Optimal orientation of cross-sections of columns of 3D steel frames in a single and multi-objective optimization

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Abstract. In most structural optimization problems, the minimization of the structure's weight is a traditional objective. Furthermore, it is expected to improve other aspects of the optimum design, leading to conflicting objective functions. This paper analyses single and multi-objective structural optimization problems of a six-story space steel frame, considering the structure's weight minimization as the first objective function and the second one concerning the maximum horizontal displacement at the top of the frame to be minimized. The design variables are the profiles assigned to the beams and the profiles assigned to the columns in which their orientations, concerning the principal axes, are also design variables. Pareto fronts with the non-dominated solutions are presented. A Multi-Tournament Decision method is adopted to extract solutions from the obtained Pareto fronts based on the decision maker's preferences. The search algorithm adopted is the Third Step Differential Evolution (GDE3) coupled with an Adaptive Penalty Method (APM) to handle the constraints.

Keywords: Multi-objective Optimization, Space Steel Frame, Differential Evolution

1 Introduction

In a structural optimization problem, it is usually desired to minimize the structure weight. Furthermore, it may also be intended to improve some mechanical aspects, such as minimizing horizontal displacements. That leads to conflicting objective functions. Therefore, this kind of problem requests a multi-objective optimization formulation. In contrast to the single-objective problem, where the evolutionary process provides one best solution, the result is a set of non-dominated solutions called Pareto front in the multi-objective problem. From this set, a decision-maker extracts one or more solutions according to his/her preferences.

In many problems concerning the optimum design of steel frames, the commercial profiles assigned to the members in the structure are design variables. Another interesting variable to be analyzed is the orientation of the members' cross-section. Beams are usually oriented in such a way that their strong axes resist the stresses due to gravitational loads. On the other hand, columns' best orientations depend on the lateral loads distribution and structure shape. Regardless of the nature of the structural optimization problem, or the adopted design variables, in most cases, the problem must satisfy constraints concerning structural and aesthetic aspects.

There is considerable literature on single and multi-objective optimization of steel frames. In 1992, Chan [1] applied the optimality criteria algorithm for tall steel building design using standard sections and satisfying inter-story drift constraints. A design procedure employing a Teaching–Learning Based Optimization (TLBO) technique was presented in Toğan [2]. The study obtained the minimum weight of planar steel frames subjected to strength and displacement requirements imposed by the American Institute for Steel Construction (AISC). A comparison of different meta-heuristics in multi-objective optimization of steel frames, with displacement and weight minimization as conflicting objectives, is conducted by Gholizadeh and Baghchevan [3]. Recently, Resende *et al.* [4] analyzed multi-objective problems considering the structure weight minimization, first natural frequency maximization, critical load factor maximization, and maximum horizontal displacement minimization.

Structural optimization problems concerning column orientation have also been explored in prior studies. In 2010, Kızılkan [5] investigated the effect of the appropriate choice of columns orientations on the minimum weight of steel frames. In Lemonge and Barbosa [6] a space frame is optimized considering multiple cardinality constraints and design variables corresponding to the orientation of cross-section of the profiles searched for the columns.

This paper is organized as follows: Section 2 describes the formulation of the single and multi-objective optimization problems discussed in this paper, Section 3 exposes considerations about reinforced concrete slabs in the structure, Section 4 presents the basic concepts of the Differential Evolution, the constraint handling technique, and briefly describes the Multi-criteria decision-making used to extract the solutions from the Pareto sets, Section 5 presents the numerical experiments and their results are analyzed in Section 6. Finally, the conclusions and future works are presented in Section 7.

2 Single and multi-objective structural problems

The structural optimization problems presented in this paper refers to find $\mathbf{x} = \{x_1, x_2, ..., x_N\}$, a set of N design variables of the steel frame. The single-objective structural problem is written as shown in Equation (1), where $of(\mathbf{x})$ is the objective function to be minimized. The multi-objective structural problem is written as shown in Equation (2), where $of_1(\mathbf{x})$ and $of_2(\mathbf{x})$ are the conflicting objective functions to be minimized.

$$\begin{aligned} \min \quad & of(\mathbf{x}) \\ \text{s.t.} \quad & \text{structural constraints} \end{aligned} \tag{1}$$

$$\begin{aligned} \min \quad of_1(\mathbf{x}) \quad \text{and} \quad \min & \quad of_2(\mathbf{x}) \\ \text{s.t.} & \quad \text{structural constraints} \end{aligned} \tag{2}$$

The structure weight, which is an objective function in the computational experiments presented in this paper, is written as shown in Equation (3), where ρ_i, L_i and A_i are the specific mass, the length and the cross-sectional area of the i-th element of the structure, respectively, and N is the number of elements.

$$W(\mathbf{x}) = \sum_{i=1}^{N} \rho_i A_i Li \tag{3}$$

Two experiments are analyzed in this paper. In the first one, the only objective is to minimize the structure weight ($W(\mathbf{x})$). In the second one, minimizing the maximum horizontal displacement ($\delta_{max}(\mathbf{x})$) is also an objective, as shown in Equation (4). The displacements can be found using $[K]\{u\} = \{p\}$, the equilibrium equation for a discrete system of bars, where $[K]$ is the stiffness matrix, $\{u\}$ are the nodal displacements and $\{p\}$ are the load components (Bathe [7]). The lower and upper bounds in the search space are denoted by $\mathbf{x}_L$ and $\mathbf{x}_U$, respectively.

$$\begin{aligned} \text{Single-objective:} \quad & \min \quad W(\mathbf{x}) \\ \text{Multi-objective:} \quad & \min \quad W(\mathbf{x}) \quad \text{and} \quad \min \quad \delta_{max}(\mathbf{x}) \\ \text{s.t.} \quad & \text{structural constraints} \\ & \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{aligned} \tag{4}$$

The constraints of the problems are the maximum inter-story drift, the first natural frequency of vibration, the critical load factor concerning the global stability, the LRFD interaction equations for combined axial force and bending moments, the LRDF shearing equation, and geometric constraints referring to column-column connection. In the single-objective problem, the maximum horizontal displacement is also a constraint.

According to Brazilian ABNT [8] and American ANSI [9] codes, the structure must present a maximum horizontal displacement ($\delta_{max}(\mathbf{x})$) and a maximum inter-story drift ($d_{max}(\mathbf{x})$) that are lower than the maximum allowable values ($\bar{\delta}$ and $\bar{d}$). These values depend on the building height (H) and on the height between two consecutive stories (h) and are given by $\bar{\delta} = H/400$ and $\bar{d} = h/500$. These constraints are described in Equations 5 and 6.

$$\frac{\delta_{max}(\mathbf{x})}{\bar{\delta}} - 1 \leq 0 \tag{5}$$

$$\frac{d_{max}(\mathbf{x})}{\bar{d}} - 1 \leq 0 \tag{6}$$

The structure must present the first natural frequency ($f_1(\mathbf{x})$) higher than a minimum allowable ($\bar{f}_1$) as described in Equation (7). The natural frequencies of vibration are determined by solving an eigenproblem concerning the mass matrix ($[M]$) and the stiffness matrix ($[K]$) of the structure (Bathe [7]).

$$1 - \frac{f_1(\mathbf{x})}{\bar{f}_1} \leq 0 \tag{7}$$

To guarantee the structure's global stability, the critical load factor ($\lambda_{crt}(\mathbf{x})$) must be higher than one, as defined in Equation (8). The load factors are determined by solving an eigenproblem concerning the stiffness matrix ($[K]$) and the geometric stiffness matrix ($[K_G]$) of the structure. The critical load factor (λ_{crt}) is the lowest eigenvalue computed (McGuire *et al.* [10]).

$$1 - \frac{\lambda_{crt}(\mathbf{x})}{1} \leq 0 \tag{8}$$

All members of the structure must also satisfy the LRDF interaction equation for combined axial and bending (Equation (9)) and the LRDF shearing equation (Equation (10)). In the equations, the subscripts r and c refer, respectively, to the required and allowable axial strength (P), flexural strength about the major axis and the minor axis (M_x and M_y) and shearing strength (V). The methodology of determining the allowable strengths are similar in both ABNT [8] and ANSI [9] and adopted in this paper.

$$\begin{cases} \dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \dfrac{P_r}{P_c} \geq 0.2 \\ \dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \dfrac{P_r}{P_c} < 0.2 \end{cases} \tag{9}$$

$$\frac{V_r}{V_c} - 1 \leq 0 \tag{10}$$

The geometric constraints refer to the column-column connection, in order to establish that the upper column must not have, neither the profile depth nor the mass, higher than the lower column. Equations (11) and (12) show the geometric constraints, where $dp_i(\mathbf{x})$ and $dp_{i-1}(\mathbf{x})$ are the depth of the W section selected for the group of columns i and $i-1$, respectively. $ms_i(\mathbf{x})$ and $ms_{i-1}(\mathbf{x})$ are the unit weight of W section selected for the group of columns i and $i-1$, respectively. NG_c is the number of groups of columns.

$$\frac{dp_i(\mathbf{x})}{dp_{i-1}(\mathbf{x})} - 1 \leq 0 \quad i = 1, NG_c \tag{11}$$

$$\frac{ms_i(\mathbf{x})}{ms_{i-1}(\mathbf{x})} - 1 \leq 0 \quad i = 1, NG_c \tag{12}$$

3 Effects of reinforced concrete slabs in the structure

Reinforced concrete slabs were considered in the whole area of the six stories of the frame. This consideration leads to some significant structural effects. The stiffness of a slab incorporated into the space frame generates a rigid diaphragm effect. It means that the slab element absorbs part of the solicitant efforts and contributes to the stability of the structure and the increasing of the natural frequencies of vibration. In addition to that, the horizontal displacements in a story can be considered the same for all nodes in that story. On the other hand, considering the slabs in the structure mass has the negative effect of decreasing the natural frequencies of vibration.

In the problems analyzed in this paper, the consideration of the slabs was made in a simplified way, by increasing the beams' lateral inertia in the stiffness matrix ($[K]$) and increasing the beams' weight in the mass matrix ($[M]$). It is important to point out that the weight of the slabs is not considered in the objective function $W(\mathbf{x})$ described in Equation 3.

4 The GDE3, constraint-handling technique, and Multi-criteria decision making

The search algorithm adopted is the Third Step Differential Evolution (GDE3), proposed by Kukkonen and Lampinen [11] as an extension of the Differential Evolution (DE) proposed by Storn and Price [12]. The GDE3 starts by generating a random population and improves it using DE's selection, mutation, and crossover operations. The GDE3 parameters are the crossover rate ($C_r \in [0, 1]$), the scale factor ($F \in \mathbb{R}$) and the population size (N).

Let P_G be a population of N_p decision vectors $\mathbf{x}_{i,G}$ in generation G, where $i \in \{1, 2, 3, \ldots, N_p\}$ is a vector index. Each $\mathbf{x}_{i,G}$ of the population in generation G is a n-dimensional vector and $\mathbf{x}_{j,i,G}$ is its j-th component ($j \in \{1, 2, 3, \ldots, n\}$). Applying mutation and crossover operations (Storn and Price [12]), each decision vector $\mathbf{x}_{i,G}$ creates a corresponding trial vector $\mathbf{u}_{i,G}$. After that, the trial vector $\mathbf{u}_{i,G}$ is compared to the decision vector $\mathbf{x}_{i,G}$ using the constraint domination concept. A vector $\mathbf{x}$ dominates a vector $\mathbf{y}$ (denoted by $\mathbf{x} \succeq_c \mathbf{y}$) if one, and only one, of the following conditions is true (i): both are unfeasible and $\mathbf{x} \succ \mathbf{y}$ in the constraint function violation space; (ii) $\mathbf{x}$ is feasible and $\mathbf{y}$ is unfeasible, or (iii) $\mathbf{x}$ and $\mathbf{y}$ are feasible and $\mathbf{x} \succ \mathbf{y}$ in the objective function space. If $\mathbf{u}_{i,G} \succeq_c \mathbf{x}_{i,G}$, the trial vector $\mathbf{u}_{i,G}$ is selected to replace the decision vector $\mathbf{x}_{i,G}$ in the next generation P_{G+1} (population in generation $G+1$). If $\mathbf{x}_{i,G} \succeq_c \mathbf{u}_{i,G}$, $\mathbf{u}_{i,G}$ is discarded and $\mathbf{x}_{i,G}$ remains in the population. Otherwise, both are included in P_{G+1}. A complete and detailed description of the entire GDE3 algorithm can be found in Vargas *et al.* [13].

The Adaptive Penalty Method (APM) proposed by Barbosa and Lemonge [14] is adopted in this paper to handle the constraints. The APM adapts the value of the penalty coefficients of each constraint by using information collected from the population, such as the mean objective function and the level of violation of each restriction. With these information, the method automatically sets a higher penalty coefficient on those constraints that seem to be more difficult to satisfy.

After obtaining the Pareto fronts of non-dominated-solutions, a Multi Tournament Decision Method (MTD) method was adopted to extract the solutions. Weights of importance (w_i) for each objective function are established by the Decision Maker and, according to them, the MTD ranks the best and the worst possible solutions in the Pareto frontier. The complete description of the MTD and a pseudo-code for this method can be found in Parreiras and Vasconcelos [15].

5 Numerical Experiments

The structural optimization problems analyzed in this paper concern the six-story spatial steel frame with 258 members and 126 joints illustrated in Figure 1. The bays in both x and y directions have a regular spacing of 3m and each story is 3m height.

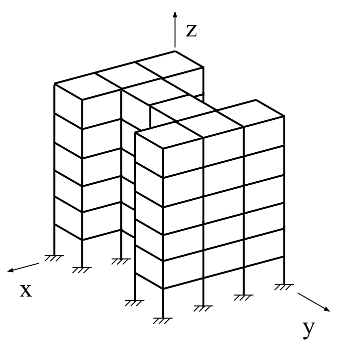

Figure 1. Six-story spatial steel frame

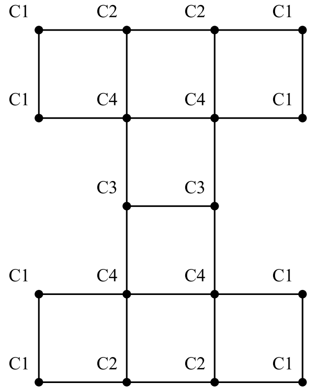

Figure 2. Member grouping for columns in plan level

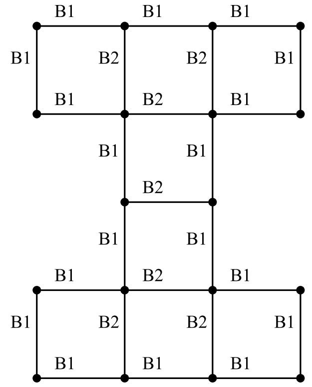

Figure 3. Member grouping for beams in plan level

The sizing design variables are to be chosen from a subset of commercial W-shape sections. The columns and beams are linked as detailed in Figures 2 and 3, respectively. The groups change for every three stories resulting in twelve sizing design variables. The column orientation of a group remains the same for all six stories, which leads to four additional design variables. That results in a total of sixteen design variables, denoted by $\mathbf{x} = \{C_1, ..., C_8, B_1, ..., B_4, O_1, ..., O_4\}$, where C_i and B_i are integer indexes that designate the commercial steel profiles for each group of columns and beams, respectively, and O_i is an index that indicates the orientation of a group of columns. If O_i equals 0, the web of the columns of the i-th group are oriented in the x direction (⊢⊣) and if O_i equals 1, the web of the columns of the i-th group are oriented in the y direction (**I**).

The gravity loads that act on the structure are 10 kN/m in the outer beams (B1 grouping) and 20 kN/m in the inner beams (B2 grouping). The horizontal loads in the top of the columns are detailed in Figures 4 and 5. The concrete slabs have a thickness of 10 cm and a specific weight of 25 kN/m^3. The maximum allowable displacement is $\overline{\delta} = 45mm$, the maximum allowable inter-story drift is $\overline{d} = 6mm$ and the minimum allowable frequency of vibration is $\overline{f_1} = 1Hz$.

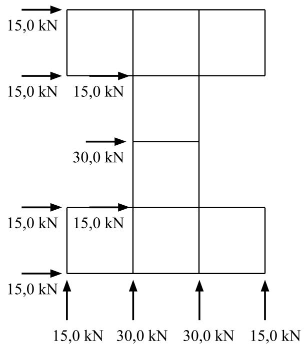

Figure 4. Horizontal loads on the stories 1 to 5

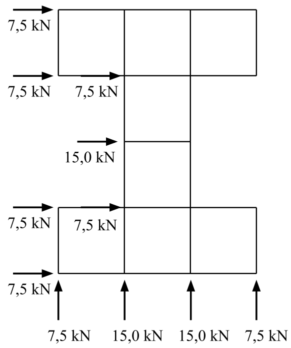

Figure 5. Horizontal loads on the story 6

Ten independent runs with 100 generations and a population of 50 candidate vectors are set for the two problems. The DE parameters adopted are the crossover ratio $C_r = 0.9$, the mutation probability $M = 0.1$ and the scale factor $F = 0.4$. In the multi-objective problem, three solutions are extracted by the Multi-criteria Tournament Decision: (i) scenario 1: w_1 = 0.75 and w_2 = 0.25; (ii) scenario 2: w_1 = w_2 = 0.5; (iii) scenario 3: w_1 = 0.25 and w_2 = 0.75. Where w_1 and w_2 are the importance weights for the two conflicting objective functions: minimization of the structure's weight and minimization of the maximum horizontal displacement, respectively.

The columns orientations of the solution of the single-objective problem are represented in Figure 6. The Pareto front of non-dominated solutions obtained for the multi-objective problem, as well as the columns orientations of the extracted solutions, are illustrated in Figure 7. Finally, the results found for the single and multi-objective problems are presented in Table 1, where $LRFD_{max}$ and V_{max} are the highest values found for the LRDF interaction and shearing equations, respectively.

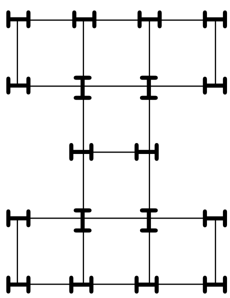

Figure 6. Layout of the best solution in the single-objective problem

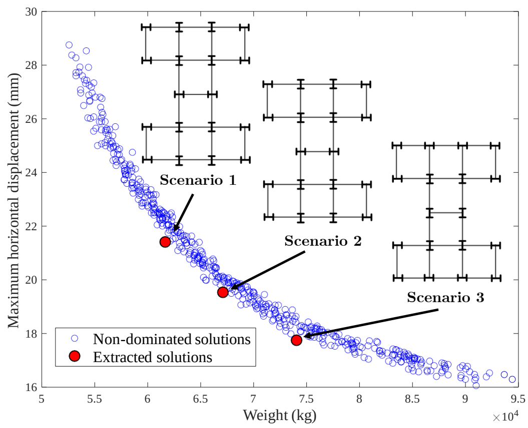

Figure 7. Pareto front for the multi-objective optimization problem with layout of the extracted solutions

Table 1. Best results found for the single and multi-objective problems presenting the profiles assigned to each member group, the columns' cross-sections orientation and constraints and objective function values.

		Single-objective		Multi-objective					
Scenario				1		2		3	
Group	Stories	W Profiles and orientations (for columns)							
C1	1-3	310x117	H	310x117	H	310x117	H	310x117	H
	4-6	310x79		310x97		310x107		310x117	
C2	1-3	310x93	H	310x117	I	310x117	I	310x117	H
	4-6	150x22.5		310x79		310x97		310x110	
C3	1-3	310x107	H	310x97	H	310x125	H	310x117	I
	4-6	310x79		310x79		310x107		310x79	
C4	1-3	310x97	I	310x117	I	310x117	I	310x117	I
	4-6	310x97		310x107		310x117		310x117	
B1	1-3	530x66		530x66		530x82		610x101	
	4-6	410x46.1		530x66		530x66		530x72	
B2	1-3	310x21		410x38.8		410x38.8		530x66	
	4-6	460x60		460x89		250x32.7		250x25.3	
Constraints and objective functions values									
$LRFD_{max}(\mathbf{x})$		0.9323		0.8141		0.7790		0.7298	
$V_{max}(\mathbf{x})$		0.1918		0.1288		0.1285		0.1108	
$d_{max}(\mathbf{x})(mm)$		6.0		5.0		4.6		4.2	
$f_1(\mathbf{x})(Hz)$		2.0945		2.2731		2.3423		2.3797	
$\lambda_{crt}(\mathbf{x})$		45.1694		52.0633		54.5901		57.3947	
$\delta_{max}(\mathbf{x})(mm)$		28.9		21.4		19.5		17.7	
$W(\mathbf{x})(kg)$		51071		61656		67100		74053	

6 Analysis of results

Table 1 presents the results found for both single and multi-objective problems. As far as the structure weight is concerned, one can observe that the lightest structure, as expected, was obtained in the single-objective problem ($W(\mathbf{x}) = 51071$ kg). In scenario 1 of the multi-objective problem the structure weight is $W(\mathbf{x}) = 61656$ kg, value that is 21% higher than in the single-objective problem. As the importance weight w_1 decreases and w_2 increases, the value of $W(\mathbf{x})$ also increases ($W(\mathbf{x}) = 74053$ kg in scenario 3, 20% higher than in scenario 1).

With regard to the horizontal displacements, one can observe that in single-objective problem, the maximum inter-story drift ($d_{max}(\mathbf{x})$) obtained was equal to the maximum allowed ($\bar{d} = 6.0$mm), thus being an active constraint. The highest horizontal displacement was also obtained in this case ($\delta_{max}(\mathbf{x}) = 28.9$ mm). In scenario 1 of the multi-objective problem the maximum horizontal displacement is $\delta_{max}(\mathbf{x}) = 21.4$ mm, value that is 26% lower than in the single-objective problem. As the importance weight w_1 decreases and w_2 increases, the value of $\delta_{max}(\mathbf{x})$ decreases ($\delta_{max}(\mathbf{x}) = 17.7$ mm in scenario 3, 17% lower than in scenario 1).

Another interesting point is that, for the four solutions described in Table 1, as the weight increases, the values of $LRFD_{max}(\mathbf{x})$, $V_{max}(\mathbf{x})$ and $d_{max}(\mathbf{x})$ decrease and the values of $\lambda_{crt}(\mathbf{x})$ and $f_1(\mathbf{x})$ increase. This is intuitive since higher weights are related to more rigid structures, which present a better mechanical behaviour. As far as column orientation is concerned, one can observe that the orientations obtained for the columns in groups C1 and C4 remained the same for the four extracted solutions. Moreover, the column orientations for scenarios 1 and 2 of the multi-objective problem were identical. That leads to the conclusion that these are significant variables.

7 Conclusions

This paper analyzed single and multi-objective structural optimization problems of a six-story space steel frame, considering the structure weight minimization as the first objective function and the second one concerning the maximum horizontal displacement at the top of the frame to be minimized. The design variables are the profiles assigned to the beams and the profiles assigned to the columns in which their orientations, concerning the principal axes, are also design variables. In the multi-objective problem, three different scenarios were considered with different importance weights for the objective functions. In the first scenario, the structure weight was given an importance of 25%, in the second 50%, and the third 75%.

The results obtained from the single and multi-objective numerical experiments analyzed in this paper presented coherent aspects, as expected. The results concerning column orientation showed that this is a significant variable. For future work, it is intended to make comparisons with other evolutionary algorithms. Furthermore, it is expected to extend the analyses to large-scale problems, considering different objective functions and other design variables.

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