Multi-objective optimization of ground structures with constraints of overlapping bars and maximum stresses

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Abstract. Structural multi-objective optimization problems are common in real-world problems of the Engineering field where one or more objective functions may be considered and desired to be optimized. In general, these functions are conflicting, leading to complex optimization problems. This paper analyses the multi-objective structural optimization of truss ground structures considering the weight minimization (or volume) as the first objective function and compliance as the second one. The constraints refer to the allowable axial stresses in the bars and also the bars overlapping. Thus, the structural optimization problems consider sizing and topology design variables simultaneously, and they can be continuous, discrete, or mixed. After obtaining the Pareto Front, one of the most important steps is defining which solution or solutions will be considered after obtaining the Pareto curve. Unfortunately, this task is not trivial, and a Multi-Tournament Decision method is applied to extract the solutions from the Pareto based on Decision-Maker preferences. The search algorithm adopted here is a modified version of the Differential Evolution called Third Evolution Step Differential Evolution (GDE3).

Keywords: Multi-objective structural optimization; ground-structures; differential evolution, multi-criteria decision-making.

1 Introduction

Topology optimization is a powerful tool that's largely used in many fields of engineering. It consists of removing material from regions where it's unnecessary, which is very interesting for lowering manufacturing costs and improving efficiency, for example, when the use of the minimal weight is desirable, such as aviation, high-performance cars, and others.

In many cases, weight is only one of the desirable aspects to be manipulated during the optimization process. The decision-maker (DM) might want to observe the behavior of one aspect regarding another, and, hence, a multi-objective problem where the objective functions are conflicting can be employed. Once the process is complete, one can analyze the Pareto and choose the preferred solution among the ones presented on it according to its preferences, assigning weights to the objective functions in order to prioritize one or another.

A survey on this theme can be related on following studies and it's respective referenced works: Zegard and Paulino [1], Mela [2], Kanno [3], Zhang et al. [4].

This paper is organized as follows: Section 2 describes the formulation of the multi-objective optimization problem. Section 3 presents the numerical experiments analyzed in this paper. The results of the numerical experiment are provided in Section 4, and finally, the conclusions and future works are presented in Section 5.

2 The multi-objective structural optimization problem

The multi-objective structural optimization problem of this study can be defined as follows:

$$\begin{aligned} \min \quad & (W(\mathbf{x}), \delta_{max}(\mathbf{x}), -f_1(\mathbf{x})) \\ \text{s.t.} \quad & constraints \end{aligned} \tag{1}$$

where **x** is the vector containing the areas of the structure, $W(\mathbf{x})$ is the weight, $\delta_{max}(\mathbf{x})$ is the maximum displacement and $f_1(\mathbf{x})$ is the first natural frequency. In this paper, two ground structures will be optimized under constraints of stresses and overlapping-bars.

- The weight of structure is obtained by the sum:

$$W(\mathbf{x}) = \sum_{i=1}^{N} \rho A_i L_i \tag{2}$$

where ρ is the specific mass of the material, A_i and L_i are the cross-sectional areas and the length of the i-th bar of the structure, respectively. The number of bars of the structure is denoted by N.

- The nodal displacements $\{\delta\}$ are obtained by the equilibrium equation for a discrete system of truss bars (3 degrees of freedom per node), which is written as:

$$[K]\{\delta\} = \{F\} \tag{3}$$

where $[K]$ is the elastic stiffness matrix of structure and $\{F\}$ is the nodal force vector.

- The natural frequencies of vibration are obtained by evaluating the eigenvalues of matrix

$$\left([K] - f_{m_f}^2 [M]\right) \phi_{m_f} = 0 \tag{4}$$

where $[M]$ is the mass matrix (consistent formulation) and ϕ_{m_f} is the m_f-th eigenvector corresponding to the m_f-th eigenvalue.

The optimization algorithm used in this study is The Third Evolution Step of Generalized Differential Evolution (GDE3) proposed by Kukkonen & Lampinen, fully described in Kukkonen and Lampinen [5]. Furthermore, to handle the constraints, the Adaptative Penalty Method was chosen, which can be related in Barbosa and Lemonge [6]. Finally, the extraction of solutions from Pareto was made via Multicriteria Tournament Decision Method (MCDM) described in Parreiras and Vasconcelos [7].

3 Numerical experiments

In this section, the numerical experiments are described. They are the L-shape (Mela [2]) and a bi-supported cantilever beam (Kanno [3]). Both experiments were performed into 20 independent runs, and the respective number of function evaluations (nfe) per run is indicated in each description.

3.1 The L-Shape domain

This experiment is the L-Shaped domain which is shown in Figure 1. It has 21 nodes and 86 non-overlapping members, where the vertical load is applied at the right middle node and equal to 400 kN. The search space of the areas is composed of the Square Hollow Sections (SHS) profiles from Table 6 of the study by Mela [2]. The Young modulus is equal to 200 GPa, the Yield Strength (fy) is equal to 450 MPa, and the specific mass is equal to 7860 kg/m^3. This experiment was performed with a population size of 100 individuals throughout 500 generations (50000 nfe).

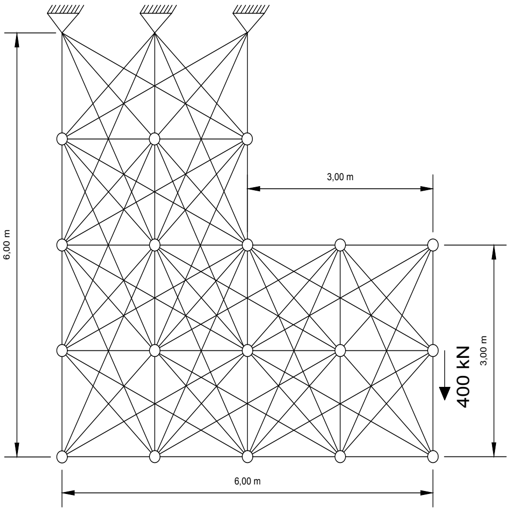

Figure 1. The L-shape domain.

The obtained results for respective weights are shown in Figures 2 - 4:

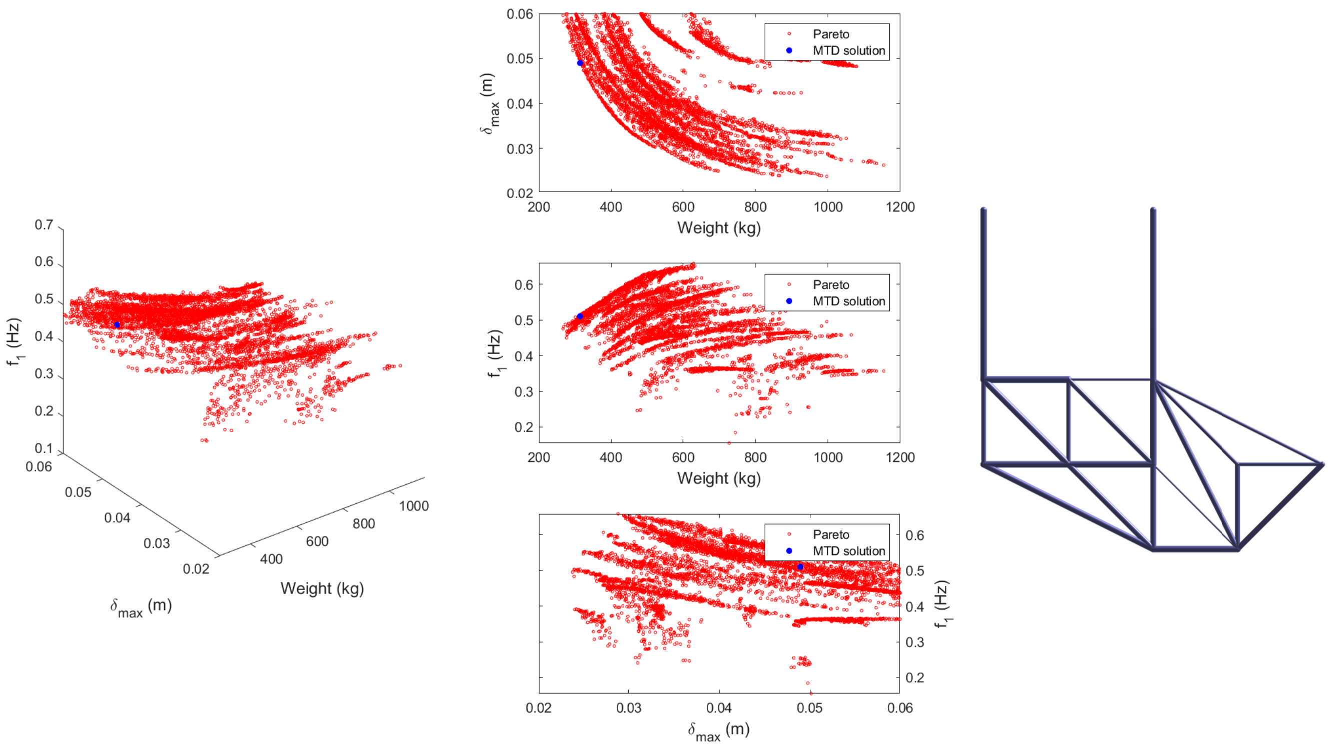

Figure 2. Pareto and the extracted solution setting w_1 = 0.90, w_2 = 0.05 and w_3 = 0.05 - L-shape.

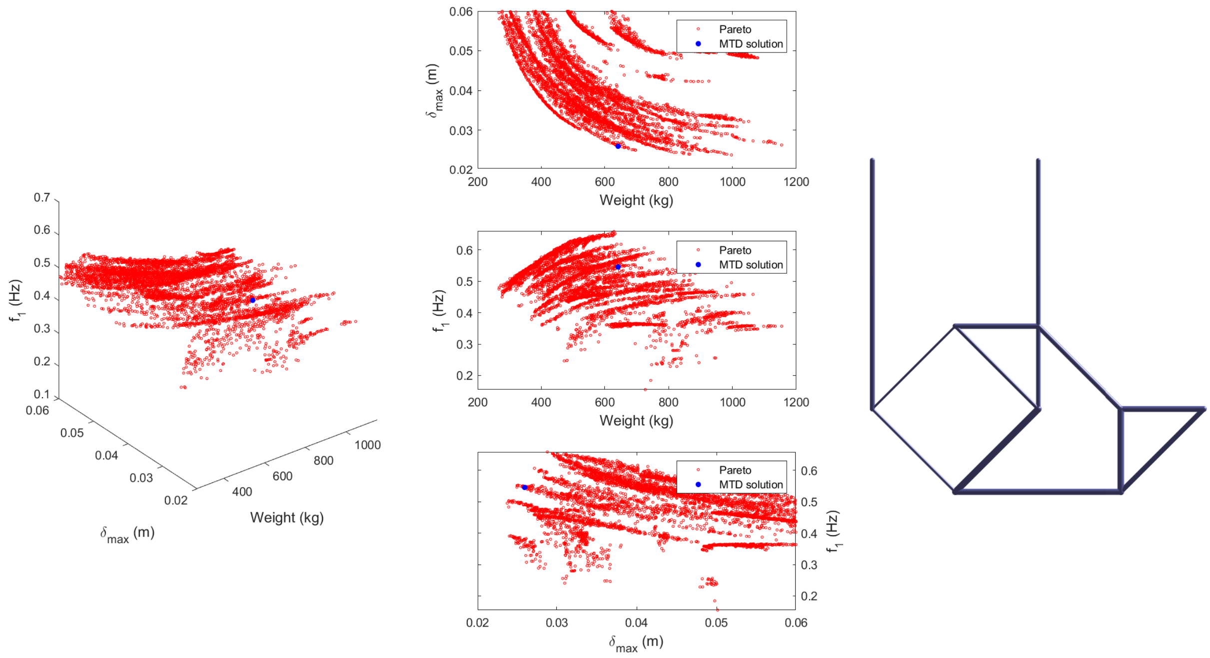

Figure 3. Pareto and the extracted solution setting w_1 = 0.05, w_2 = 0.90 and w_3 = 0.05 - L-shape.

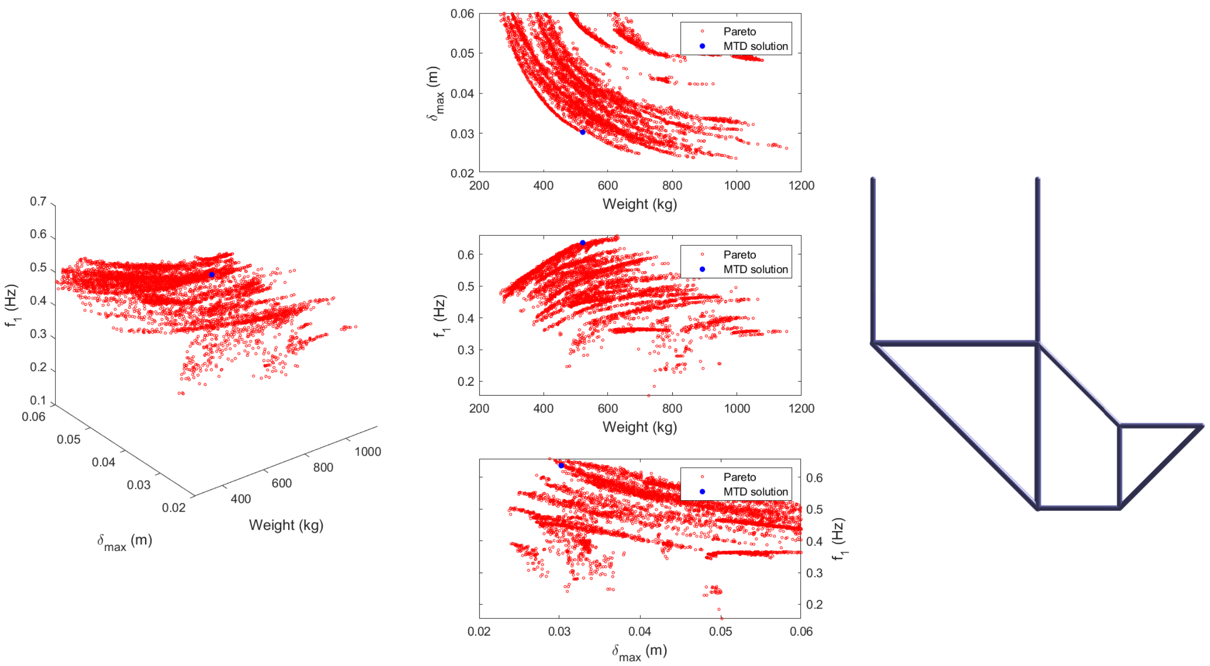

Figure 4. Pareto and the extracted solution setting w_1 = 0.05, w_2 = 0.05 and w_3 = 0.90 - L-shape.

3.2 The bi-supported cantilever domain (bscd)

This experiment is the ground structure shown in Figure 5. The grid is made up of 5 x 5 nodes leading to 200 non-overlapping bars as design variables. The continuous search space for the areas is bounded by 1.0 and 20.0 cm^2. The Young modulus is equal to 200 GPa, the Yield Strength (fy) is equal to 450 MPa, and the specific mass is equal to 7860 kg/m^3. This experiment was performed with a population size of 200 individuals throughout 1000 generations (200000 nfe).

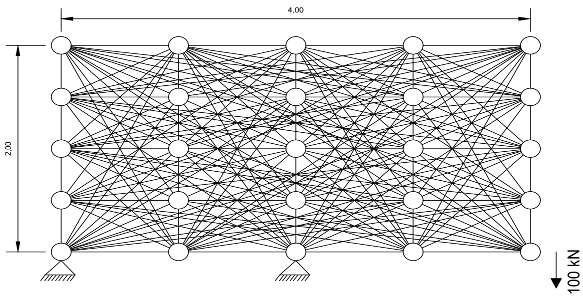

Figure 5. The Cantilever domain.

The obtained results for respective weights are shown in Figures 6 - 8.

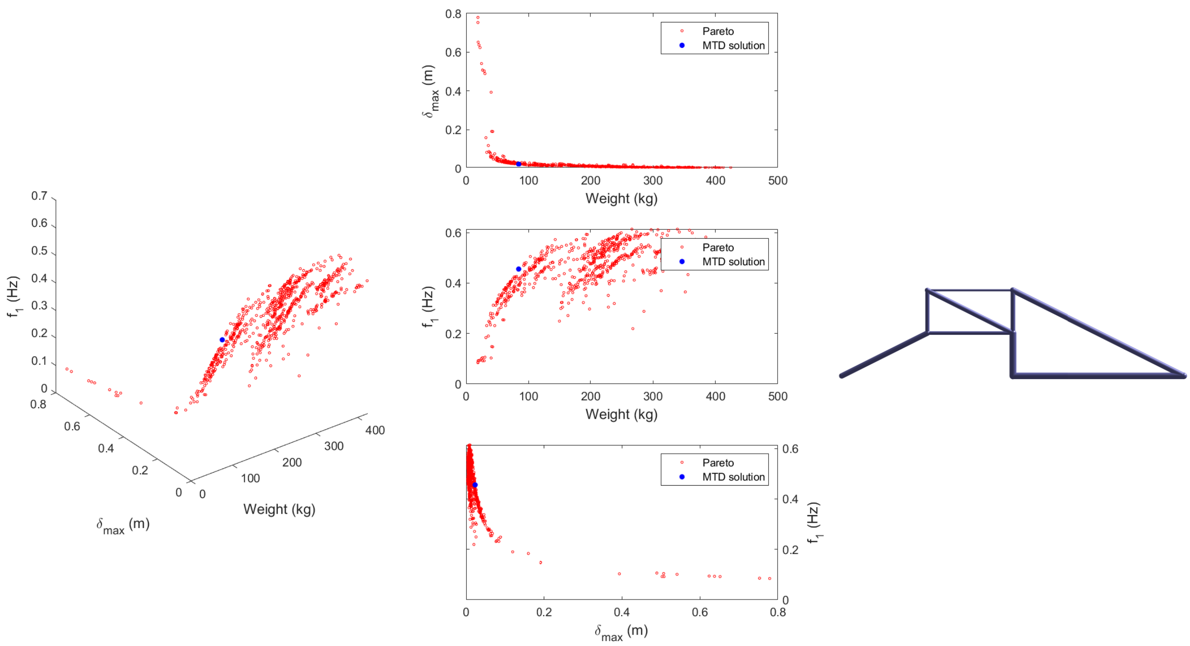

Figure 6. Pareto and the extracted solution setting w_1 = 0.90, w_2 = 0.05 and w_3 = 0.05 - bscd.

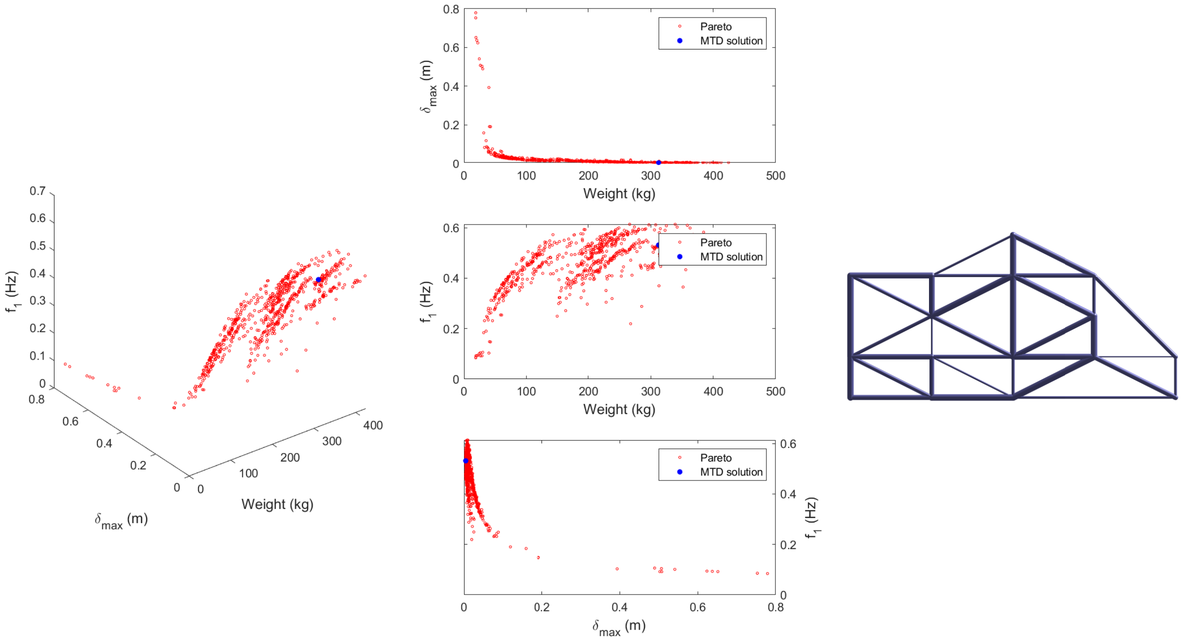

Figure 7. Pareto and the extracted solution setting w_1 = 0.05, w_2 = 0.90 and w_3 = 0.05 - bscd.

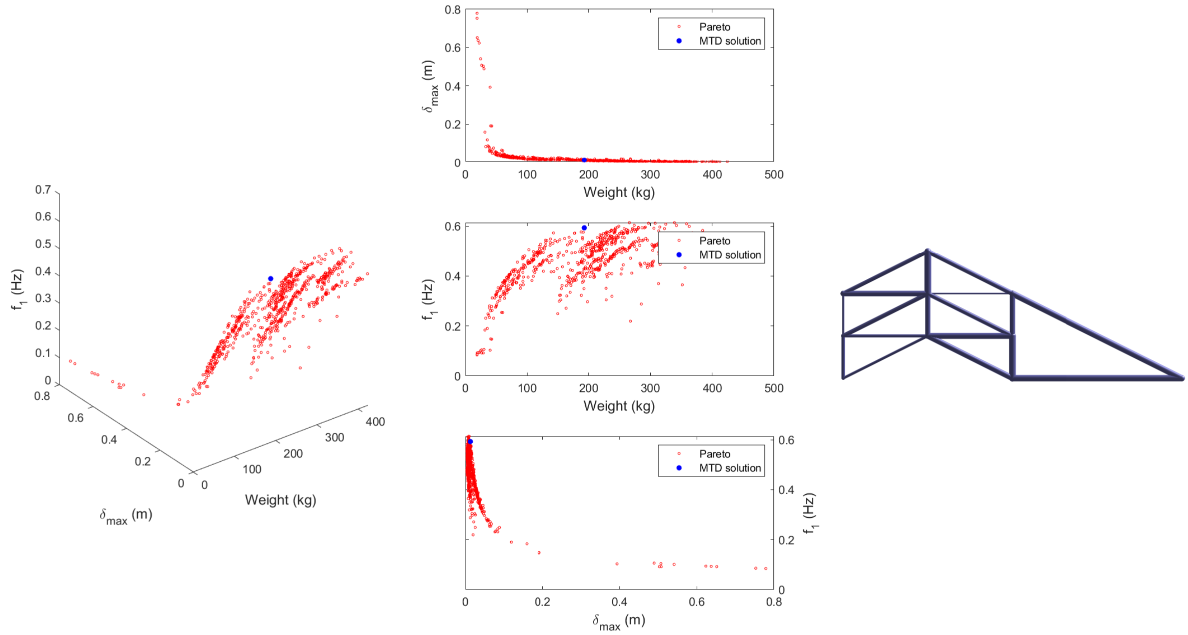

Figure 8. Pareto and the extracted solution setting w_1 = 0.05, w_2 = 0.05 and w_3 = 0.90 - bscd.

4 Results

The results obtained are interesting and visually attractive for engineering manufacturing if it was the case. In addition, the constraints of not allowing overlapping bars allow models that are more reliable in two dimensions if compared to designs where overlapping bars are allowed.

The Pareto fronts obtained for both experiments show translated curves representing different topologies obtained during the optimization process. For both cases when prioritizing maximize the first natural frequency (w_1 = 0.05, w_2 = 0.05 and w_3 = 0.90), the obtained results showed topologies with bars located in the "path" when the load is transferred from the load point to the supports. Prioritizing the weight (w_1 = 0.90, w_2 = 0.05 and w_3 = 0.05), one could intuitively expect a small number of bars, which occurred only for the bscd model; for the L-shape problem, the solution for this preference presented a large number of bars (with small areas) if compared to the other two extractions.

The L-shape problem presented a Pareto with a large number of non-dominated solutions compared to the bscd, so this first set of results provides a wide offer of different topologies to be chosen by the Decision-Maker to extractions that are very sensitive to the weights.

5 Conclusions and future works

The proposed methodology showed interesting results for the problems analyzed. The chosen structures via the extraction method show coherent points in Pareto. The obtained topologies are visually attractive and intuitive when looking at the respective original ground structure models. As futures works, it is intended to consider many objective functions in the optimization problem formulation and model the structures like frames, leading to conclusions on whether a model best describes this type of problem.

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References

[1] T. Zegard and G. H. Paulino. Grand — ground structure based topology optimization for arbitrary 2d domains using matlab. *Structural and Multidisciplinary Optimization*, vol. 50, pp. 861–882, 2014.

[2] K. Mela. Resolving issues with member buckling in truss topology optimization using a mixed variable approach. *Structural and Multidisciplinary Optimization*, vol. 50, pp. 1037–1049, 2014.

[3] Y. Kanno. Mixed-integer second-order cone programming for global optimization of compliance of frame structure with discrete. design variables. *Structural and Multidisciplinary Optimization*, vol. 54, pp. 301–316, 2016.

[4] X. Zhang, A. S. Ramos Jr, and G. H. Paulino. Material nonlinear topology optimization using the ground structure method with a discrete filtering scheme. *Structural and Multidisciplinary Optimization*, vol. 55, n. 6, pp. 2045–2072, 2017.

[5] S. Kukkonen and J. Lampinen. Gde3: the third evolution step of generalized differential evolution. In *2005 IEEE Congress on Evolutionary Computation*, volume 1, pp. 443–450 Vol.1, 2005.

[6] H. J. C. Barbosa and A. C. C. Lemonge. An adaptive penalty scheme in genetic algorithms for constrained optimization problems. In *GECCO'02: Proceedings of the Genetic and Evolutionary Computation Conference.*, pp. 287–294, New York, 2002.

[7] R. Parreiras and J. Vasconcelos. Decision making in multiobjective optimization aided by the multicriteria tournament decision method. *Nonlinear Analysis: Theory, Methods & Applications*, vol. 71, n. 12, pp. e191–e198, 2009.