DETERMINATION OF THE FIRST NATURAL FREQUENCY OF VIBRATION OF A STEEL POLE, UNDER THE EFFECT OF GEOMETRIC NONLINEARITY, USING OPTIMIZATION TECHNIQUES

Paulo H. dos S. Matos[1], Marcelo A. da Silva[2],

[1] *Structural specialist by PECE-Poli in University of São Paulo*
Street Maria Moura da Conceição, 397 (Belcito Garden), Cond. Mendoza, Bl. 01, Apt. 203, 04855-257, São Paulo-SP, Brasil
paulo_hdsm@hotmail.com
[2] *PhD in structural engineering and structure teacher in PECE-Poli in University of São Paulo*
Street Arcturus, 03 (Antares Garden), Building Delta, 09606-070, São Paulo-SP, Brasil
marcelo.araujo@ufabc.edu.br

Abstract. This work aims to find a procedure to obtain an alternative formulation that represents the first mode of vibration of slender steel poles considering the effect of geometric non-linearity, using the Reyleigh-Ritz method, trigonometric formulations with optimization techniques and a finite element mathematical model of an existing polygonal steel pole. In order to consider the geometric non-linearity in the calculation of the natural frequencies of the respective structure, the concept of initial stiffness, geometric stiffness and effective stiffness computed by the Rayleigh method for vibration problems in mechanical systems was used. So, to optimize the time to obtain the modal response in the dynamic analysis of the described structure, without replacing the precision of the results of a rigorous analysis with sophisticated methodologies, alternative formulations to those described in NBR 6123 (1988) will be presented in this work.

Keywords: Dynamic Analysis, Vibration, Steel Poles, Geometric Non-Linearity, Vibration Mode, Rayleigh method.

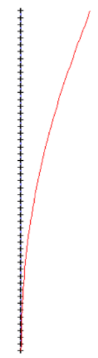
1 Introduction

Steel poles with hollow cross-section have been developed for the implementation of telecommunication systems in reduced spaces and are usually designed with a height of up to 60 meters, support loads of up to 30 square meters of wind exposure area and have a better cost-benefit ratio when compared to other solutions on the market, they are manufactured with prismatic sections in a dodecagonal, octadecagonal or circular shape. Due to the area of exposure to the wind, this type of structure presents considerable dynamic behavior as indicated by Brasil and Silva (2015), to obtaining it's dynamic response is highly important to characterize the vibrant behavior of the structure. The analysis of the problem discussed here aims to present the procedure to obtain an alternative formulation that represents the first vibration mode of slender steel pole with geometric nonlinearity effect. The objective is to carry out a study with the Reyleigh-Ritz method, trigonometric formulations with optimization technique and a finite element mathematical model of a polygonal steel pole. To consider the geometric nonlinearity in calculating the frequencies of the respective structure, the concept of initial stiffness, geometric stiffness and effective stiffness given by the Rayleigh method for vibration problems in mechanical systems will be presented, with its formulation found in Wahrhaftig (2017). In this work, the theories for modal analysis were used in a polygonal metal pole of 50 meters high, top diameter of 0.45 meters and base diameter of 1.50 meters, as shown in Figure 2.

2 Theoetical basis and results

The studies by Brasil and Silva (2015), show that among the modes of the structural typology described here, more than 90% of the dynamic response is contained in the first vibration mode (φ_1) presente in Figure 1

Figure 1 - Typology of the studied vibration mode.



Brasil and Silva (2015) and ABNT NBR 6123 (1988) indicate that tall structures with a reduced cross section, of high slenderness, usually present vibration modes with frequencies below 1Hz, showing the need for a dynamic analysis. The first vibration mode of the simplified dynamic model supported by ABNT NBR 6123 (1988), can be written as (1), where z_i is the height of the mass, H is the total height of the structure and the exponente γ is given by Table 1.

$$\varphi_i = \left(\frac{z_i}{H}\right)^{\gamma}. \tag{1}$$

Table 1 - Parameters for determining dynamic effects by ABNT NBR 6123 (1988).

Type of constructions	γ
Concrete towers and chimneys, variable section	2,7
Steel towers and chimneys, uniform section	1,7
γ is the exponent of the function that represents the vibration mode;	

Wahrhaftig (2017) reports “Once the effect of the axial compressive force is to reduce the stiffness of the members of the structure, the approach to aspects involving a concept of geometric stiffness is related, at the same time, to the analysis of the elastic stability of structural systems. Timoshenko (1985) presents problems of elastic instability of prismatic bars where it is possible to verify reduction of stiffness due to the presence of normal efforts. For the buckling load, the structure does not offer resistance to any disturbance that occurs on it and when such disturbance occurs, the displacements in the configuration continue to increase without the need to additional loads. This suggests that when this time is reached, the displacements of the structure, for the critical load, will grow indefinitely, meaning that, on the other hand, the rigidity of the structure has become null", so it is possible to write the rigidity of the structure by next difference.

$$[K] = [K_0] - [K_g]. \tag{2}$$

Where K_0 represents the elastic stiffness in a function of the mechanical properties of the structure and the term K_g it's the geometric stiffness in function of mechanical properties and axial loading. Geometric stiffness can be calculated with the Rayleigh method based on the principle of energy conservation. Its formulation for continuous systems contains the form function ϕ, that represents the mode of vibration and its application requires that the modal mass of the structure be.

$$m = \int_0^H \bar{m}\phi^2 dz \rightarrow m = \sum_{i=1}^{n} \bar{m}_i {\phi_i}^2 \Delta z_i. \quad ((3)$$

the modal elastic rigidity

$$K_0 = \int_0^H EI\left(\frac{d^2\phi}{dz^2}\right)^2 \bar{m}dz \rightarrow K_0 = \sum_{i=1}^{n} EI_i \left(\frac{d^2\phi_i}{{dz_i}^2}\right)^2 \Delta z_i. \quad ((4)$$

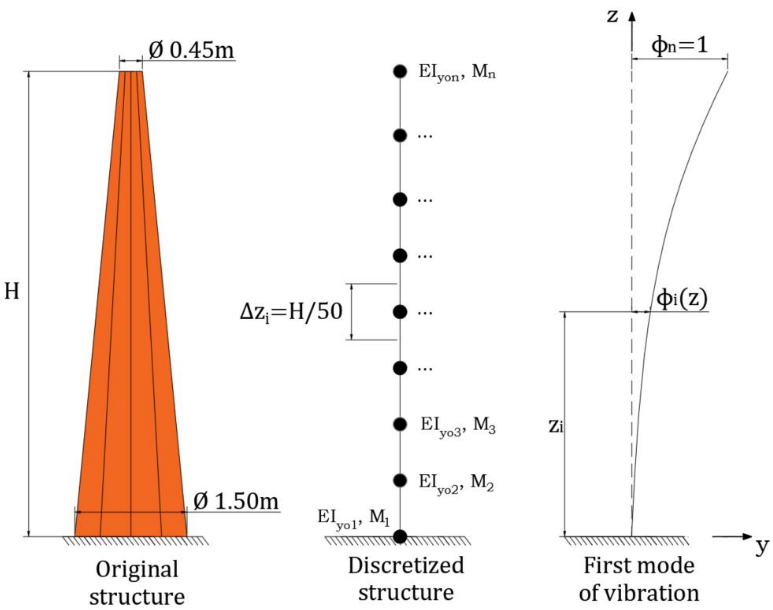
and modal geometric stiffness

$$K_g = \int_0^H N\left(\frac{d\phi}{dz}\right)^2 \bar{m}dz \rightarrow K_g = \sum_{i=1}^{n} N_i \left(\frac{d\phi_i}{dz_i}\right)^2 \Delta z_i. \quad ((5)$$

wehre $\bar{m}$ is the mass of the structure, dz is the infinitesimal length of the axis of the bar that represents the body of the structure and N is the normal axial force.

The structure discussed has a behavior similar to a cantilever, so the shape functions must meet the boundary conditions, Wahrhaftig et. al (in press) indicate that in numerical solutions of differential equations by the technique of 'test functions or form functions', they have functions that are only approximations, or trials, and not exact solutions. Different functions, even meeting the boundary conditions of the problem, can lead to different results. Theoretically, in the Rayleigh method, it is enough to respect the conditions of the first type, those of Dirichlet, as visible ni Figure 2.

Figure 2 – Relationship between physical model and developed mathematical model.



To optimize the natural frequencies, the minimization of errors was performed with the Solver tool of Microsoft Exc. Brasil e Silva (2019) report that "Excel's Solver can be used to solve optimization problems where non-linear continuous problems, Solver uses a version of GRG ("Generalized Reduced Gradient"). Two shape functions were used for the first vibration mode, that meet the conditions imposed in the Figure 2 and that will be applied in the equations ((3), ((4) e ((5) through a discretized model, as visible Figure 2. The shape functions are (1) and the following trigonometric function with factors a an b.

$$\phi_i(z_i) = \left(1 - cos\left(\frac{\pi z_i}{2H}\right)\right)\left(a\frac{z_i}{H} + b\right). \tag{6}$$

According to Wahrhaftig et. al (in press) the factors a and b can be obtained by optimizing the error, between their real values and imposed initial values, thus composing the vector $\{c\}$.

$$\{c\}^T = [a \quad b]. \tag{7}$$

which must be determined in order to minimize the function

$$f(c) = r\sum_{i=1}^{n}\left[\phi_i(\{c\}, z_i) - \phi_{i_{MEF}}(z_i)\right]^2. \tag{8}$$

or

$$f(c) = r\sum_{i=1}^{n}\left[f(\phi_i(\{c\}, z_i)) - f_{MEF}(\phi_{i_{MEF}}(z_i))\right]^2. \tag{9}$$

where r is a penalty factor used to adjust the magnitude of errors. The values of $\phi_{i_{FEM}}(z_i)$ until $\phi_{n_{FEM}}(z_n)$, are the modes obtained at each coordinate discretized from the finite element model (FEM). The frequencies f and f_{FEM} are provided by the proposed formulation and the frequency obtained with the finite element model. The restriction, described in Figure 2, imposed to represent the first mode of vibration is

$$\phi_n(\{c\}, H) = 1. \tag{10}$$

In order for this restriction to be satisfied, we start with the following values for the vector$\{c\}$

$$\{c\}^T = [0 \quad 1]. \tag{11}$$

Similar to function (8) the function (12) and (34) were used to optimized γ in function (1)

$$f(\gamma) = r\sum_{i=1}^{n}\left[\phi_i(\gamma, z_i) - \phi_{i_{MEF}}(z_i)\right]^2. \tag{12}$$

or

$$f(\gamma) = r\sum_{i=1}^{n}\left[f(\phi_i(\gamma, z_i)) - f_{MEF}(\phi_{i_{MEF}}(z_i))\right]^2. \tag{13}$$

where its starting value is present in Table 1.

To apply the Rayleigh method it's necessary to solve the first and second derivatives of the trial function in relation to vertical length, where the first and second derivatives for the function (1) are.

$$\frac{d\phi_i}{dz_i} = \frac{\gamma}{H^\gamma}{z_i}^{\gamma-1}; \frac{d^2\phi_i}{d{z_i}^2} = \frac{\gamma(\gamma-1)}{H^\gamma}{z_i}^{\gamma-2}. \tag{14}$$

Similarly, the first derivative of the function (6), is

$$\frac{d\phi_i}{dz_i} = \frac{\pi}{2H} sin\left(\frac{\pi z_i}{2H}\right)\left(\frac{az_i}{H} + b\right) + \left(1 - cos\left(\frac{\pi z_i}{2H}\right)\right)\frac{a}{H}. \tag{15}$$

and the second

$$\frac{d^2\phi_i}{{dz_i}^2} = \frac{\pi^2}{4H^2} cos\left(\frac{\pi z_i}{2H}\right)\left(\frac{az_i}{H} + b\right) + \frac{a\pi}{H^2} sin\left(\frac{\pi z_i}{2H}\right). \tag{16}$$

Numerically, the values of elastic stiffness, geometric stiffness and modal mass for the γ in table 1 can be obtained for discussed structure, their values are shown in the Table 2. In the hypothesis of the trigonometric function (6) with coefficients in the equation (11), the dynamic parameters were obtained and are presente in Table 3. In table 4 are presente the modal parametrês obtained with FEM model. In the hypothesis of considering geometric nonlinearity influenced frequencies and periods, the geometric nonlinearity was considered by multiplying the modulus of elasticity by the factor 0.97, which represents the ratio between the effective elastic stiffness K and the initial elastic stiffness K0 in the evaluated structure.

Table 2 – Parameters obtained for each coefficient given by NBR 6123.

γ =	2.700	γ =	1.700
K0 (Nm²)	19161.896	K0 (Nm²)	23193.833
Kg (Nm²)	582.729	Kg (Nm²)	662.508
K (Nm²)	18579.167	K (Nm²)	22531.325
K=K0-Kg			
f1 (s-1)	3.803	f1 (s-1)	3.436
f1 (Hz)	0.605	f1 (Hz)	0.547
T1 (s)	1.652	T1 (s)	1.828
K=K0			
f1 (s-1)	3.862	f1 (s-1)	3.487
f1 (Hz)	0.615	f1 (Hz)	0.555
T1 (s)	1.627	T1 (s)	1.802

Table 3 – Parameters obtained with the trigonometric function.

K0 (Nm²)	24815.843
Kg (Nm²)	680.434
K (Nm²)	24135.409
K=K0-Kg	
f1 (s-1)	3.569
f1 (Hz)	0.568
T1 (s)	1.761
K=K0	
f1 (s-1)	3.619
f1 (Hz)	0.576
T1 (s)	1.736

Table 4 – Frequency and period obtained with FEM model (Finite Element Method).

K=K0-Kg		K=K0	
f1 (s-1)	3.245	f1 (s-1)	3.297
f1 (Hz)	0.516	f1 (Hz)	0.525
T1 (s)	1.937	T1 (s)	1.905

In possession of dynamic parameters and vibration modes for the conditions of the Table 2, Table 3 and Table 4 it's possible to apply the optimization methods discussed later in this study. The optimization process adopted minimizes the errors between the mode or frequency, obtained between the finite element model and the modes or frequencies provided by the functions (1) and (6).

Table 5 – Resume of errors between the frequencies obtained and those given by the MEF model

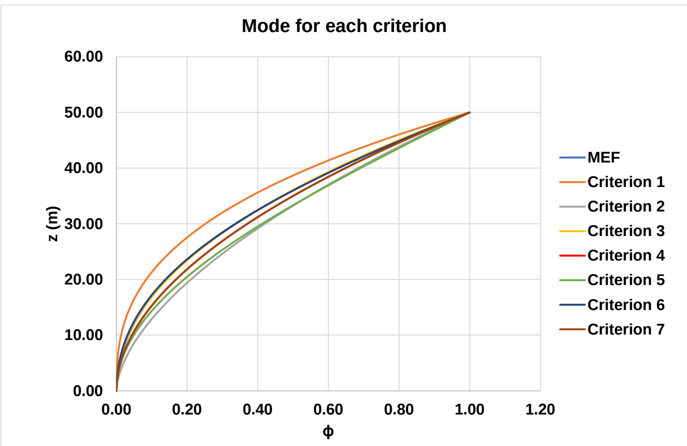
				Analysis of results		Error	Error
Criteria	γ	a	b	f1 para K=K0 (Hz)	f1 para K=K0-KG (Hz)	K0	K0-KG
1	2.700	-	-	0.615	0.605	17.12%	17.20%
2	1.700	-	-	0.555	0.547	5.74%	5.91%
3	2.122	-	-	0.554	0.546	5.65%	5.65%
4	1.943	-	-	0.550	0.541	4.78%	4.83%
5	-	-	-	0.576	0.568	9.74%	9.99%
6	-	0.451	0.548	0.563	0.559	7.25%	8.27%
7	-	0.300	0.700	0.562	0.555	7.10%	7.51%
8	-	-	-	0.525	0.516	0.00%	0.00%

Table 6 – Description of situations for each optimization.

Criteria	Description
1	First Tentative function of the NBR
2	Second tentative function of the NBR
3	Optimization in ϕ1 regarding the tentative function of the NBR
4	Optimization in f1 regarding the tentative function of the NBR
5	Trigonometric function
6	Optimization in ϕ1 by the trigonometric function with factors a and b
7	Optimization in f1 by trigonometric with factors a and b
8	FEM model

The criteria 3, 4, 6 and 7 in Table 6 were obtained with the help of the Solver tool of Microsoft Excel, where the mathematical representation of the optimization performed is present in the functions (8), (9), (12) and (13). The procedure performed has the vibration mode restriction necessarily provide the value 1 in the position corresponding to the top of the structure and the first derivative of the form function to provide the value of $\varphi'(0) = 0$ at the base of the structure. In criteria 3 and 6 the error reduction was performed directly in vibration mode, but in the criteria 4 and 7 the error reduction was in the frequencies. In Figure 3 all forms for the first mode of vibration studied in this work are presented.

Figure 3 – List of shapes of the first studied modes of vibration.



3 Conclusion

Based on the results shown, it is possible to conclude that the analyzed structure has a significant dynamic behavior, the vibration frequency for the first mode is in the order of 0.5 Hz. The effects due to geometric stiffness generated approximately 1% smaller differences in frequencies and, as expected, in the hypothesis of its consideration, a reduction in the stiffness of the structure is observed. This difference will be more accentuated with the increase of vertical actions, which cause normal efforts to the structure.

It is possible to check through the Table 5 an error of approximately 17%, in relation to the FEM model, to obtain the first natural frequency of vibration of the respective structure when using $\gamma = 2.7$, given by NBR 6123. This error decreases to 6% when used $\gamma = 1.7$, also given by the same standard. The use of the trigonometric function showed an error 10% with respect to FEM model.

Using optimization techniques, the exponential function presented the smallest error for calculating the frequency, about 5%, with obtaining an optimized γ equal to 1.943. For the trigonometric function, an error of 8% in relation to FEM model, obtaining the optimal values of $a = 0.3$ and $b = 0.7$.

So, it is concluded that the use of optimization techniques has significantly improved, in the order of 20%, the calculation of the fundamental frequency of the structure. Thus, in simulations of very complex problems, which require a high computational cost, the use of this method can be very useful to obtain values with very reasonable precision, without the need for all the formulation and computational time of the FEM.

It is suggested for future studies a comparison between displacements and dynamic solicitation efforts obtained by the FEM model, in problems with several degrees of freedom, with those obtained with the aid of this simplified analysis, with only one degree of freedom, using the modes determined by optimization techniques.

Acknowledgements. The authors thank's the Dr. Reyolando M.L.R.F. Brasil, for the orientations in the reviews of this research and to Letícia Paixão for the collaboration in the instrumentation of this study. This research was carried out in the Continuing Education Program of the Polytechnic School of the University of São Paulo (PECE).

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. ABNT NBR 6123 – Errata 2: Forces due to wind in buildings. Rio de Janeiro, 2013.
[2] BRASIL, Reyolando Manoel Lopes Rebello da Fonseca; SILVA, Marcelo Araújo da. *Introduction to Structures Dynamics: For Civil Engineering*. 2ª ed. São Paulo, 2015.
[3] WAHRHAFTIG, Alexandre De Macêdo. *Wind Action On Slender Structures with Geometric Effect*. 2ª ed. São Paulo 2017.
[4] THIMOSHENKO, Sthephen P.; GERE, James M. *Theory of Elastick Stability*. 17th. Co – Singapore, 1985.
[5] BRASIL, Reyolando Manoel Lopes Rebello da Fonseca; SILVA, Marcelo Araújo da. *Optimization of engineering projects.* 1ª ed. São Paulo, 2019.