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Noise control in ducts using local resonator acoustic metamaterial arrays.

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Abstract. Acoustic local resonators (Helmholtz, side branch, expansion chamber etc.) are usually used to control noise in acoustic systems. Noise attenuation in local resonators occurs due to impedance mismatch of incident, transmitted, and reflected waves induced by the addition of resonators. The frequency band and the attenuation level generated by the resonator depend on its geometry and behave as a narrow-band filter. However, if they are distributed properly (periodically or not) along the duct length, a much larger frequency band, called bandgap, is obtained. In this paper, acoustic metamaterial noise attenuation is investigated using the transfer matrix method and assuming plane wave propagation. Also, a study about the arrangements of the local resonators (serial and parallel) is conducted to verify the method accuracy and efficiency related to band width enlargement and transmission loss. Simulated results are obtained for different examples and geometries.

Keywords: Local resonators, Acoustic metamaterial, Transfer matrix method.

1 Introduction

Helmholtz resonators are passive silencers that can be coupled to ducts to control noise in industrial ventilation systems, energy stations and automobiles. These devices are composed of a small tube opened in both sides (neck) and a large tube closed in one side (cavity) connected in sequence and are able to provide a high transmission loss in a narrow frequency band, with a peak on its resonance frequency. Several studies showed that coupling these devices in certain arrays (periodic or not) along the duct length, expands the attenuated frequency band, generating what is called a band gap. The band gaps occur due to two different effects that happen simultaneously at the duct resonator system: local attenuation due to resonator's geometry, as shown by Ingard [1], and Bragg's scattering, due to resonator periodicity, as studied by Shen [2]. Since then, analytical, and numerical methods have been developed and used to describe a periodic duct-resonator system. In this paper, the transfer matrix method is applied to control the variables of the acoustic problem, despite the difficulty in accurately incorporating the acoustic field behavior at the area discontinuities between the duct and the resonator [3]. This problem has been softened by adding an extra length to the duct physical length, as shown by Munjal [4], and to the interfaces between neck-duct and neck-cavity, as shown by Wen [5]. Since the resonators system is focused on the attenuation of low frequency noise, the hypothesis of only plane wave propagation inside the duct and the resonator is acceptable [4]. Furthermore, a complex wavenumber is used in order to incorporate damping in the air, mainly by the interaction with the duct wall (Sing [6]). This paper, considering all above, performs a review of serial and parallel array proposed by Seo [7].

2 Transfer Matrix Method - TMM

The Transfer Matrix Method (TMM) assumes that only plane waves propagate inside the environment [4]. This hypothesis is acceptable since the silencers being investigated attenuate low frequency noises, which is characteristic of plane waves. Publications of Ingard [1], Sing [6], Kinsler [8] report an expression to calculate the acoustic duct cut-on frequency ($f_c = 1.8412c/2\pi a$), that is the frequency below which only plane waves propagate inside the duct. To include the energy dissipation between air and duct walls, a complex wavenumber is used as $\hat{k} = k(1 - j\eta)$, where η is the loss factor, j is the imaginary unity and $k = c/\omega$ is the wavenumber with c as the air sound speed and ω as the circular frequency. Also, Ingard [1], Sing [6], Kinsler [8] describes the corrections in the physical length of the acoustic ducts based on its extremities (flanged or unflanged end).

2.1 Duct-HR modeling

Figure 1 shows a scheme of a Helmholtz resonator coupled to a main duct.

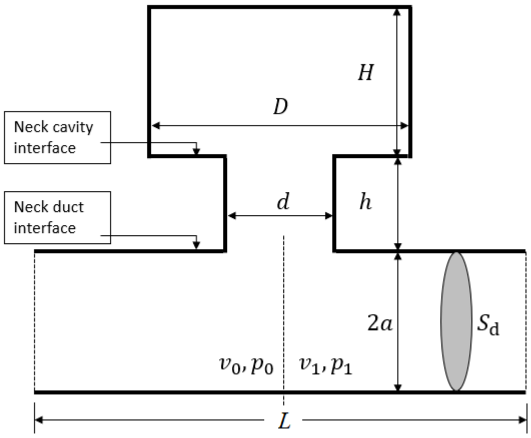


Figure 1. Scheme of the acoustic circular duct coupled with a Helmholtz resonator.

To obtain the complete transfer matrix model of the duct-HR (Helmholtz Resonator) system, the scheme in Fig. 1 can be divided in three acoustic elements: (1) duct segment before-HR, (2) duct segment after-HR, and (3) the Helmholtz resonator.

From the book of Munjal [4], the TMM model of an acoustic uniform circular duct of length L can be written as the relationship of the acoustic pressure (p_r) and volume velocity (v_r) between the input ($\{p_0 \quad v_0\}^T$) and output ($\{p_L \quad v_L\}^T$) of the duct, given by:

$$\begin{Bmatrix} p_0 \\ v_0 \end{Bmatrix} = \underbrace{\begin{bmatrix} cos(\hat{k}l_{eff}) & j\frac{\rho c}{S}sin(\hat{k}l_{eff}) \\ j\frac{S}{\rho c}sin(\hat{k}l_{eff}) & cos(\hat{k}l_{eff}) \end{bmatrix}}_{\mathbf{T}_d} \begin{Bmatrix} p_L \\ v_L \end{Bmatrix}, \tag{1}$$

where, $l_{eff} = L + 0.6a$ is the duct effective length and $\mathbf{T_d}$ is the duct transfer matrix.

The TMM model of a HR resonator can be obtained using the impedance approach [7], in which the relationship between two points immediately before and immediately after the resonator is given by the following expression:

$$\begin{Bmatrix} p_0 \\ v_0 \end{Bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_r} & 1 \end{bmatrix}}_{\mathbf{T}_{hr}} \begin{Bmatrix} p_1 \\ v_1 \end{Bmatrix}, \tag{2}$$

where $Z_r = -jZ_c cot(kH) + Z_h$ is the acoustic impedance of the HR, $Z_c = \rho c/S$ is the cavity impedance, $Z_h = (\rho c/S_p)[0.0072 + jk(h + 0.75d)]$ is the neck impedance including the neck physical length correction and $\mathbf{T}_{HR}$ is the HR transfer matrix. An important parameter to be evaluated is the frequency of the attenuation peak generated by the duct-HR system, which is the HR natural frequency given by [4], $f_{HR} = (c/2\pi)\sqrt{d^2/(h_{eff}D^2H)}$.

The metamaterial model for the coupled system (duct-HR system) is obtained by multiplying the transfer matrices in the sequence that the wave faces the acoustic elements, which gives the metamaterial duct-HR transfer matrix as:

$$\mathbf{T}_{dhr} = \mathbf{T}_{d1}\mathbf{T}_{hr}\mathbf{T}_{d2} \tag{3}$$

where $\mathbf{T}_{d1}$ and $\mathbf{T}_{d2}$ are the duct segments before-HR and after-HR transfer matrices, respectively.

2.2 Series and parallel duct-HR modeling

To maximize the noise attenuation with the duct-HR system, parallel and series arrays of this acoustic metamaterial are proposed. The series system is composed of two or more HRs distributed along the longitudinal direction on the main duct, in such a way that the transfer matrix of a system with n HRs in series can be written as:

$$\begin{Bmatrix} p_0 \\ v_0 \end{Bmatrix} = \mathbf{T}_d \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_{r_1}} & 1 \end{bmatrix}}_{\mathbf{T}_{hr_1}} \mathbf{T}_d \cdots \mathbf{T}_d \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_{r_n}} & 1 \end{bmatrix}}_{\mathbf{T}_{hr_n}} \mathbf{T}_d \begin{Bmatrix} p_n \\ v_n \end{Bmatrix}. \tag{4}$$

The parallel system is composed of two or more HRs coupled at the same longitudinal position, but distributed along the circumferential direction of the main duct, in such a way that the transfer matrix of a system with N resonators is given by:

$$\begin{Bmatrix} p_0 \\ v_0 \end{Bmatrix} = \mathbf{T}_d \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{N}{Z_r} & 1 \end{bmatrix}}_{\mathbf{T}_{hr_N}} \mathbf{T}_d \begin{Bmatrix} p_L \\ v_L \end{Bmatrix}. \tag{5}$$

2.3 Transmission Loss

The effectiveness of a noise control system can be measured by its ability to increase the loss of sound transmission along its length. Transmission Loss (TL) is the most common analysis parameter, once it can be obtained directly from the final transfer matrix of the acoustic system. According to Munjal [4], the transmission loss can be calculated as:

$$TL = \frac{20}{N}\log\left|\frac{(T_{11} + \frac{S}{\rho c}T_{12} + \frac{\rho c}{S}T_{21} + T_{22})}{2}\right|, \tag{6}$$

where T_{mn} denotes the elements at the m^{th} row and at the n^{th} column of the transfer matrix of the acoustic system.

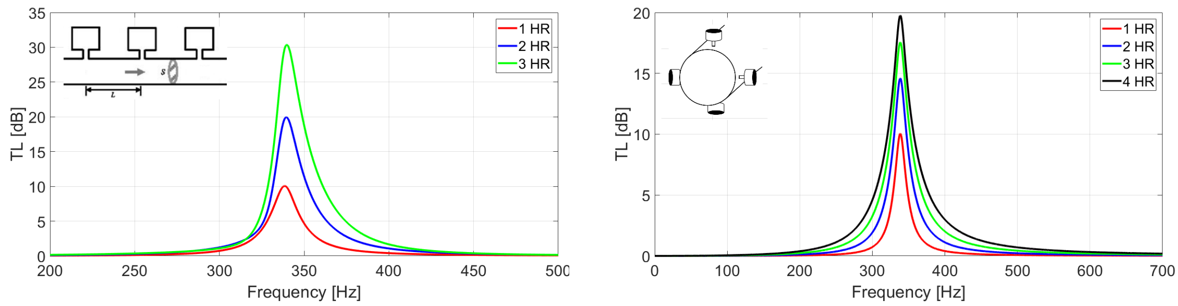
3 Simulated Results

A TMM code was implemented in the Matlab environment. A verification example was performed to calculate the transmission loss (TL) of series and parallel arrays of duct-HR using the same geometry and air properties (Tab. 1) as presented in reference [7]. For all simulated examples the loss factor $\eta = 0$.

Figure 2 shows the TLs calculated with the implemented TMM to the series and parallel duct-HR metamaterials. Figure 2(a) presents the TLs for a metamaterial including $n = \{1, 2, 3\}$ duct-HR in series, while Fig. 2(b) shows TLs for a metamaterial including $n = \{1, 2, 3, 4\}$ duct-HR in parallel. If compared, the results in reference [7] and the results obtained in the example are in good agreement.

Table 1. Metamaterial geometry and air properties.

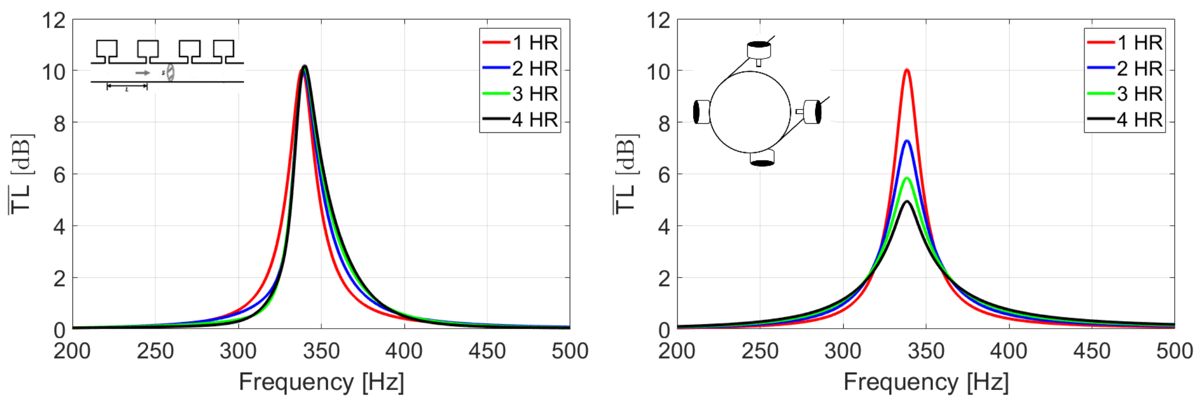
Geometry/Property	Value
Unit-cell length (L)	0.100 m
Duct radius (a)	0.050 m
Neck diameter (d)	0.010 m
Neck length (h)	0.025 m
Cavity height (H)	0.0485 m
Cavity diameter (D)	0.040 m
Air sound velocity (c)	343.0 m/s
Air mass density (ρ)	1.2041 kg/m^3



(a) Series (b) Parallel

Figure 2. Transmission loss verification of arrays of duct-HR system in: a) series; and b) parallel.

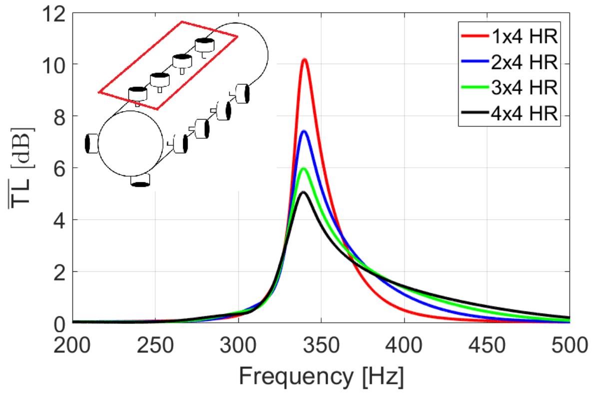
From the TL results in Figure 2, as the number of HRs increases in the metamaterial, the TL magnitude also increases, which seems to be unreal. Based on these results Wang and Ming [9] proposed the average transmission loss $\overline{\mathsf{TL}} = TL/N$ to calculate periodic acoustic metamaterials, where N is the number of HRs. Based on this approach, the verification example (Fig. 2) is recalculated for metamaterial arrays including $n = \{1, 2, 3, 4\}$ duct-HR in series and in parallel. Figure 3 shows that for both cases the obtained $\overline{\mathsf{TL}}$ presents more reasonable values when compared with that of the conventional TL. It can be seen that the main effect of the series-HR is the enlargement around HR resonance frequency as the number of HRs increases, while for the parallel-HR the band enlargement around HR resonance frequency is also observed, but the $\overline{\mathsf{TL}}$ magnitude increases as the number of HRs increases.



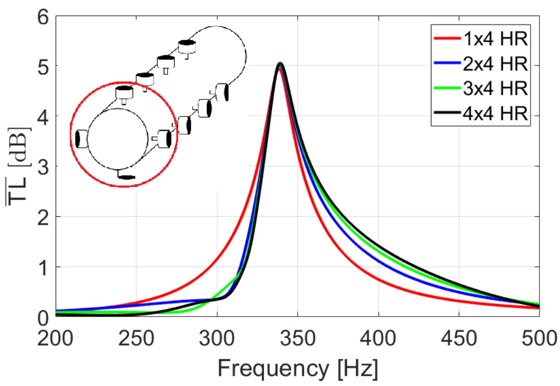
(a) Series (b) Parallel

Figure 3. Average transmission loss of duct-HR array system in: a) series; and b) parallel.

To verify which combination is more relevant to the metamaterial, the $\overline{\mathsf{TL}}$ for a combination of four arrays with 4-HR in series are calculated. Starting with 01 array with 4-HRs in series up to 04 array with 4-HR in series (Fig. 4(a)). Similarly, the $\overline{\text{TL}}$ for a combination of four arrays with 4-HRs in parallel are calculated. Starting with 01 array with 4-HRs in parallel up to 04 arrays with 4-HRs in parallel (Fig. 4(b)). From the Fig. 4 it can be seen that for both combinations (series and parallel) an enlargement of the frequency band is obtained, while for the $\overline{\text{TL}}$ the magnitude for the series combination increases as the number of 4-HRs arrays increases. However for the parallel combination the magnitude of $\overline{\text{TL}}$ remains constant.



(a) Series



(b) Parallel

Figure 4. Average transmission loss for a combination of 01 up to 04 arrays with 4-HRs in: a) series; and b) parallel.

The final example consists of a metamaterial system composed by a combination of 04 arrays with 04-HRs, but now with different HR resonance frequencies along the duct length. Frequency variation is produced by changing the HR neck length as $h = \{0.050, 0.075, 0.010, 0.125\}$ m. Figure 5 shows that a combination of $\overline{\text{TL}}$ occurs, which produces a larger attenuation frequency band with a reasonable magnitude. .

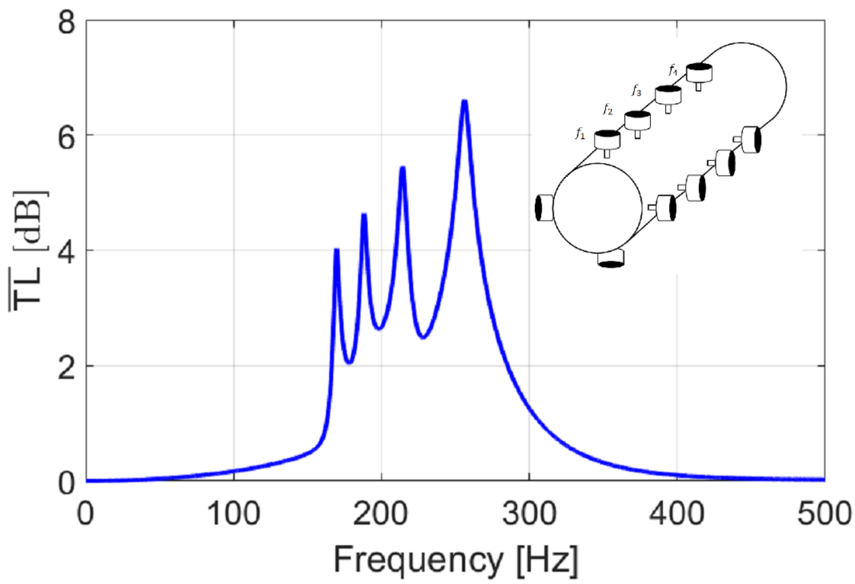


Figure 5. Average transmission loss for a combination of 04 arrays with 4-HRs with different resonance frequency along the duct length.

4 Final remarks

Based on the obtained results, its possible to conclude that the correct coupling of Helmholtz resonators provides a significant noise canceling and attenuation. For further works, its possible to investigate deeply the interfaces corrections, as well as the disposition on the parallel groups in different spacings. These are preliminary results and experimental investigation needs to be performed to confirm the theory and effectiveness of the systems.

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