



## Single and multi-objective optimization of spatial steel frame considering different bracing systems.

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**Abstract.** In a structural optimization problem, the objective may consist in minimizing its cost and maximizing its performance according to horizontal displacements, dynamic behavior, structural stability, etc. In addition to that, finding the best bracing system configuration which presents the best results according to the objectives of the problem is not a trivial task. This work is based on multi-objective optimization of steel frames considering different bracing systems, in which distinct configurations of space frames compete in the same evolutionary process. An integer index variable defines which type of bracing system configuration is adopted for each candidate solution. The output of a multi-objective problem is a Pareto-front curve, from where the designer must choose the most suitable solution according to objectives that are taken into account. A Multi-Tournament method based on the preferences of the decision-maker is used to extract the solutions. The search algorithm adopted is the Third Step Differential Evolution (GDE3) coupled with an Adaptive Penalty (APM) Method to handle the constraints.

**Keywords:** Multi-objective optimization, bracing systems, steel frames, differential evolution.

## 1 Introduction

Most structural optimization problems considering steel frames take as a unique objective to minimize the structure's total cost, or in a simplified way, its weight. However, in real-world engineering problems, the objective may be minimizing the structure's weight and enhancing its performance, such as minimizing horizontal displacements due to the wind or improving its dynamic behavior or its global stability. In addition to that, it's not possible to know beforehand which configuration of the bracing system leads to the best results according to a specific objective, and, in general, it is determined according to the engineer's experience. Bracing systems are necessary for tall buildings to stiffen the structure, making it work as a vertical truss, which redistributes the internal forces in a more balanced way and enhances the whole structural performance in horizontal displacements and vibrations.

This work consists of single- and multi-objective optimizations of 3D steel frames considering different configurations of bracing systems as design variable. Some consolidated configurations of bracing elements such as diagonal, "Z", "V", and "X" are encoded by an integer index in the candidate solution. Besides, in a multi-

objective problem, the result is a Pareto front of several solutions, from where the designer must extract the most suitable ones according to the “importance” of each objective. For that, a multi-criteria decision making developed by Parreiras and Vasconcelos [1] is employed. The search methodology used in this work is the GDE3 proposed by Kukkonen and Lampinen [2], and the constraint handling technique is the APM developed by Barbosa and Lemonge [3]. One can cite some relevant studies that consider different bracing systems configuration in multi-objective problems such as Kicinger and Arciszewski [4], Kicinger et al. [5], Richardson et al. [6] and Babaei and Sanaei [7]. However, there is no proven better way of considering different configurations of bracing systems, and the present work presents an alternative way of what is done in the cited works.

The remainder of this paper is organized as follows: Section 2 describes the formulation of the optimization problem. Section 3 briefly presents the search method and the constraint handling technique. The multi-criteria decision-maker is described in Section 4. Numerical experiment and its analysis are detailed in Sections 5 and 6, respectively. Finally, conclusions and future works are reported in Section 7.

## 2 Formulation of the optimization problem

The structural optimization problem presented in this paper consists in finding a bracing system configuration and a set of commercial steel profiles, designated by an integer index vector  $\mathbf{x} = \{I_1, I_2, \dots, I_i\}$  (design variables), in which the first index indicates which configuration of bracing elements will be applied and the others point to commercial profiles. This vector is a candidate solution, and have to minimize the first objective function ( $of_1(\mathbf{x})$ ) as well as the second conflicting objective function ( $of_2(\mathbf{x})$ ), subjected to structural design constraints, in the case of a single-objective problem, only one objective function is subjected to minimization (eq. (1)).

$$\begin{aligned}
 \text{Single-objective:} & \quad \min \quad of_1(\mathbf{x}) \\
 \text{Multi-objective:} & \quad \min \quad of_1(\mathbf{x}) \quad \text{and} \quad \min \quad of_2(\mathbf{x}) \\
 & \quad \text{s.t.} \quad \text{structural constraints} \\
 & \quad \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U
 \end{aligned} \tag{1}$$

The first objective function ( $of_1(\mathbf{x})$ ) is the structure’s total weight, defined by eq. (2). Where  $L_i$ ,  $A_i$ , and  $\rho_i$  are the length, the cross-sectional area, and the specific mass of the  $i$ -th member, respectively, and  $N$  is the number of elements. The second objective function is the maximum displacement at the top of the building obtained by solving a direct stiffness method equation system (eq.(3)). Where  $\delta(\mathbf{x})$  are the nodal displacements,  $\mathbf{K}$  is the stiffness matrix and  $\mathbf{f}$  is the force vector. After defining both of the objectives functions it is possible to rewrite the general formulation (eq. (1)) leading to a specific formulation for the problems treated in this work in eq. (4). Where  $\mathbf{x}^L$  and  $\mathbf{x}^U$  are vectors of lower and upper bounds for the variables.

$$W(\mathbf{x}) = \sum_{i=1}^N \rho_i A_i L_i \tag{2}$$

$$\delta(\mathbf{x}) = \mathbf{K}^{-1} \mathbf{f} \tag{3}$$

$$\begin{aligned}
 \text{Single-objective:} & \quad \min \quad W(\mathbf{x}) \\
 \text{Multi-objective:} & \quad \min \quad W(\mathbf{x}) \quad \text{and} \quad \min \quad \delta_{max}(\mathbf{x}) \\
 & \quad \text{s.t.} \quad \text{structural constraints} \\
 & \quad \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U
 \end{aligned} \tag{4}$$

The constraints of the problem are the maximum horizontal displacement, the inter-story drift, the LRFD (Load and Resistance Factor Design) interaction equations for combined axial force and bending moments, the LRDF shearing equation, the first natural frequency of vibration, the critical load factor concerning the global stability, and geometric constraints referring to column-column connection. It is important to point out that when a constraint becomes an objective function, it stops being a constraint.

The displacement constraints are the maximum horizontal displacement and the maximum inter-story drift. Equations (5) and (6) define the displacement related constraints, in which  $\delta_{max}(\mathbf{x})$  is the maximum horizontal displacement computed,  $\bar{\delta}$  is the maximum allowable horizontal displacement,  $d_{max}(\mathbf{x})$  is the maximum inter-story drift computed and  $\bar{d}$  is the maximum allowable inter-story drift. The values of the maximum allowable horizontal displacement and the maximum inter-story drift are taken as  $\bar{\delta} = H/400$  and  $\bar{d} = h/500$ , according

with both Brazilian ABNT [8] and American ANSI [9] codes. Where  $H$  is the building height and  $h$  is the height between two consecutive stories. It is important to highlight that, when the maximum horizontal displacement is taken as an objective functions, it stops being a constraint.

$$\frac{\delta_{max}(\mathbf{x})}{\bar{\delta}} - 1 \leq 0 \quad (5)$$

$$\frac{d_{max}(\mathbf{x})}{\bar{d}} - 1 \leq 0 \quad (6)$$

The first natural frequency of vibration is determined by solving the eigenvalue problem that involves the stiffness and mass matrix (Bathe [10]). The structure must present the first natural frequency ( $f_1(\mathbf{x})$ ) higher than a minimum allowable ( $\bar{f}_1$ ). Equation (7) describes the natural frequency of vibration constraint.

$$1 - \frac{f_1(\mathbf{x})}{\bar{f}_1} \leq 0 \quad (7)$$

To ensure the structure's global stability, the critical load factor ( $\lambda_{crt}(\mathbf{x})$ ) must be higher than one, as defined in eq. (8). The critical load factor is obtained by solving an eigenvalue problem concerning the elastic and geometric stiffness matrices (McGuire et al. [11]).

$$1 - \frac{\lambda_{crt}(\mathbf{x})}{1} \leq 0 \quad (8)$$

The members of the structure are designed to satisfy the LRDF interaction equation for combined axial and bending effects (eq.(9)), and the LRDF shearing equation (eq. (9)).  $P_r$ ,  $M_{rx}$ , and  $M_{ry}$  are the required axial strength, required flexural strength about the major axis and the minor axis, respectively. The available axial and flexural members strength are named as  $P_c$ ,  $M_{cx}$ , and  $M_{cy}$ . For the allowable shearing strength equation,  $V_r$  is the required shearing strength, and  $V_c$  is the available shearing strength. The methodology of determining the allowable strengths are similar in both ABNT [8] and ANSI [9] and adopted in this paper.

$$\begin{cases} \frac{P_r}{P_c} + \frac{8}{9} \left( \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) - 1 \leq 0 & \text{if } \frac{P_r}{P_c} \geq 0.2 \\ \frac{P_r}{2P_c} + \left( \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) - 1 \leq 0 & \text{if } \frac{P_r}{P_c} < 0.2 \end{cases} \quad (9)$$

$$\frac{V_r}{V_c} - 1 \leq 0 \quad (10)$$

The geometric constraints refer to the column-column connection, in order to establish that the upper column must not have, neither the profile depth nor the mass, higher than the lower column. Equations (11) and (12) show the geometric constraints, where  $dp_i(\mathbf{x})$  and  $dp_{i-1}(\mathbf{x})$  are the depth of the W section selected for the group of columns  $i$  and  $i - 1$ , respectively.  $ms_i(\mathbf{x})$  and  $ms_{i-1}(\mathbf{x})$  are the unit weight of W section selected for the group of columns  $i$  and  $i - 1$ , respectively.  $NG_c$  is the number of groups of columns.

$$\frac{dp_i(\mathbf{x})}{dp_{i-1}(\mathbf{x})} - 1 \leq 0 \quad i = 1, NG_c \quad (11)$$

$$\frac{ms_i(\mathbf{x})}{ms_{i-1}(\mathbf{x})} - 1 \leq 0 \quad i = 1, NG_c \quad (12)$$

### 3 Search algorithm and constraint handling technique

The search algorithm adopted in this work is the Third Evolution Step of Generalized Differential Evolution (GDE3), proposed by Kukkonen and Lampinen [2]. It is an extension of the previously proposed Differential Evolution (DE) by Storn and Price [12]. The search methodology starts with a population randomly generated, which improves along an evolutionary process of DE, consisting of selection, mutation, and crossover operations. The parameters of the algorithm are the crossover rate ( $C_R \in [0, 1]$ ), the mutation factor ( $F \in \mathbb{R}$ ) and the population size ( $N_p$ ).

Being  $P_G$  be a population of  $N_p$  decision vectors  $\mathbf{x}_{i,G}$  in generation  $G$ , where  $i \in \{1, 2, 3, \dots, N_p\}$  is a vector index. Each  $\mathbf{x}_{i,G}$  of the population in generation  $G$  is a  $n$ -dimensional vector and  $\mathbf{x}_{j,i,G}$  is its  $j$ -th component

( $j \in \{1, 2, 3, \dots, n\}$ ). A decision vector  $\mathbf{x}_{i,G}$  creates the corresponding trial vector  $\mathbf{u}_{i,G}$  applying mutation and crossover operations (Storn [13]). After that, the trial vector  $\mathbf{u}_{i,G}$  is compared to the decision vector  $\mathbf{x}_{i,G}$  using the constraint domination concept. A vector  $\mathbf{x}$  dominates a vector  $\mathbf{y}$  (denoted by  $\mathbf{x} \succeq_c \mathbf{y}$ ) if one, and only one, of the following conditions is true: (i) both are unfeasible and  $\mathbf{x} \succ \mathbf{y}$  in the constraint function violation space; (ii)  $\mathbf{x}$  is feasible and  $\mathbf{y}$  is unfeasible, and (iii)  $\mathbf{x}$  and  $\mathbf{y}$  are feasible and  $\mathbf{x} \succ \mathbf{y}$  in the objective function space. The trial vector  $\mathbf{u}_{i,G}$  is selected to replace the decision vector  $\mathbf{x}_{i,G}$  in the next generation  $P_{G+1}$  (population in generation  $G + 1$ ) if  $\mathbf{u}_{i,G} \succeq_c \mathbf{x}_{i,G}$ . If  $\mathbf{x}_{i,G} \succeq_c \mathbf{u}_{i,G}$ ,  $\mathbf{u}_{i,G}$  is discarded and  $\mathbf{x}_{i,G}$  remains in the population. Otherwise, both are included in  $P_{G+1}$ . A complete and detailed description of the entire GDE3 algorithm can be found in Vargias et al. [14].

The Adaptive Penalty Method (APM) proposed by Barbosa and Lemonge [3] is adopted in this paper to handle the constraints. Through statistical information of the whole population, the method balances the penalty coefficients according to the difficulty of constraints to be satisfied.

## 4 Multi-criteria decision maker

The result of a multi-objective optimization problem is a Pareto front with several non-dominated solutions, which causes the extraction of a best-fitting solution to be a non-trivial task. One way of transcending this problem is extracting solutions based on a predefined methodology, in which it is possible to determine importance weight coefficients for each objective. The decision-making in this paper is aided by a multi-criteria tournament proposed by Parreiras and Vasconcelos [1]. According to the objective functions and their respective importance weights ( $w_i$ ), established by the Decision-Maker, a Multi Tournament Decision Method (MTD) ranks the best and the worst possible solutions in the Pareto front. The complete and detailed description of the MTD method can be found in Parreiras and Vasconcelos [1] and examples in multi-objective structural optimization in Carvalho et al. [15].

## 5 Numerical Examples

The numerical experiments conducted in this paper concern single- and multi-objective optimizations of a six-story spatial steel frame with beams and columns three meters long. The problem consists of minimizing both the structure's total weight and its maximum horizontal displacement on the top story. Also, a single-objective experiment to minimize the structural weight is conducted for comparison matters. The bracing system configuration is a variable of this problem, and the structure can assume four different configurations: (i) a 90 bars diagonally braced frame; (ii) a 90 bars "Z" braced frame; (iii) a 114 bars "V" braced frame and (iv) a 126 bars "X" braced frame. The first index guides the configuration in the candidate vector, assuming values one to four. The other variables concern the profile employed on columns and beams. The search spaces for members are composed of 29 "H" profiles for the columns and 56 "I" for beams, all of them part of the AISC profile tables. Figure 1 illustrates the candidate vector and the corresponding phenotype of the frame according to the first index.

The structure is subjected to gravity loads of 10 kN/m on the outer beams and 20 kN/m on the inner beams. The wind pressure acts on the larger facade, resulting in a mean load of 3.17 kN/m for the corner columns and 6.34 kN/m for the outer columns, calculated for a reference velocity of 35 m/s in accordance to ABNT [16]. The minimum allowed frequency of vibration is  $f_1 = 2$  Hz, and the maximum allowed inter-story drift is  $\bar{d} = 6$  mm. The members of the frame are linked as follows: CC (corner columns), OC (outer columns), OB (outer beams), IB (inner beams), and BC (bracers). The group changes for every three stories for beams and columns, resulting in eight groups plus one extra group for the bracing members, totaling nine groups.

The experiments are conducted with ten independent runs of 200 generations with 50 candidate vectors, for both single- and multi-objective problems. In multi-objective problem, the solutions are extracted according to three different scenarios: (i) scenario 1: the extracted solution has the structure's weight  $w_1 = 0.2$  of importance and  $w_2 = 0.8$  for the maximum horizontal displacement at the top story; (ii) scenario 2: the extracted solution has both objective functions with the same importance i.e.  $w_1 = w_2 = 0.5$ ; (iii) scenario 3: the extracted solution has the structure's weight  $w_1 = 0.8$  of importance and  $w_2 = 0.2$  for the maximum horizontal displacement. The best results found are displayed on Table 1 detailing the bracing system, the profiles for each group and the values for constraints and objective functions. Figure 2 illustrates the Pareto front trade-off curve, from where are extracted solutions for each of the three scenarios described before. The structural configuration of each solution is also illustrated for an easy comprehension.

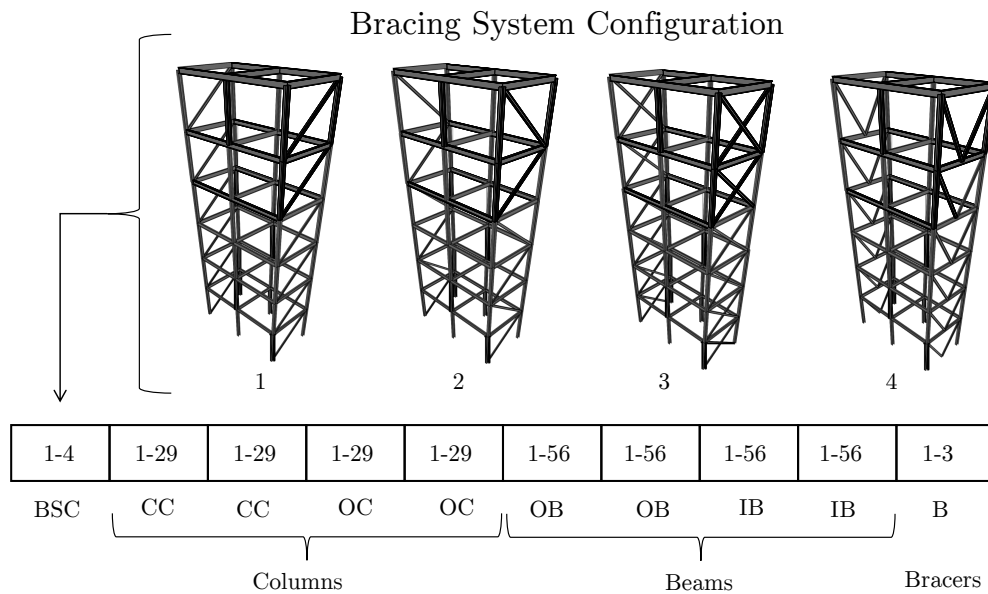


Figure 1. Candidate vector and bracing systems configurations.

Table 1. Best results found for the three scenarios of the multi-objective problem and the single scenario of the single-objective problem, presenting details of the profiles assigned to each member group, constraints, and objective function values.

	Multi-objective			Single-objective
	Extracted Solutions			Single Solution
Scenario	1	2	3	Single Scenario
Bracing System	X	X	Z	D
Group (Stories)	W Profiles			
CC (1-3)	310x125	360x122	150x37	150x29.8
CC (4-6)	250x89	200x46.1	150x22.5	150x22.5
OC (1-3)	360x122	360x91	360x122	200x59
OC (4-6)	360x122	150x22.5	360x91	150x22.5
OB (1-3)	310x21	250x17.9	250x17.9	250x17.9
OB (4-6)	610x125	360x72	150x13	200x26.6
IB (1-3)	610x125	310x21	610x125	410x38.8
IB (4-6)	610x125	310x21	610x113	310x26.6
BC (1-6)	150x24	150x24	150x24	150x13
Constraints and objective functions values				
$LRFD_{max}(\mathbf{x})$	0.33	0.95	0.98	0.99
$V_{max}(\mathbf{x})$	0.08	0.22	0.35	0.23
$d_{max}(\mathbf{x})$ (mm)	1	2	3	6
$f_1(\mathbf{x})$ (Hz)	2.14	2.01	2.01	2.02
$\lambda_{crit}(\mathbf{x})$	24.11	12.02	9.07	4.85
$\delta_{max}(\mathbf{x})$ (mm)	7	10	15	42
$W(\mathbf{x})$ (kg)	24821	15844	10506	7056

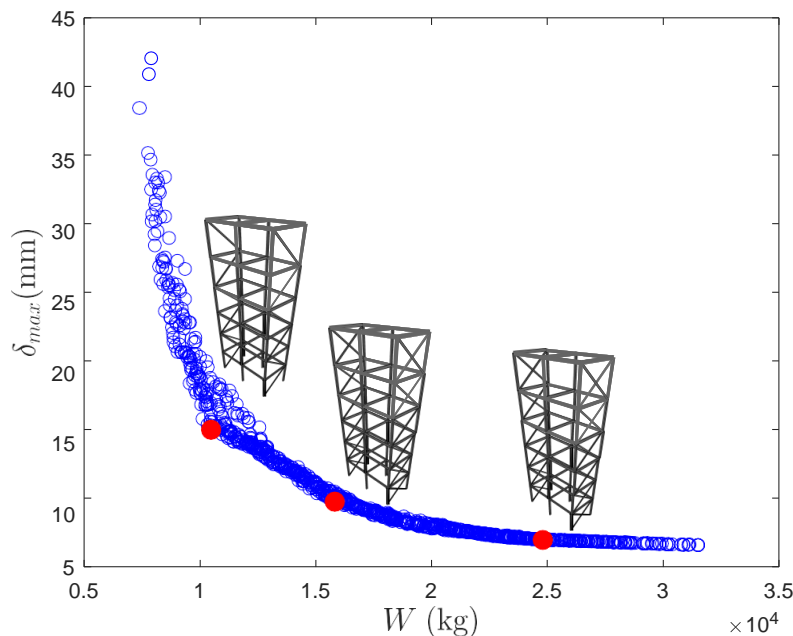


Figure 2. Trade-off curve total weight versus maximum displacement with extracted solutions.

## 6 Analysis of the results

Analyzing the Table 1 it is possible to make some interesting observations. By looking at the multi-objective solutions, extracted from three different scenarios previously defined, one can observe that the weight decreases ( $W_1(\mathbf{x}) = 24821$  kg,  $W_2(\mathbf{x}) = 15844$  kg and  $W_3(\mathbf{x}) = 10506$  kg) and the maximum horizontal displacement increases ( $\delta_{max1}(\mathbf{x}) = 7$  mm,  $\delta_{max2}(\mathbf{x}) = 10$  mm and  $\delta_{max3}(\mathbf{x}) = 10506$  mm) from scenarios 1 to 3. It is expected, as they are conflicting objectives and the importance of the weight function increases from scenarios 1 to 3. It is also notable that, as the structure becomes lighter, the critical load factor according to global stability ( $\lambda_{crt}$ ) reduces and the interaction equation for combined flexural and bending strength ( $LRFD$ ) increases. The second statement leads to a more effective structure in terms of material using. Also, it is verified that the first natural frequency of vibration of all scenarios is near  $f_1 = 2$  Hz, suggesting that the frequency constraint is almost active in this problem. Considering the bracing system configuration, it is possible to observe that, on the one hand, the “Z” configuration leads to a lighter solution with higher displacement. On the other hand, the “X” configuration leads to a heavier solution with lower horizontal displacements. The analysis of which bracing system fits best the desired objective depends on the geometry of the whole structure. In other words, the “X” configuration could provide lighter solutions for a different problem with a different spatial frame.

As expected, the single-objective solution leads to the lightest structure ( $W(\mathbf{x}) = 7056$  kg) with the highest displacement ( $\delta_{max}(\mathbf{x}) = 42$  mm) when comparing with the multi-objective solutions. It happens because, in the single-objective problem, the maximum horizontal displacement is only a constraint. Without a secondary objective function, the whole effort of the problem is to minimize the structural weight, not taking into account if the displacements are low or high, provided that they are within the constraint limit. Due to that, the maximum inter-story drift is higher, and the critical load factor is lower compared to the other solutions. In addition to that, the interaction equation ratio for combined bending and flexural effects is now active ( $LRFD_{max} = 0.99$ ) as the frequency remains active also. The bracing system configuration that provides the lightest solution in this work is found in the single-objective problem solution as the diagonal (“D”) bracing configuration.

## 7 Conclusions

This work is about single- and multi-objective optimization of spatial steel frames considering different configurations of bracing systems as design variables. The search methodology employed is the GDE3 with the APM to handle the constraints. Two numerical experiments were conducted with a six-story steel frame, where the found results show that it is important to consider different configurations of bracing systems, as they are non-trivial to be predefined and, in general, are designed based on engineer’s experience. Also, multi-objective problems demon-

strate that a Pareto front of solutions provides a wide range of possible structures, from where the Decision-Maker can extract whichever frame fits the designer requirements by previously defining importance weights for the objective functions. This work has plenty of future possibilities to be enhanced, considering more objective functions such as the first natural frequency of vibration and the global stability through the critical load factor, the orientation of the columns as a design variable, and semi-rigid connections. It is important to highlight that this work does not have the scope of making comparative analysis of different search algorithms, in spite of that, future works will carry out these comparisons.

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