^+ - i\,\omega\, z_{f_2}\, \bar{w}_2^-\, \mathbf{n}, \tag{9}$$

where the cost function can be written as the ordinary least squares norm:

$$\sum_{j=1}^{M} [real\,\{\mathbf{r}_j\}]^T [real\,\{\mathbf{r}_j\}] + [imag\,\{\mathbf{r}_j\}]^T [imag\,\{\mathbf{r}_j\}]. \tag{10}$$

11. Repeat steps 2–10, perturbing the entries in $\mathbf{p}^{(0)}$, one at a time, to assemble the Jacobian Matrix $\mathbf{J}$:

$$J_{ij} = \frac{r_i(p_1^{(0)},...,p_j^{(0)} + \epsilon,...,p_N^{(0)}) - r_i(p_1^{(0)},...,p_j^{(0)},...,p_N^{(0)})}{\epsilon}. \tag{11}$$

12. Each residual r_j represented in the cost function 10 may be thought of as a linear function of the minimization parameters vector. Therefore, the cost function 10 is quadratic with respect to the minimization parameters and has thus only one (global) minimum, and the Jacobian (sensitivity) matrix associated to the problem is not a function of the minimization parameters. In this case, the minimization of 10 can be performed straightforwardly as

$$\mathbf{p}^* = \mathbf{J}^{(0)} - [\mathbf{J}^T\mathbf{J}]^{-1}\mathbf{J}^T\mathbf{r}(\mathbf{p}^{(0)}), \tag{12}$$

where $\mathbf{p}^*$ is the solution for the problem.

From the direct resolution of the problem, a deep neural network is then trained with the scattered reflected field as input and a random damage field, generated from stochastic Guassian fields, as output. Furthermore, in the inverse analysis, we use noisy scattered data to try to recover the damage field associated with the response using the trained neural network.

3 Results

The analysed physical system corresponds to a structure composed by stainless steel, copper and aluminum, bonded by thin epoxy adhesives and immersed in water, the whole system and its measures are shown in Fig. 2. Furthermore, the mechanical properties of each constituent material are presented in table 1.

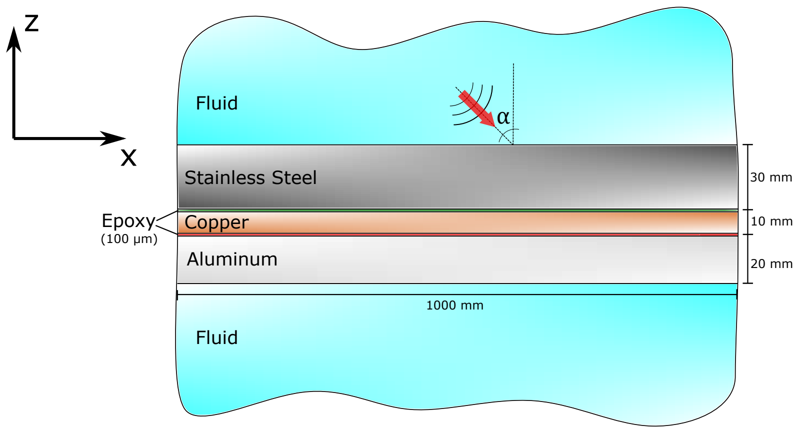

Figure 2. The physical system. Where α corresponds to the orientation of the incident field (angle of incidence).

For the results shown In Figure 3, we inspect and consider only defects at the interface between the copper and aluminum layers, in the x-direction (red interface, see Figure 2). Furthermore, the proposed incident field is a

Material	c_p $[m/s]$	c_s $[m/s]$	ρ $[kg/m^3]$
Stainless Steel	5790	3100	7900
Copper	4660	2660	8930
Aluminum	6320	3130	2700
Epoxy	2150	1030	1200
Water	1480	0	1000

Table 1. Mechanical properties of the constituent materials of the system shown in Fig.2. Note: c_p and c_s stands for P-wave and S-wave speeds, respectively.

time harmonic rectangular wave, with an oblique incidence of 4.1^o and main frequency of 102.8 kHz. In Figure 3) the results for a Flawless interface, a Gaussian damage and a rectangular damage using a deep neural network as the inversion procedure.

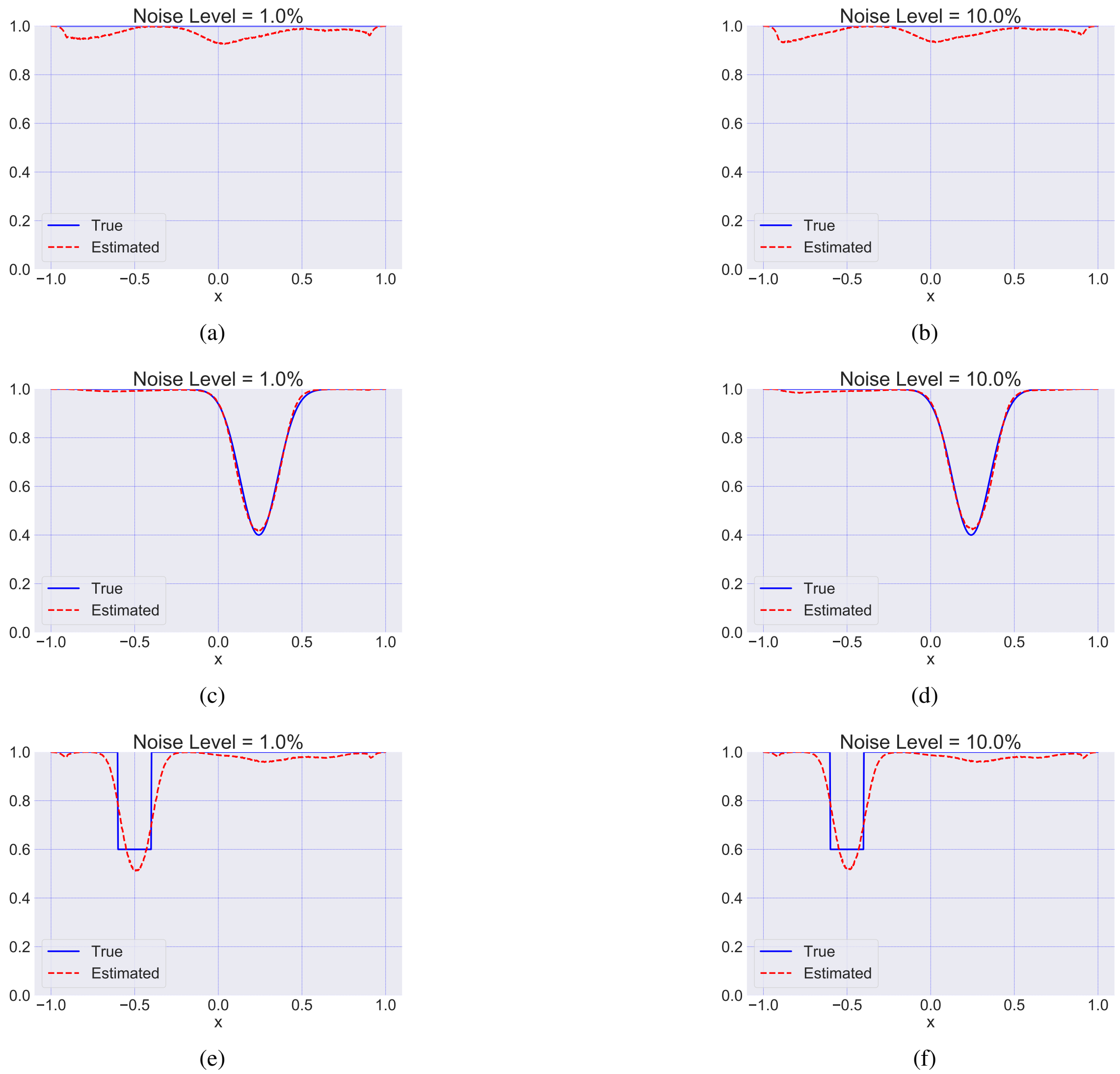

Figure 3. Damage field prediction between the copper and aluminum layers in the x-direction. The blue line represents the true field, while the red dashed line represents the estimated field. Here x represents the inclined x-axis in respect to the angle of incidence α normalized to the half of the length of the represented structure ($\bar{x} = X/L$).

4 Conclusions

The idea of a deep learning inverse analysis came from the fact that, traditionally, inverse scattering problems with objective-function approach have a very high computational cost, since the direct problem is already an interactive process, i.e., it is costly. Take the work of Leiderman and Castello [14], as an example, where they adopt a specific parametrization to describe the damage, reducing the computational cost, but not enough to predict defects in a suitable time. In fact, solving the whole inverse problem using an objective-function approach takes a lot of time. In contrast, even though the process of training the deep network demands a considerable time, once trained the approach has the potential to predict a whole damage field, without the need for parameterization, in real-time and with accurate results, as can be seen in the present work.

Acknowledgements. This work was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. Daniel A. Castello also would like to express his gratitude to the financial support provided by CNPq under grant number 312355/2020-3.

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References

[1] J.-B. Ihn and F.-K. Chang. Pitch-catch active sensing methods in structural health monitoring for aircraft structures. *Structural Health Monitoring*, vol. 7, n. 1, pp. 5–19, 2008.

[2] S. Sarkar, K. K. Reddy, and M. Giering. Deep learning for structural health monitoring: A damage characterization application. In *Annual Conference of the PHM Society*, volume 8, 2016.

[3] N. Zobeiry, J. Reiner, and R. Vaziri. Theory-guided machine learning for damage characterization of composites. *Composite Structures*, vol. 246, pp. 112407, 2020.

[4] A. Kaddour, M. Hinton, P. Smith, and S. Li. A comparison between the predictive capability of matrix cracking, damage and failure criteria for fibre reinforced composite laminates: Part a of the third world-wide failure exercise. *Journal of Composite Materials*, vol. 47, n. 20-21, pp. 2749–2779, 2013.

[5] J. Reiner and R. Vaziri. 8.4 structural analysis of composites with finite element codes: An overview of commonly used computational methods. *Comprehensive composite materials II*, pp. 61–84, 2018.

[6] X. Chen, Z. Wei, M. Li, and P. Rocca. A review of deep learning approaches for inverse scattering problems. *Progress In Electromagnetics Research*, vol. 167, pp. 67–81, 2020.

[7] Y. LeCun, Y. Bengio, and G. Hinton. Deep learning. *Nature*, vol. 521, pp. 436–444, 2015.

[8] H. Li, J. Schwab, S. Antholzer, and M. Haltmeier. Nett: solving inverse problems with deep neural networks. *Inverse Problems*, vol. 36, n. 6, pp. 065005, 2020.

[9] P. Maass. Deep learning for trivial inverse problems. *Compressed Sensing and Its Applications*, pp. 195–209, 2019.

[10] H. Liu, S. Liu, Z. Liu, N. Mrad, and H. Dong. Prognostics of damage growth in composite materials using machine learning techniques. In *2017 IEEE International Conference on Industrial Technology (ICIT)*, pp. 1042–1047, 2017.

[11] T. Ritto and F. Rochinha. Digital twin, physics-based model, and machine learning applied to damage detection in structures. *Mechanical Systems and Signal Processing*, vol. 155, pp. 107614, 2021.

[12] A. Santos, E. Figueiredo, M. Silva, C. Sales, and J. Costa. Machine learning algorithms for damage detection: Kernel-based approaches. *Journal of Sound and Vibration*, vol. 363, pp. 584–599, 2016.

[13] C. S. N. Pathirage, J. Li, L. Li, H. Hao, W. Liu, and P. Ni. Structural damage identification based on autoencoder neural networks and deep learning. *Engineering Structures*, vol. 172, pp. 13–28, 2018.

[14] R. Leiderman and D. A. Castello. Detecting and classifying interfacial defects by inverse ultrasound scattering analysis. *Wave Motion*, vol. 65, pp. 119–129, 2016.

[15] F. Cakoni, D. Colton, P. Monk, and J. Sun. The inverse electromagnetic scattering problem for anisotropic media. *Inverse Problems*, vol. 26, n. 7, pp. 074004, 2010.

[16] K. P. Gaikovich and P. K. Gaikovich. Inverse problem of near-field scattering in multilayer media. *Inverse Problems*, vol. 26, n. 12, pp. 125013, 2010.

[17] R. Leiderman, B. F. Junqueira, D. A. Castello, and A. M. Braga. Identifying the ultrasonic inspecting fields that most strongly interact with adhesive bonding defects. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 40, n. 4, pp. 227, 2018.

[18] J. L. Rose. *Ultrasonic guided waves in solid media*. Cambridge university press, 2014.
[19] A. M. B. Braga. Wave propagation in anisotropic layered composites. *PhDT*, 1990.
[20] R. Leiderman and D. Castello. Scattering of ultrasonic waves by heterogeneous interfaces: Formulating the direct scattering problem as a least-squares problem. *The Journal of the Acoustical Society of America*, vol. 135, n. 1, pp. 5–16, 2014.