Modified Improved Harmony Search Applied to Reinforced Concrete Beams

Fernando L. T. Junior[1], Moacir Kripka[2]

[1]*Programa de Pós-Graduação em Engenharia Civil e Ambiental, Universidade de Passo Fundo*
São José, 99052-900, Passo Fundo, Rio Grande do Sul, Brasil
161547@upf.br
[2] *Programa de Pós-Graduação em Engenharia Civil e Ambiental, Universidade de Passo Fundo*
São José, 99052-900, Passo Fundo, Rio Grande do Sul, Brasil
mkripka@upf.br

Abstract. During the structural design, engineers can use optimization algorithms as efficient tools to conceive structures with lower cost and environmental impacts compared to the traditional trial and error process. This paper presents the cost optimization of reinforced concrete beams, composed by the cost of concrete, steel bars and formworks. The design variables were the cross-section dimensions, position, and diameter of the reinforcement. The constraints applied in the problem are those imposed by the Brazilian Standard NBR 6118 (2014), checking the ultimate and serviceability states. To achieve that, a formulation to analyze reinforced concrete beams was implemented in the software developed in this research. The optimization method used was a variant of Harmony Search, a metaheuristic inspired by the jazz musical improvisation process, called Modified Improved Harmony Search (MIHS). Aiming to verify the efficiency of the developed software, as well as to investigate the performance of the optimization method, beams with different spans, loads and concrete strengths were optimized. The obtained results were satisfactory and competitive, although they show the need for improvements in the program.

Keywords: beams, cost, Harmony Search, optimization, reinforced concrete.

1 Introduction

Reinforced concrete is a composite material widely used to build structures, given its versatility, mechanical characteristics, and durability. During the design of these structures, the engineer comes across several possible solutions, and it is his duty to seek the one with lowest cost while bearing the loads. This sizing process is usually done by a manual, iterative and slow trial and error method, where the designer's previous experiences have great influence on decision making. Besides that, there is no certainty that the results obtained are the optimal solution for the problem.

With the computational development, it became possible to use structural sizing software, which speed up the process and assist in the design of more economical structures. This cost minimization can be even greater with the combined use of structural optimization algorithms, which are efficient tools to find optimal solutions, using a more rational approach through a systematic search. To ensure that the optimized structure is feasible, it must meet the sizing requirements, verified by the ultimate limit state (ULS) and service limit state (SLS) accordingly to its respective design code.

Among the optimization techniques, the Harmony Search (HS) is a metaheuristic able to converge to optimal results using fewer iterations and smaller populations compared to other methods, provided that its internal parameters are well calibrated to the problem. The method is inspired by jazz musical improvisation, where the musicians propose new combinations of notes for each instrument, based on prior knowledge or repeated attempts. In this analogy, the instruments represent the variables of the optimization problem, which must be combined in best possible way to find the perfect harmony, that is, the global optimum. Since the method's original publication, improvements have been proposed aiming to increase the algorithm's performance, such as the Improved Harmony Search (IHS), one of the most notable.

This paper aims to minimize the cost of reinforced concrete beams by optimizing their cross section. For this, it was developed a program that finds the optimal configurations for the section of simply supported beam, performing the verification of the ULS and SLS accordingly to the Brazilian code ABNT NBR 6118/2014 [1]. The problem variables are the dimensions of the beam section, the amount of rebars and their diameters, chosen from commercial diameters. The method used for the optimization is the Modified Improved Harmony Search (MIHS), proposed by Medeiros and Kripka [2].

2 Modified Improved Harmony Search

Optimization methods can be deterministic, through mathematical algorithms that involve the calculation of derivatives, or probabilistic, with heuristic algorithms that normally refer to processes in nature. In structural engineering, the functions involved in the sizing stage are usually non-linear, non-convex, discontinuous and with several points of local minimum or maximum, causing the deterministic methods to have their efficiency reduced or even become inapplicable to the problem.

Therefore, heuristic methods are widely applied in the optimization of structures, with the disadvantage of requiring a greater number of objective function calculations and, consequently, greater computational effort. Among the heuristics, the most applied are genetic algorithms, simulated annealing, bee colony, ant colony, particle swarm, tabu search, harmony search, among many others.

The Harmony Search is a metaheuristic, originally proposed by Lee, Geem and Loganathan [3], which makes an analogy to jazz musical improvisation. It consists in an iterative process based on memorization and improvisation. First, an initial set of solutions is proposed, which is then compared to the new solution generated by the method. The best solutions are stored while the worst ones are discarded throughout the iterations. The general algorithm can be described in 5 basic steps, as shown in Fig. 1.

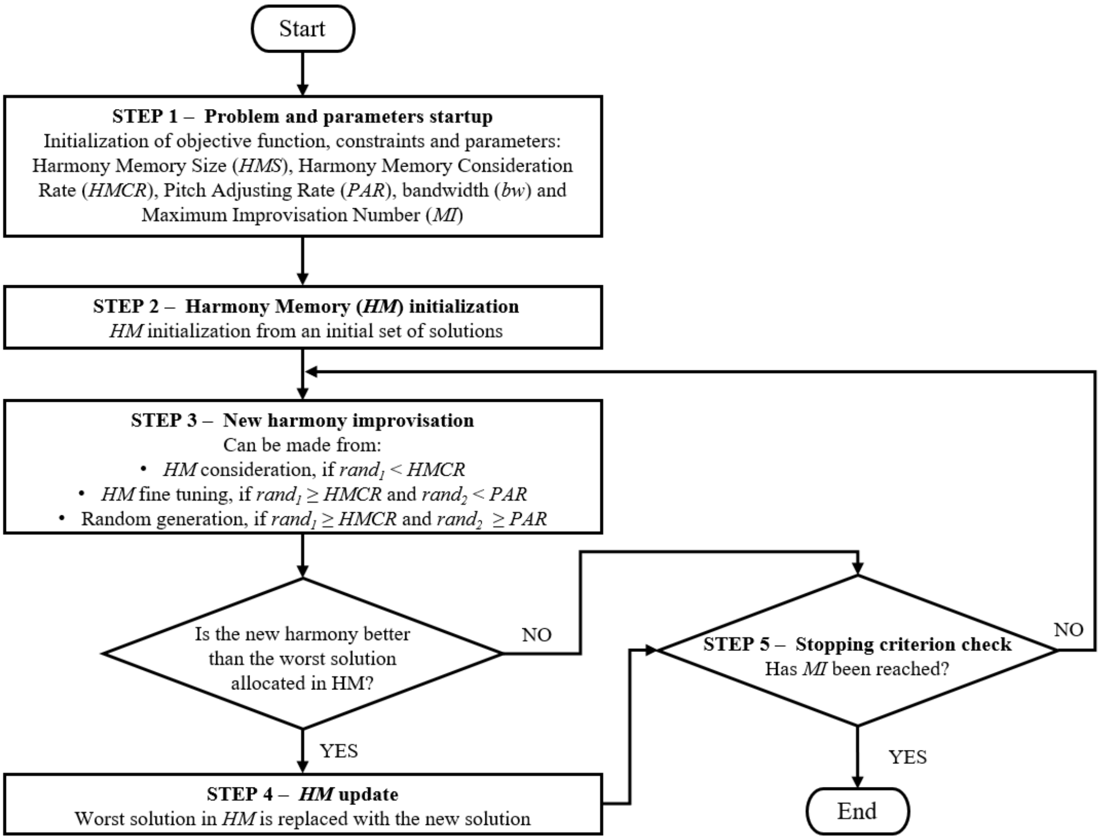

Figure 1. Basic flowchart of the HS method

Since the method's publication, some improvements to it have been proposed, aiming to obtain a better performance with the algorithm. The most notable of these is the work of Mahdavi, Fesanghary and Damangir [4], who proposed the Improved Harmony Search (IHS), where the parameters *PAR* and *bw* dynamically vary over the iterations. As shown in Fig. 2, *PAR* grows linearly with the iterations, while *bw* decreases exponentially.

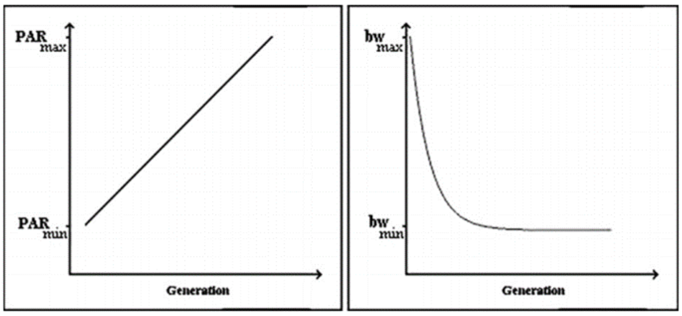

Figure 2. *PAR* and *bw* variation along generations

Based on the IHS, Medeiros and Kripka [2] proposed the Modified Improved Harmony Search (MIHS) which includes a full or partial reset of the Harmony Memory when the values of the best and the worst harmony become too close, seeking to prevent the algorithm form converging to a local optimum. The goal is to make the method less dependent on the size of the Harmony Memory. In addition, the authors highlight the possibility of creating a new stopping criterion: if after a stipulated number of resets the best solution does not change, the method ends before the maximum number of iterations is met, reducing running time of the algorithm with no improvements to the objective function value.

It is possible to verify the efficiency of the method based on recent works that used the HS and its improvements to optimize structural engineering problems. Medeiros and Kripka [5] applied the HS algorithm to reduce the monetary and environmental costs of reinforced concrete columns. From numerical applications, it was found that the method achieved better or at least equal results to those obtained by conventional sizing and other optimization methods. It was found that the monetary cost optimization of these structures is, in general, closely related to the environmental cost optimization, since the solutions for one objective are also satisfactory for the other.

Kaveh and Ghafari [6] performed the geometric and parametric optimization of steel roof structures using nine different heuristic methods, comparing the results. From a model where the optimal global solution was known, it was attested that the heuristics present good efficiency in optimizing the type of structure studied. Among the methods studied, the ones that presented best performance were Grey Wolf Optimizer (GWO), Colliding Bodies Optimization (CBO) and Harmony Search.

Shaqfa and Orban [7] optimized reinforced concrete beams following the EC2 European standard. Different design cases and boundary conditions were optimized using a modification of the HS called Parameter-Setting-Free Harmony Search (PSFHS). The modified method proved to be stable and robust in optimizing high-dimensional problems, managing to escape local minima. Although it is mentioned that there is still a need for further refinements and investigations in the method, the authors continue to recommend the method for optimizing reinforced concrete structures, with a large number of design variables.

Nehdi, Keshtegar and Zhu [8] used a dynamic self-adjusting version of the HS to predict the bond stress of the reinforcing rebar in the cementitious matrix, comparing the results with other non-linear regression models based on experimental data. Models that used the improved HS method performed better compared to those that did not.

Arama et al [9] optimized the cost and CO2 emissions of reinforced concrete cantilever soldier piles, using the HS method. The authors found both the solutions with the lowest cost and those with the lowest carbon emissions, with the excavation depth being the factor with the greatest influence on the design.

Yucel et al. [10] used an Adaptive-Hybrid Harmony Search (AHHS) algorithm to optimize reinforced concrete retaining walls, seeking for structures with less material consumption and, consequently, less environmental impact. The improvements implemented in the method proved to be positive, reaching competitive solutions using a reduced population, which reduces the optimization processing time.

3 Optimization Problem Formulation

The objective of this work is to reduce the cost of reinforced concrete beams, considering the cost of concrete, steel, and formwork. Thus, the objective function of the optimization problem is obtained by adding the products of the unit cost of each material and the respective quantity consumed in the beam. The optimization problem can be represented mathematically as follows:

$$minimize\ C_T = V_C \cdot C_C + A_F \cdot C_F + P_A \cdot C_A, \tag{1}$$

where C_T is the total structure cost, V_C is the concrete volume in m³, C_C is the concrete cost per m³, A_F is the formwork area in m², C_F is the formwork cost per m², P_A is the steel mass in kg e C_A is the steel cost per kg.

The constraints of the problem involve the verification of the ultimate and service limit states, according to the guidelines of the Brazilian code ABNT NBR 6118/2014 [1]. The first constraint, shown in Eq. (2), concerns the rebars horizontal spacing (e_h), which must be greater than the minimum spacing (e_{min}) calculated according to the previously mentioned standard.

$$e_h \geq e_{mín}. \tag{2}$$

For the Ultimate Limit State (ELU), the stresses are verified by determining the resisting capacity to simple bending (M_{ud}), as exposed by Araújo [11]. For the solution to be feasible, this value must be equal to or greater than the acting bending stress (M_d), giving rise to the first inequality constraint of the problem, disregarding the compression reinforcement

$$\frac{M_{ud}}{M_d} \geq 1. \tag{3}$$

M_{ud} value is determined by

$$M_{ud} = 0{,}8 \cdot b \cdot x \cdot (d - 0{,}4 \cdot x) \cdot \sigma_c, \tag{4}$$

where x is the depth of neutral axis, d is the reinforcement's gravity axis depth, and σ_c corresponds to 0.85 of the concrete's design compressive strength (f_{cd}). The tensile reinforcement is also verified by Eq. (3), since it is used to determine the position of the beam's neutral axis and, therefore, the beam's resisting capacity to simple bending. Further details explaining the determination of the beam's neutral axis are shown by Araújo [11].

In the Service Limit State (SLS), crack opening (ELS-F and ELS-W from NBR 6118 [1]) and excessive deformation (ELS-D) are checked. If cracks are formed, their thickness (w_k) must be less than the limit ($w_{k,lim}$) of 0.03 cm, according to the aforementioned standard. The structure's deflection (δ) is obtained considering the long-term effects, and the displacement limit (δ_{lim}) is relative to the visual effects, with a value equal to 1/250th of the span. The normalized equations that represent the described constraints are respectively presented below:

$$\frac{w_{k,lim}}{w_k} \geq 1, \tag{5}$$

$$\frac{\delta_{lim}}{\delta} \geq 1. \tag{6}$$

Aiming to reduce the cost of reinforced concrete beams, the following variables were defined, also represented in Fig. 3: b is the beam base in cm; h is the beam height in cm; Nb_{int} is the number of internal rebars; $Øe$ is the diameter of the outer rebars; and $Øi$ is the diameter of the inner rebars. All variables of the problem are discrete and can assume the pre-established values shown in equations 7 to 10:

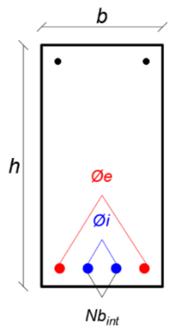

Figure 3. Optimization problem variables

$$b = \{14, 15, 16, \cdots, 38, 39,40\}\ in\ \text{cm}, \tag{7}$$

$$h = \{20, 25, 30, \cdots, 140, 145,150\}\ in\ \text{cm}, \tag{8}$$

$$Nb_{int} = \{1, 2, 3, \cdots, 8, 9, 10\}, \tag{9}$$

$$Øe, Øi = \{6{,}3;\ 8;\ 10;\ 12{,}5;\ 16;\ 20;\ 25;\ 32;\ 40\ \}\ in\ \text{mm}. \tag{10}$$

The values of the optimization method's internal parameters were determined based on previously performed tests, and they are presented in Tab. 1. As for the new parameters that concerns the improvements implemented in the MIHS, it was defined that 40% of the worst harmonies have their values recalculated on each HM reset. The HM reset, on the other hand, is performed when the difference between the best and worst solutions is less than 0.5%. Furthermore, the problem's initial solution is generated randomly within the possible solution space.

Table 1. Coefficients in constitutive relations

Parameter	Value
MI	50000
HMCR	0,9
HMS	5
bw_{min}	2
bw_{max}	5
PAR_{min}	0,5
PAR_{max}	0,9

4 Program Validation and Numerical Applications

First, seeking to attest the developed program efficiency, as well as the improvements implemented to the original HS method, simply supported beams were optimized. The chosen models are the same ones used by Medeiros and Kripka [12], who applied the Simulated Annealing algorithm in the optimization problem. The spans of the beams vary between 2 and 10 m, and three different concrete strengths were tested: 20, 30 and 45 MPa. In addition, two load situations were considered, which cover most cases of beams in residential buildings. The minimum load corresponds to a permanent load of 9.86 kN/m and a live load of 2 kN/m, while the maximum load is 16 kN/m and a 7 kN/m live load. The unit costs of each material are the same as the reference paper, that have been kept to allow the results comparison, although they are out of date. Such values are shown in Tab. 2.

Table 2. Unit costs of materials

Material	Unit	Cost (R$)
Steel (CA-50)	kg	3.97
Formworks	m²	8.68
20 MPa Concrete	m³	213.07
30 MPa Concrete	m³	252.70
45 MPa Concrete	m³	303.71

The optimization results are shown in Fig.4, and the optimal costs are similar to those found by Medeiros and Kripka [12], even though the values achieved in this paper are around 10% higher. However, this increase in the objective function value was already expected since commercial rebar diameters were used to compose the steel area of the beam, instead of using the steel area as a continuous variable of the optimization problem, which was done at the work in comparison. Some examples of the beam's optimal cross section achieved in this paper are shown in Fig. 5. It is important to mention that the difference between the values would be even greater if this paper had also considered the transverse reinforcements of the beam. Nevertheless, it can be concluded that the developed program is able to reach competitive solutions, validating its applicability.

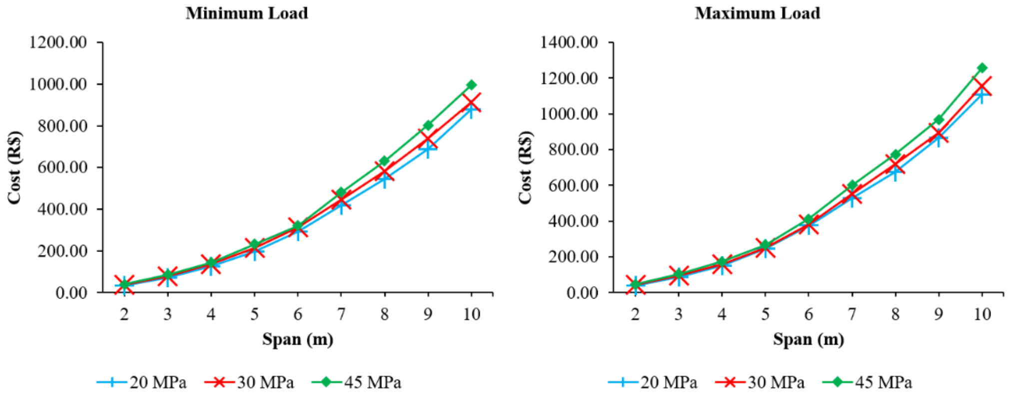

Figure 4. Optimal costs

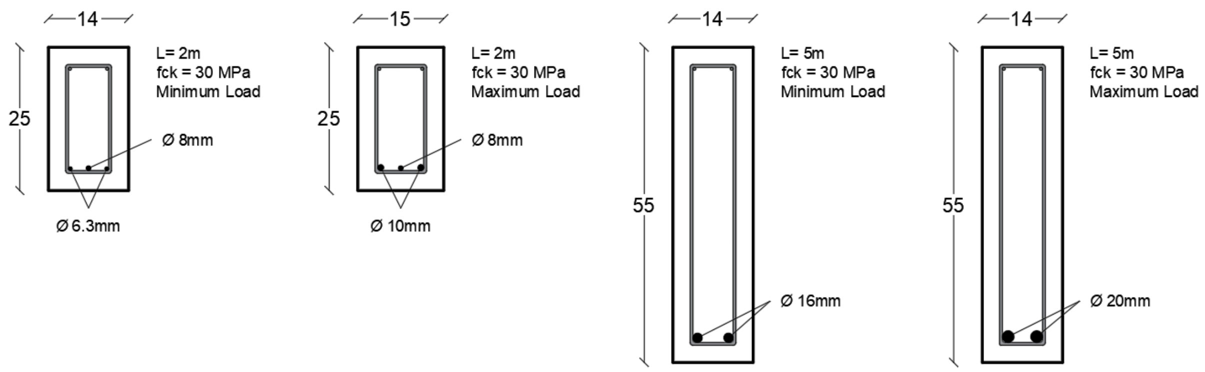

Figure 5. Optimal cross section of the beam

For smaller spans, from 2 to 4 meters, the optimal cost for each concrete resistance is virtually the same, for both studied loading cases. As the span grows, the difference between the costs for each f_{ck} increases, with the higher-strength concretes being more expensive. This shows that the increase in the concrete strength is not enough to offset the increase of the material's unit cost, as also pointed out by Medeiros and Kripka [12]. However, it is necessary to assess whether this behavior is repeated considering current costs.

Regarding the improvements implemented in the MIHS algorithm, the results obtained with the method are the same as the ones reached with the IHS, in all the studied situations. This could be due to the low number of variables and possible solutions to the problems, where both algorithms reach a possible global optimum. Therefore, it can be concluded that the MIHS converges to results at least equal to the IHS, although tests with more complex problems are needed to verify the method's improvements. Figure 6 shows the differences between the worst and best solutions along the improvisations for both methods, considering the best runs of each algorithm.

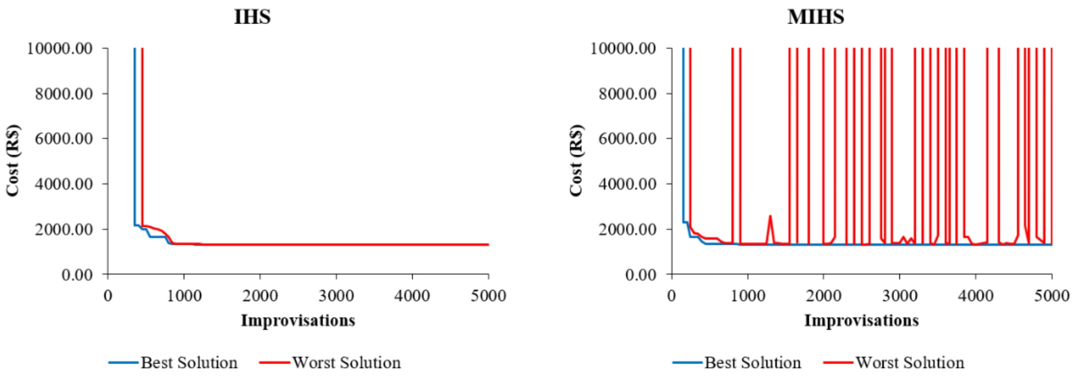

Figure 6. Comparison between MIHS and IHS solutions along the improvisations

5 Conclusions

Based on the obtained and presented results, it is possible to conclude that the developed program manages to reach feasible and competitive solutions, in comparison to a similar work. As already expected, the optimal solutions achieved in this paper had slightly higher costs, with the use of commercial diameters of rebars being the greatest responsible for this, although it corresponds better to practical situations. The results also show that the higher-strength concretes tend to be more expensive, as the increase of its strength does not pay off its higher unit costs. Improvements in the program and in the definition of the optimization problem's variables are steel needed to improve the results. The possibility of generating sections with more than one steel layer is an example of this. Finally, several future implementations can be proposed, such as the consideration of transverse reinforcement, the possibility of using double reinforcement, verification of other acting stresses and the implementation of the concrete strength as a variable to be optimized.

Acknowledgements. The authors acknowledge the financial support of the Coordination for the Improvement of Higher Education Personnel (CAPES) to the first author, and the financial support of the Brazilian National Council for Scientific and Technological Development (CNPq) to the last author

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