Dynamic Analysis of Aircraft Floors

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Abstract. In this paper, we present a study of the dynamic analysis of aircraft floors. These are usually constructed of either composite materials plates or metal plates resting on longitudinal spars. In either case, bending of multiple materials layers must be considered. As we are studying vibrations, that is, small displacements about a static equilibrium configuration, small strains are also to be expected, leading to validity of the assumption of linear behavior of all involved materials. Further, we also consider valid Bernoulli-Euler's Hypothesis that plane sections remain plane after bending, that is, strains are linearly proportional to the distance of a particular layer to the neutral axis of the section, whose position must be determined via equilibrium considerations. Thus, an equivalent Young's Modulus can be determined. After this phase of the analysis, usual linear vibration frequencies and modes computation can be carried out either via the Finite Element Method or closed form solutions available in literature. We also present forced vibrations due to people motion on the floor using a Fourier approach.

Keywords: aircraft floors, composite materials, dynamic analysis.

1 Introduction

We propose a scheme for the dynamic analysis of aircraft floors. These are usually constructed of either composite materials plates or metal plates resting on longitudinal spars. In either case, bending of multiple materials layers must be considered. Here we follow the classical approach of Timoshenko (1940). As we are studying vibrations, that is, small displacements about a static equilibrium configuration, small strains are also to be expected, leading to validity of the assumption of linear behavior of all involved materials. Further, we also consider valid Bernoulli-Euler's Hypothesis that plane sections remain plane after bending, that is, strains are linearly proportional to the distance of a particular layer to the neutral axis of the section, whose position must be determined via equilibrium considerations. Thus, an equivalent Elastic Modulus can be determined. After this phase of the analysis, usual linear vibration frequencies and modes computation can be carried out either via the Finite Element Method or closed form solutions available in literature, such as in Leissa (1969). We also present forced vibrations due to people motion on the floor using a Fourier approach, as suggested by Bachmann and Amman (1989).

2 Bending of multiple materials layers plates

2.1 Theory

Consider a plate strip composed of 2 layers, with unit width, height *h*, in pure bending. The Young's modulus adopted for these two materials are E_U and E_L., for the upper and lower layers, respectively.

It is postulated the validity of the Bernoulli-Euler hypothesis, that the cross sections remain plane after bending, resulting in that the deformations vary linearly with the distance from the neutral line, where the stresses are null, and have equal value across the entire width. in each fiber.

Let's adopt Timoshenko's (1940) technique of replacing the composite section with an equivalent one of a single material. For example, consider the section to be composed of just the E_U material, set as reference, with the other part replaced by a layer of the reference material the of same thickness with an appropriately determined width *b*.

According to Hooke's Law and Euler-Bernoulli hypothesis, the deformation in a fiber, which does not depend on the material, is

$$\varepsilon = \frac{\sigma_U}{E_U} = \frac{\sigma_L}{E_L} \tag{1}$$

resulting

$$\frac{\sigma_U}{\sigma_L} = \frac{E_U}{E_L} \tag{2}$$

The forces resulting from these stresses in each area element of the transformed section must be equal

$$\sigma_U dA_U = \sigma_L dA_L \tag{3}$$

$$dA_U = \frac{\sigma_L}{\sigma_U} dA_L = \frac{E_L}{E_U} dA_L = n_{LU} dA_L \tag{4}$$

As the thickness of the layer of second material converted to the reference one does not change, the width *b* of this layer has to be multiplied by n_{LU}. Thus, as the original width of the plate was adopted as unitary, we will have a T section, with the width of the reference material layer still unitary, and the other material layer with width numerically equal to n_{LU} , the relationship between the modulus of elasticity of the two materials .

2.2 Equivalent Young's Modulus

Calculating the moment of inertia of the equivalent section made only of the reference material with respect to the neutral line, the equivalent elastic modulus of the plate is given by equality

$$E_U I_{eq} = E_{eq} I \qquad E_{eq} = E_U\, I_{eq}/I \tag{5}$$

where *I* is the actual moment of inertia of the section. If it is a simple unity width, *h* height, plate strip,

$$I = \frac{h^3}{12} \tag{6}$$

As a numerical example, let's consider the reference material upper layer thickness to be 100 mm and its elastic modulus equals to 38.3 GPa. The lower layer is adopted to be 160 mm thick and its elastic modulus to be 9.9 GPa. Applying the above theory, the resulting one material section has an upper layer 1 m wide and 100 mm thick, as it previously was, and a lower layer 258.5 mm wide and 160mm thick. The distance from the bottom of the section to the neutral line is 172 mm. The moment of inertia of the equivalent section made of only the reference material is 6.66 x 10^{-4} m^4. The moment of inertia of the original unit width plate strip is 0.015 m^4, leading to an equivalent modulus of elasticity of 17.4158 GPa

3 Frequency and modes of free undamped vibrations

The analyzed sample plate has dimensions in plan of 7 m x 27 m. It considered clamped along one of the 27 m faces, simply supported on the opposite 27 m face, and free on the 7 m orthogonal faces.

A finite element analysis was performed, in which Figure 1 shows the discretization and boundary conditions considered. Discretization took place in square elements with sides equal to 0.25 m. The two free edges are parallel to the x2 axis.

In addition to the structure's own mass, adopted as 1,362 kg/m³, an additional mass of 100 kg/m² was considered. Thus, the computed natural vibration frequencies are shown in Tables 1.

For comparisons, the tabulated solution for the first frequency of rectangular plates with the boundary conditions of the case, of thickness *t*, linear elastic material, provided by Leissa/NASA (1969) and Brazil (2019), in Hz, is

$$f = \frac{\pi}{2a^2}\sqrt{\frac{2{,}56\,D}{\rho}} \tag{7}$$

where

$$D = \frac{Et^3}{12(1-\nu^2)} \tag{8}$$

ρ is the material density, a the plate span and ν the Poisson coefficient. The value obtained from Eq. (7) for the adopted numerical values is 10.4 Hz, which compares well with the first frequency value in Table 1.

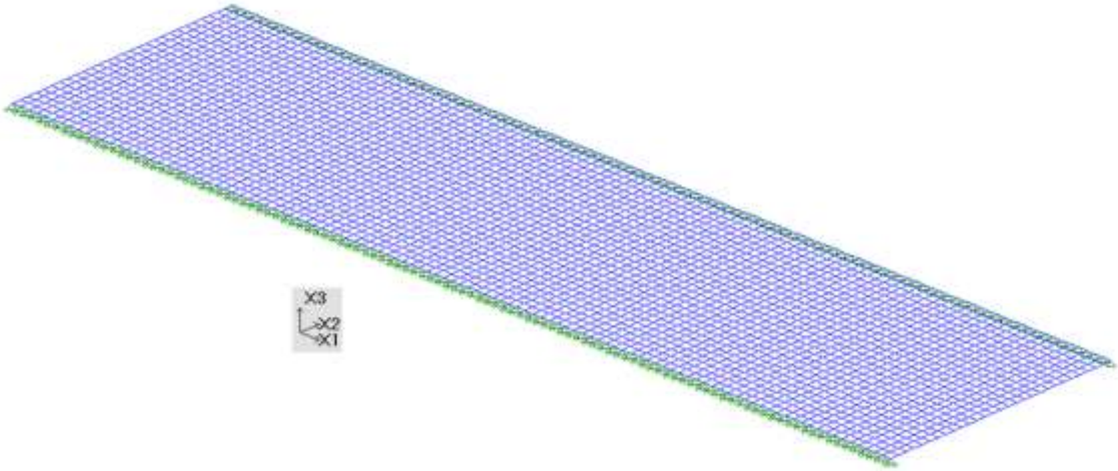

Figure 1: Plate FEM model

Table 1 – Natural free frequencies of vibration

Mode N.º	Eigenvalue (Omega**2)	Frequency (Hertz)	Period (s)	Maximum Displacment Node-DOF
1	3887.831	9.9237	0.10077	2937-3
2	4106.315	10.1987	0.09805	465-3
3	4889.938	11.1294	0.08985	465-3
4	6404.390	12.7368	0.07851	465-3
5	8984.424	15.0857	0.06629	465-3
6	13126.064	18.2342	0.05484	2937-3
7	19500.963	22.2253	0.04499	436-3
8	28960.822	27.0848	0.03692	2909-3
9	41658.301	32.4841	0.03078	262-3
10	42535.723	32.8244	0.03047	2909-3

For the sake of completeness, the first two vibration modes are displayed in Figures 2 and 3.

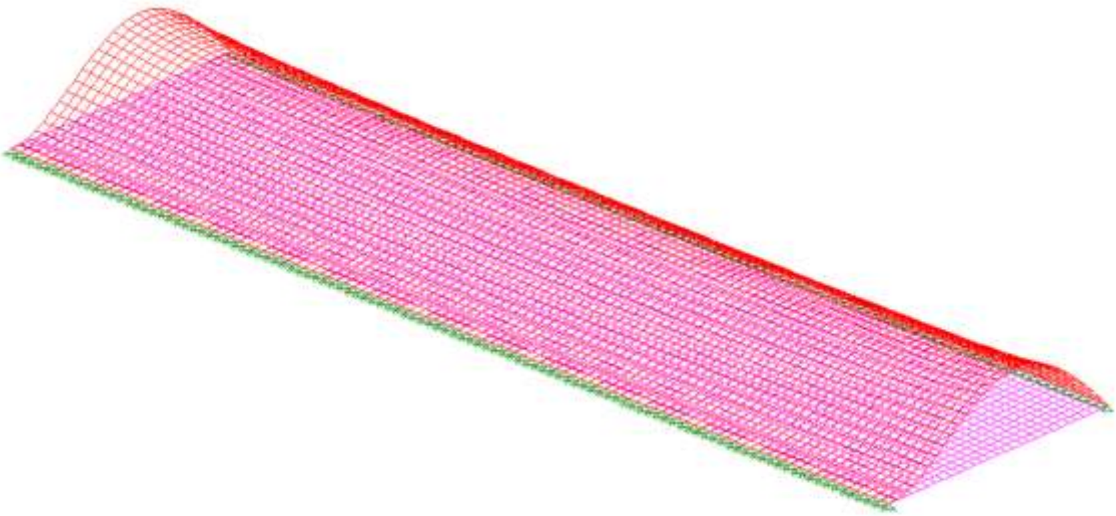

Figure 2: First vibration mode

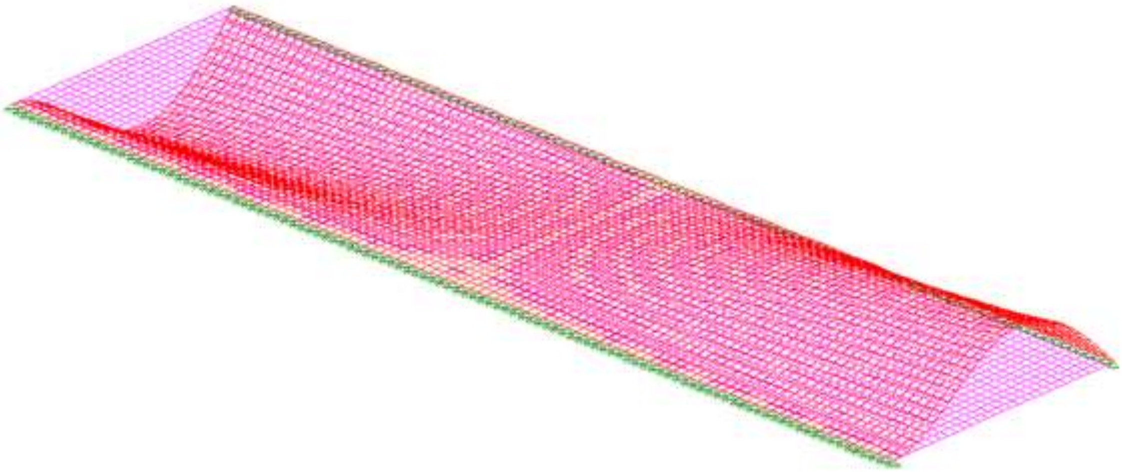

Figure 3: Second vibration mode

4 Forced vibrations

According to Bachmann & Ammann (1995) and Brasil & Silva (2015), the floating (dynamic) part of the loading due to pedestrian movement, a function of time t, is periodic and given by the superposition of Fourier harmonics, according to Eq. (9).

$$F_{fl}(t) = G\sum_{k=1}^{3} a_k \sin\left(2k\pi f_p\, t - \varphi_k\right) \quad (9)$$

where

G : person's weight per m²;

f_P : adopted forcing fundamental frequency of walking, 2 Hz;

a_k : Fourier coefficients for each harmonic: $a_1 = 0{,}4$, $a_2 = 0{,}1$ and $a_3 = 0{,}115$;

φ_k: phase angle for each harmonic, equal to 0 for the first e $\pi/2$ for the second and third.

Adopted G=700 N/m², one person per m².

Forced vibration analysis via a FEM software resulted a steady state RMS acceleration around 0.4 m/s², an acceptable value as far as person's comfort is considered, according to Bachmann & Ammann (1995).

5 Conclusions

We presented a scheme for the dynamic analysis of composite materials aircraft floors excited by people motions. First, we determined an equivalent elastic modulus for multi materials layered plates, based on Bernoulli-Euler bending theory, supposing the materials display linear elastic behavior, due to the assumption that vibrations excited by this kind of loads are of small amplitude. Next, regular FEM procedures were carried out to compute the frequencies and modes of free undamped vibrations. Finally, a forcing function of excitation due to people motion was written based on Fourier decomposition coefficients available the literature.

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