Tuned Mass Damper Passive Control of Aircraft Wings Vibrations

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Abstract. In this paper we study passive control of aircraft wings vibrations using simple Tuned Mass Damper (TMD). This kind of structures are prone to large amplitude oscillations excited by time varying lift and drag forces, especially in turbulent conditions. These are low frequency vibrations, mainly in the first mode of vibration, as the stiffness is also low due to large cantilever spans and small thickness. We propose to include a thin cantilever metal bar inside the wing, carrying a point mass at its tip. This subsystem will be tunned so to be approximately in resonance with the first wing vibration mode. Numerical modeling and simulation of the system, via the Finite Element Method (FEM), will be performed to check the efficiency of this proposed Tuned Mass Damper.

Keywords: TMD, aircraft wings, passive control, FEM.

1 Introduction

In the design of aircraft wings, one of the concerns in their conception is in relation to the control of vibrations in the structure, since such elements can present large amplitudes, depending on the variables of lift and drag, being enhanced in various situations of aircraft operation such as turbulence conditions. Such vibrations, in addition to being a problem related to passenger comfort, represent a concern in the structural range, as they can lead to material fatigue or even the collapse of the structure.

In many cases, a more economical way to mitigate vibration problems is the damping introduction in the system, which basically consists of a mechanism responsible for dissipating the energy stored by the structure, considerably reducing its oscillations. There is an alternative, also very common, for mitigating vibrations, which is the introduction of a system known as Tuned Mass Damper (TMD). The idea of TMD originates from Frahm [1] and has been influenced by Ormondroyd and Den Hartog [2], who introduced a dashpot and presented the classical optimum frequency and damping for a harmonic load. In its simple form, a TMD, as show in Figure 1, consists of a small mass in a spring that moves in the direction of the vibrations of the main structure, adding an additional degree of freedom to the system (Tophøj *et. al.* [3]).

The TMD must be placed at the same vibration frequency of the structure that want to minimize vibrations, in the particular case of aircraft wings vibrations are normally of low frequency, mainly in the first vibration mode. After tuning the TMD, the energy from the vibrations of the main structure is quickly transferred to it, which, a posteriori, is responsible for dissipating it. In this way, it is possible to reduce the oscillations of the main structure efficiently and practically (Brasil and Silva [4]).

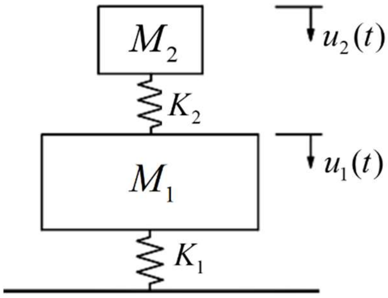


Figure 1. Simple TMD system

This paper proposes a method for passive control aircraft wings vibration, using a simple Tuned Mass Damper (TMD). An additional degree of freedom will be introduced into the original structure through a thin metal bar cantilevered inside the wing, carrying a point mass at its tip. This subsystem will be tuned to be approximately in resonance with the first wing vibration mode. A numerical modeling and simulation of the system was developed, via the Finite Element Method (FEM), to verify the efficiency of the system. A mathematical model was also developed, using Rayleigh method to compare the results.

2 Mathematical model of the aircraft wing

The cross-section of the analyzed wing (NACA 2412) is shown in Figure 2, being made of Aluminum Alloy 6061 with a Young's modulus $E = 69$GPa, and a linear mass density $p = 2700$kg/m^3. The chord length of the airfoil is 1m and wing length is 5.5m.

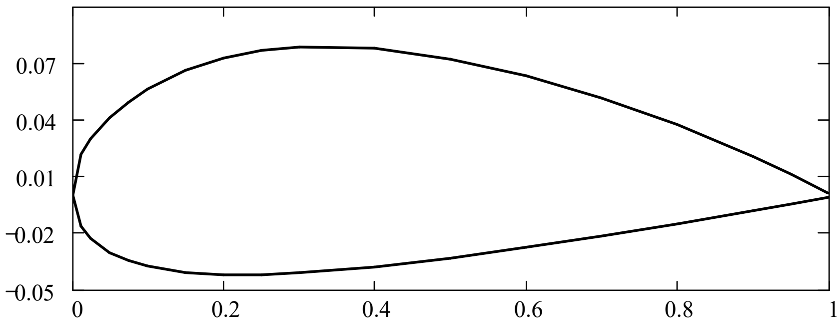


Figure 2. Aircraft (NACA 2412) wing cross-section.

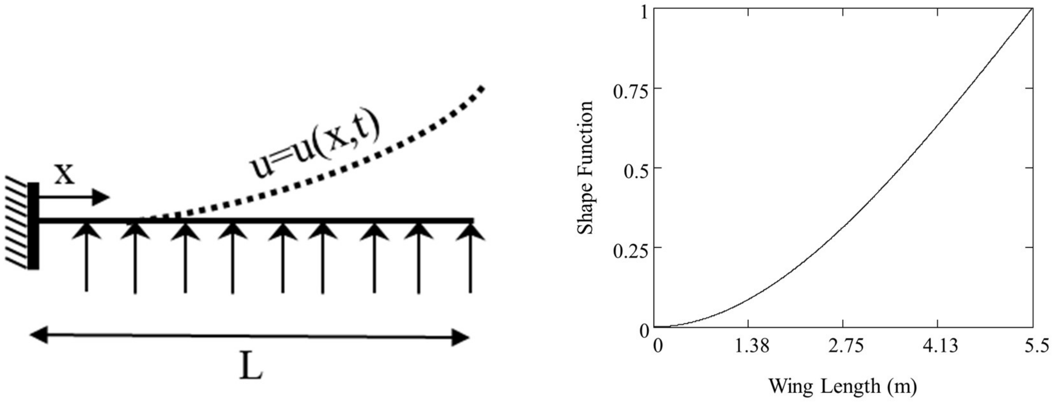
The proposed mathematical model is based on the Rayleigh method, first presented by John William Strutt, 3rd Baron of Rayleigh [5], which has become very efficient for the analysis of complex dynamical systems. The aircraft wing can be modeled as a fixed and free beam, as shown in Figure 3. The transverse horizontal vibration displacement of its *x* axis is a function of both the spatial *x* coordinate and time, and may be written as:

$$u = u(x,t) = \phi(x)q(t)\,, \tag{1}$$

where $\phi(x)$ is a dimensionless shape function, that must satisfy the geometric boundary conditions and assume unitary value where the $q(t)$ generalize coordinate is chosen. The shape function that will be used to describe the structure's behavior is:

$$\phi(x) = \frac{3x^2}{2L^2} - \frac{x^3}{2L^3} \tag{2}$$

that obey all the boundary conditions of the structure.



(a) Idealized model of the wing (b) Shape function

Figure 3. Idealized model and shape function.

The first vibration frequency of the structure (W_{wing}) is calculated by:

$$W_{wing} = \sqrt{\frac{K_{wing}}{M_{wing}}} \tag{3}$$

with,

$$K_{wing} = EI \int_0^{L_{wing}} \left(\frac{d^2}{dx^2} \phi(x) \right)^2 dx\,, \tag{4}$$

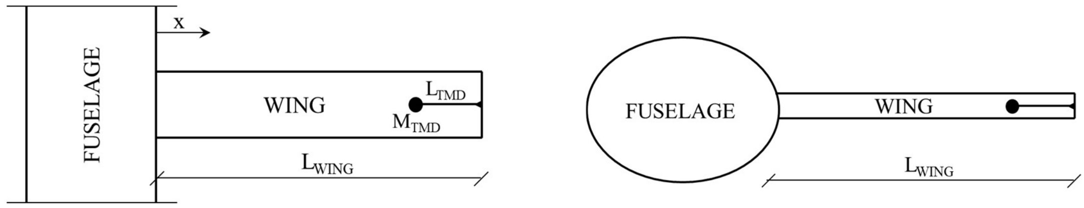
where E is the Young's modulus, I the inertia of the cross-section and $\phi(t)$ the shape function. The mass (M_{wing}) is given by:

$$M_{wing} = \bar{m} \int_0^{L_{wing}} \left(\frac{d^2}{dx^2} \phi(x) \right)^2 dx \tag{5}$$

where $\bar{m}$ is the mass per unit length.

3 Passive wing aircraft control vibration using TMD

A simple TMD vibration absorber is a conveniently tuned spring mass system, coupled to the original structure to minimize oscillation. Consider Figure 4, L_{TMD} being the length of the TMD, and M_{TMD} its mass.



(a) Plan

(b) Elevation

Figure 4. Wing visualization

As the equation of motion of the vibrator coupled, neglecting the damping:

$$\begin{bmatrix} M_{wing} & 0 \\ 0 & M_{TMD} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} M_{wing} + M_{TMD} & -M_{TMD} \\ -M_{TMD} & M_{TMD} \end{bmatrix} \begin{Bmatrix} u_{wing} \\ u_{TMD} \end{Bmatrix} = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} sin(wt) \tag{6}$$

Defining U as the amplitude of harmonic motion. The solution to the Equation (6) is:

$$\begin{Bmatrix} u_{wing} \\ u_{TMD} \end{Bmatrix} = \begin{Bmatrix} U_{wing} \\ U_{TMD} \end{Bmatrix} sin(wt) \tag{7}$$

Where, U_{wing} and U_{TMD} represent the amplitude of harmonic motion for the main structure (aircraft wing) and the TMD, respectively. Substitution of these steady-state forms into Equation (6) yields:

$$\begin{bmatrix} M_{wing} + M_{TMD} - M_{wing} w^2 & -M_{TMD} \\ -M_{TMD} & M_{TMD} - M_{TMD} w^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} sin(wt) = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} sen(wt) \tag{8}$$

Performing some algebraic manipulations:

$$U_{wing} = \frac{(K_{TMD} - M_{TMD} w^2) F_0}{\left(K_{wing} + K_{TMD} - M_{wing} w^2\right)\left(K_{TMD} - M_{TMD} w^2\right) - K^2{}_{TMD}} \tag{9}$$

The vibration amplitude of the TMD being given by:

$$U_{TMD} = \frac{K_{TMD} F_0}{\left(K_{wing} + K_{TMD} - M_{wing} w^2\right)\left(K_{TMD} - M_{TMD} w^2\right) - K^2{}_{TMD}} \tag{10}$$

Making the coefficient of F_o of equation (9) equal to zero:

$$w_{TMD} = \sqrt{\frac{K_{TMD}}{M_{TMD}}} \tag{11}$$

Thus, the idea is to make the U_{wing} value approach zero, equaling the values of the first frequency vibration of the structure with that of the TMD, this step being carried out by modifying the TMD parameters.

4 Numerical model by Finite Element Method

To compare the results with the analytical model, the wing system with and without the TMD was modeled via the Finite Element Method (FEM) using the SOLIDWORD. A NACA 2412 being made of Aluminum Alloy 6061 with a Young's modulus E = 69GPa, and a linear mass density p = 2700kg/m^3, with 2mm thickness was presented in Figure 5.

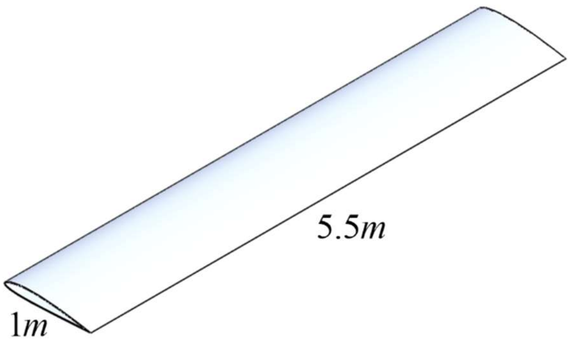


Figure 5. Numerical model of Aircraft (NACA 2412) wing cross-section.

The TMD was modeled as a beam with a circular section with a diameter of 25mm, L= 1m made of A36 steel with E=210GPa with a mass of 15.47Kg fixed at its end (Figure 6).

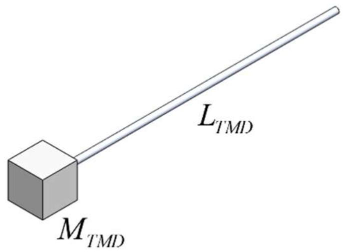


Figure 6. Numerical model of Aircraft (NACA 2412) wing cross-section.

The TMD was considered coupled to the wing, fixed it to the free end. Thus, it was possible, after excitation of the system, to establish the element vibrations without the TMD and with the TMD. Figure 7 show the system with the coupled TMD to the wing.

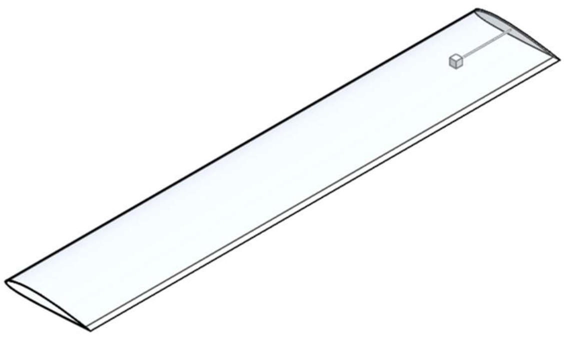


Figure 7. Numerical model of Aircraft (NACA 2412) wing cross-section with TMD.

5 Results and discussions

Figure 8 and Figure 9 show the results of displacement for the wing without the TMD and after insertion of the TMD, respectively. The adopted seismic motion is fictitious and at the same frequency of the structure to consider the worst case, i.e., resonance.

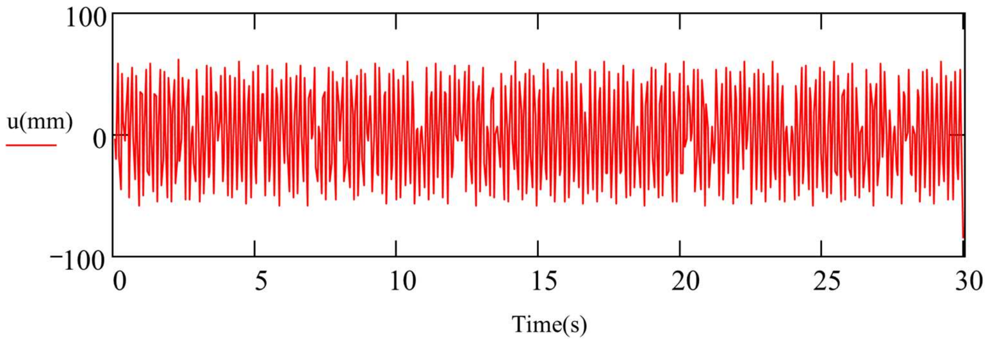


Figure 8. Displacement Graphic u(mm) x Time (s) without TMD

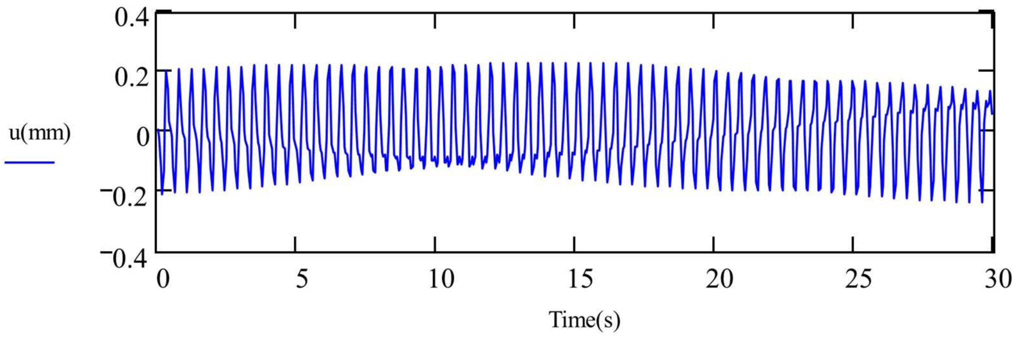


Figure 9. Displacement Graphic u(mm) x Time (s) with TMD.

Figure 10 presents a comparison between the displacements.

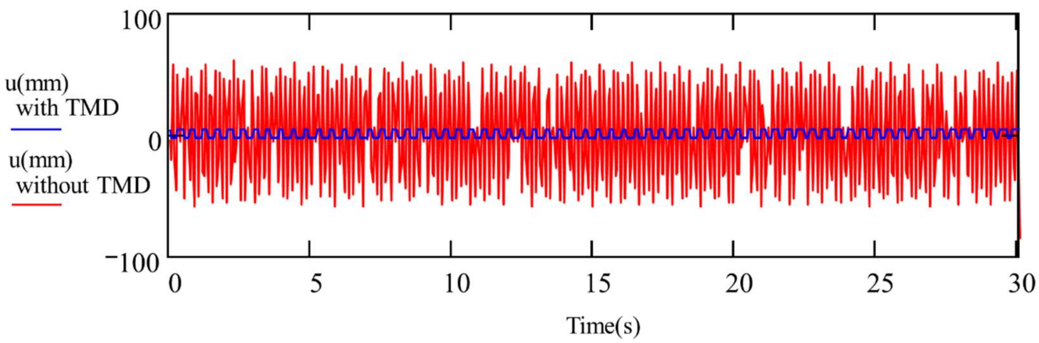


Figure 10. Comparison of results

6 Conclusions

In this paper, a passive control of aircraft wings vibrations using simple Tuned Mass was performed. A simple vibration absorption action was verified using a thin cantilever metal bar inside the wing, carrying a point mass at its tip. This subsystem has been tunned so to be approximately in resonance with the first wing vibration mode and was modeled and simulated using the Finite Element Method (FEM). It was concluded that this device is very effective to reduce vibration displacements amplitude.

A proposal for future studies should be the development of a more complex and realistic wing model, in addition to the application of optimization techniques to reach the optimal parameters of mass and bar length for these TMDs.

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References

[1] H. Frahm, Device for damping vibrations of bodies, US No. Patent 989958, 1909.
[2] J. Ormondroyd and J. P. Den Hartog, "The theory of the dynamical vibration absorber". *ASME Journal of Ap-plied*, Vol. 1, n. 50 (1928): pp. 9-22, 1928.
[3] L. Tophøj, N. Grathwol, S. O. HANSEN, "Effective mass of tuned mass dampers". *Vibration*, v. 1, n. 1, pp. 192-206, 2018.
[4] R. M. L. R. F. BRASIL, M. A. da Silva, *Introdução à dinâmica das estruturas: para a engenharia civil: para a Engenharia Civil.* 2. ed. São Paulo: Blucher, 2015.
[5] Rayleigh. *Theory of sound*. Dover Publications, New York, 1877. Reissued in 1945