Genetic Algorithm applied on the Optimization Problem for the Synthesis of a Walking Mechanism

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Abstract. This paper deals with the synthesis of articulated mechanisms employed to simulate the walking motion, which can be applied on robots, toys, vehicles and other legged locomotion systems. A six-bar mechanism comprised of a four-bar linkage with an embedded pantograph is the planar mechanism chosen for this study. The synthesis is based on the desired curve to be described by the mechanism, which is generated with the four-bar linkage and enlarged by the pantograph. The mechanism rendered by this synthesis is compact and presents links with small dimensions in comparison with the motion step. An optimization technique is used to generate the curve described by the mechanism. A computational procedure based on genetic algorithm and differential evolution is implemented to optimize the curve generation for the selected planar mechanism.

Keywords: Synthesis of mechanisms, four-bar linkage, walking mechanism, genetic algorithm.

1 Introduction

Wheeled vehicles are very effective for use on paved surfaces or surfaces with few irregularities. However, they become very ineffective on irregular surfaces. On the other hand, legged walking vehicles can effectively move on several natural terrains, presenting high energy efficiency on soft soils. This work presents a numerical procedure to perform the synthesis of a walking mechanism designed by the combination of a four-bar linkage, and a pantograph. The four-bar linkage is considered one of the simplest and most flexible closed-chain mechanisms in engineering, encountering applications in hydraulic pumps, electrical shavers, opening devices for doors, etc. [1].

Figure 1 presents a typical four-bar linkage with the symbols and numbers employed to describe their links and joints. In this figure, the rigid links are indicated by the numbers 1, 2, 3, and 4, and the four joints are indicated by the letters a, b, c, and d. Link 1 is the ground link. Link 2 is called crank, link 3 is known as a coupler, and 4 is the output link. The possible planar curves associated with a point P of the coupler can be described by a polynomial of order 6 [2]. The curves of the coupler point can be employed to produce some special movement that performs work. The problem is to determine the appropriate lengths of all links that will allow the mechanism to perform the desired curve.

The synthesis of mechanisms can be divided into three types [3]: Path, motion, and function generation. This work aims at estimating the lengths of a four-bar linkage capable of generating a curve that is adequate to reproduce the foot motion during a walk, which consists of a path generating problem. The simplest solution scheme is based on the graphic analysis, however, the solutions are generally less accurate and unfeasible for more complex problems. The method has been proved to be useful to obtain up to five precision points of a curve. Also, up to five prescribed points, the algebraic analytical solution can be obtained [3]. For more position points, the equations are highly non-linear and difficult to solve. The Newton-Raphson method can be used to implement procedures to render the analytical solution [4], however, in most cases, presents convergence problems. The homotopy method [5] has been a numerical method applied to find all the solutions to a given problem; nevertheless, some solutions can lead to complex solutions or provide defective mechanisms. Another problem is the prediction of the precision points for the 4-bar mechanism since that nine points are the maximum that can be specified.

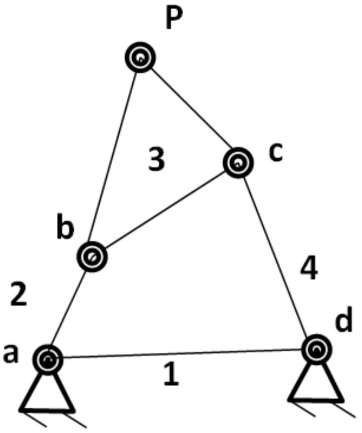

Figure 1. A schematic drawing of a four-bar linkage.

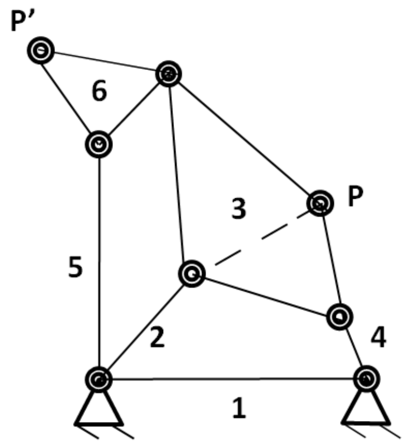

Figure 2. Six-bar linkage formed by the junction of the four-bar linkage and the skew pantograph mechanism.

Optimization techniques have been applied to the problem of curve generation [6], based on the minimization of an objective function that represents the Euclidean distance between the desired points and the predicted points. The parameters to be optimized are the link lengths. The methodology does not guarantee the accuracy of the prescribed points, but for most practical cases of curve generation, errors are small and can be neglected. There is no limit for the number of prescribed points and it is also possible to add restrictions during the search. The application of genetic algorithms in optimization is demonstrated by [7]. The proposed algorithm is based on differential evolution, which allows keeping the parameters of the mechanism as real numbers. As well as other optimization methods, it is possible to introduce restrictions and there is no limitation in the number of precision points. Another advantage of a heuristic method is that no close solution of mechanism is required to start the search; however there is the possibility of not obtaining the optimal solution.

The present work aims at estimating the dimensions of a four-bar linkage capable of generating a curve that is adequate to reproduce the movement of a foot during the walking motion. The desired characteristics for the curve are: a straight line with constant velocity during the step, a longer period in the step than in the return phase, and low foot accelerations during the beginning and end of contact with the ground. The synthesis process was carried out using the genetic algorithm based on differential evolution. The obtained four-bar mechanism is associated with a "skew" pantograph mechanism, which amplifies and rotates the curve. Figure 2 depicts this mechanism, which is a cognate four-bar linkage with a total of 6 links [8]. Point P describes the curve generated by the mechanism with point P' describing the curve enlarged and rotated by the pantograph.

2 Methodology

The genetic algorithm based on differential evolution [7] can be used in the search for optimal solutions of articulated mechanisms. The objective function, constraints, optimization parameters, and the implementation of the genetic algorithm are discussed in this section. The obtained mechanisms are subjected to kinematic analysis of velocity. The desired mechanism must have a near-constant speed during the step, to avoid slipping of the foot.

2.1 Objective Function and constraints

The objective function, eq.(1), defines the summation of Euclidean distance from the desired points to the points computed for the mechanism. For the synthesis process, the lowest value is desirable.

$$F_{obj} = \sum_{i=1}^{n} [\,(P_{xd}^{i}(X) - P_{x}^{i}(X))^2 + (P_{yd}^{i}(X) - P_{y}^{i}(X))^2\,] + h_1.M_1 + h_2.M_2 + h_3.M_3 + h_4.M_4 \quad (1)$$

P_{xd}- Coordinate of the desired point on the x-axis;
P_{yd} - Coordinate of the desired point on the y-axis;
P_x- Coordinate computed on the x-axis;
P_y - Coordinate computed on the y-axis;
n - Number of prescribed points;
h_1- equal 0, if Grashof condition is true, or 1, if not ;
h_2- equal 0, if sequence condition is true, or 1, if not ;
h_3- equal 0, if transmission angle is between [min, max] is true, or 1, if not ;
h_4- equal 0, if the position of the fixed pivot is between [min, max] is true, or 1, if not ;
M's are the constants that penalize the goal function when the associated constraint fails.

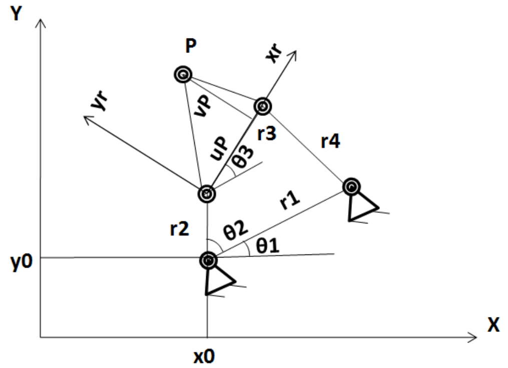

Figure 3.Parameters that define the 4-bar linkage in the model.

The optimized parameters for a four-bar linkage are: x_0, y_0, θ_1, θ_2, r_1, r_2, r_3, r_4, Up and Vp, which are depicted by Fig. 3. The parameters r_1, r_2, r_3, r_4, Up and Vp describe the lengths of the mechanism links, while parameters x_0, y_0, θ_1 are associated with the orientation of the mechanism in the plane. The crank angle θ_2 is treated as an optimization parameter in path generation problems with prescribed time.

To compute the objective function using the existing parameters, it is necessary to calculate θ_3 and θ_4, as well as the position of the coupler in the plane. The angles θ_3 and θ_4 can be estimated from the lengths of the links and the angle θ_2 using Freudenstein Equation [8]. For each angle θ_2 there are two possible mounting positions: Open position and Cross position.

The position of the coupler point relative to the local axis (x_r and y_r) can be defined by eq.(2) and eq.(3), and relative to the global axis (X and Y) by eq.(4).

$$P_{xr} = r_2 \cos\theta_2 + \mathrm{Up}\cos\theta_3 - \mathrm{Vp}\,\mathrm{sen}\,\theta_3. \tag{2}$$

$$P_{yr} = r_2\,\mathrm{sen}\,\theta_2 + \mathrm{Up}\,\mathrm{sen}\,\theta_3 + \mathrm{Vp}\cos\theta_3\,. \tag{3}$$

$$\begin{bmatrix} P_x \\ P_y \end{bmatrix} = \begin{bmatrix} \cos\theta_0 & -\mathrm{sen}\,\theta_0 \\ \mathrm{sen}\,\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} P_{xr} \\ P_{yr} \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}. \tag{4}$$

The constraints for the objective function are described as follows.

2.1.1 Grashof´s condition

The input link of the mechanism must describe a constant rotation, provided by a driving motor. To generate a crank-rocker linkage, the inequality given by eq. (5) must be satisfied, in which link r_2 is the crank length.

$$(r_1 + r_2 < r_3 + r_4) \quad \mathrm{e} \;\; r_2 < r_3 < r_4 < r_1. \tag{5}$$

2.1.2 Continous motion

The description of the continuous motion of the crank requires a criterion that contemplates a unidirectional increase in the crank position, which is established by eq. (6):

$$\theta_{2(i+1)} - \theta_{2(i)} > 0. \qquad i = 1, \dots, (n-1) \tag{6}$$

2.1.3 Transmission angle

The maximum and minimum transmission angles, $\mu 1$ and $\mu 2$, are obtained by eq.(7) and eq.(8):

$$\mu 1 = \arcos\left(\frac{(r_3{}^2 + r_4{}^2 - (r_1 + r_2)^2)}{2r_3 r_4}\right). \tag{7}$$

$$\mu 2 = \arcos\left(\frac{(r_3{}^2 + r_4{}^2 - (r_1 - r_2)^2)}{2r_3\, r_4}\right). \tag{8}$$

The optimal value of transmission angle is 90 degrees. In the proposed algorithm, acceptable values are introduced into a vector [μmin, μmax]. The importance of controlling the transmission angle is to prevent the possibility of locking.

2.1.4 Fixed pivot constraint

The values of x_0 and y_0 are limited by minimum and maximum values [min, max].

2.1.5 Link length ratio

To ensure that the links do not have very high length ratios, r_1, r_2, r_3, r_4, Up and Vp are subject to a maximum and minimum length [min, max], when values are assigned to these parameters.

2.2 Genetic Algorithm

The genetic algorithm proposed is based on differential evolution, where it is possible to work with parameters in the form of real numbers. Three basic operations are used to search for solutions: Selection, reproduction and mutation [7]. Using the algorithm definition, the population is formed by a set of mechanisms, each mechanism being an individual of the population. Each individual carries a set of parameters that are considered the individual genes. The operations used are described below:

2.2.1 Selection

In differential evolution, two random individuals and the best individual from the population are selected. They are combined in the form described by eq. (9).

$$V = X_{best} + F.(X_{aleat\ 1} - X_{aleat\ 2}) \tag{9}$$

X_{best} - vector corresponding to the best individual;
$X_{aleat\ 1}$ - vector corresponding to random individual 1;
$X_{aleat\ 2}$ - vector corresponding to random individual 2;
F - Constant of the disturbance vector, with a value between 0 and 1.

2.2.2 Reproduction

The generation of new individuals is performed by exchanging genes between the vector *V* with each individual of the population. There is a probability PC of the gene exchange occurs, which can be a value between 0 and 1. Each individual generated is compared to the previous individual. If the adaptability is greater, it takes the place of the relative, otherwise, it is discarded. The number of individuals in the population is always kept constant.

2.2.3Mutation

Mutation occurs by changing the value of a gene randomly, between a minimum and maximum value [x_i, x_i±lim]. There is a need for two different limits, the first being limit 1 corresponding to the lengths, and the second being limit 2 associated with the angles. Crossover mutation occurs with a probability indicated by PM, which has a value between 0 and 1 and should be much smaller than PC. Mutation plays an important role in avoiding local solutions.

The process occurs iteratively until the value of the objective function reaches an acceptable value or the number of iterations exceeds the maximum limit. The algorithm has been implemented in Matlab®. Due to the requirement for constant velocity only in the step phase, the proposed algorithm can mix curve generation with and without prescribed time.

2.3 Velocity analysis

To prevent foot slippage, the condition of the coupler moving with nearly constant velocity must be evaluated to select a proper mechanism. An expression for the coupler velocity [1], with respect to a local axis, is given by eqs.(10) and (11), and with respect to the global axis is given by eq.(12).

$$\mathrm{Vpx} = (-(\dot{\theta}_2) \times \mathrm{r2} \times \mathrm{sen}(\theta_2) - \dot{\theta}_3 \mathrm{Up}\,\mathrm{sen}(\theta_3) - \dot{\theta}_3 \mathrm{Vp}\cos(\theta_3)) \tag{10}$$

$$\mathrm{Vpy} = (\dot{\theta}_2 \times \mathrm{r2} \times \cos(\theta_2) + \dot{\theta}_3\ \mathrm{Up}\cos(\theta_3) -\ \dot{\theta}_3\ \mathrm{Vp}\,\mathrm{sen}(\theta_3)) \tag{11}$$

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \cos\theta_0 & -\mathrm{sen}\,\theta_0 \\ \mathrm{sen}\,\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} V_{px} \\ V_{py} \end{bmatrix} \tag{12}$$

Where $\dot{\theta}_2, \dot{\theta}_3$ are the angular velocities of the links.

3 Results

This section presents the results obtained with the algorithm implemented in this work. The algorithm was previously tested in the synthesis of similar mechanisms described by [7], rendering similar results. A combination of six points with prescribed time during the step phase and no prescribed time during the rest of the movement is used. A kinematic evaluation is performed to assess the velocity of the foot during the movement. The best solution found by the algorithm is presented.

3.1 Simulation Parameters

In the description of a straight line parallel to the X-axis, five prescribed points are used: (10,10), (15,10), (20,10), (25,10) and (30,10) . Another point was added in the middle of the curve (20,0) to force the solution to have a lifting height of the foot in the return phase. The coupler point velocity between these points should be constant, and therefore the same variation of crank angle is expected (uniform rotation), also a longer time of contact of the foot with the ground is desired. It was observed that among the possible solutions, over 150 degrees of the crank the curves tend to close, not existing space for foot lifting, and therefore a crank angle of 140° was designated during the step. As the constant velocity is of interest only during the step, the proposed algorithm allows to perform the mixture of curve generation with and without prescribed time. For the last point, the θ_2 angle is treated as a free parameter, increasing the number of possible solutions. The greatest limitation is related to the position of the fixed pivot. As observed by [11], the fixed pivot tends to remain on the concave side of the generated curve, however for the desired objective, the pivot must be positioned outside the curvature, which greatly limits the solutions. The parameters employed in the simulation are described as follows.

X=Design variables: $[\mathrm{r1}, \mathrm{r2},\ \mathrm{r3}, \mathrm{r4}, \mathrm{Up}, \mathrm{Vp}, \mathrm{X0}, \mathrm{Y0}, \theta1, \theta2]$
Prescribed points: Pd(x,y): [(10,10) (15,10) (20,10) (25,10) (30,10) (20,0)]
Δθ2=[35°, 35°, 35°, 35°], itermáx=1800, F=0.4, PC=0.8, PM=0.3,
NP - number of indivuduals =100

The other parameters used in the search are summarized in tab.1.

After a few iterations, the best solution was chosen:
r1= 74.9 , r2=7.34 , r3=38.84, r4=43.73, Up=22.21, Vp=35.53, X0=-17.74 Y0=-8.41 e θ1=301.2º
Figure 4 shows the curve generated by the chosen four-bar linkage and also the prescribed points.

Table 1 – Limits and search parameters

iter max	1800
F	0.4
PC	0.8
PM	0.3
limite1	0.2
limite2	5
r1,r3,r4[min,max]	[20,100]
r2[min,max]	[5,20]
X0[min,máx]	[-100,100]
Y0[min,máx]	[-160,5]
Up,Vp[min,max]	[-75,75]
ϴ1,ϴ2[min,max]	[0,360]
ang transm [min max]	[10,170]

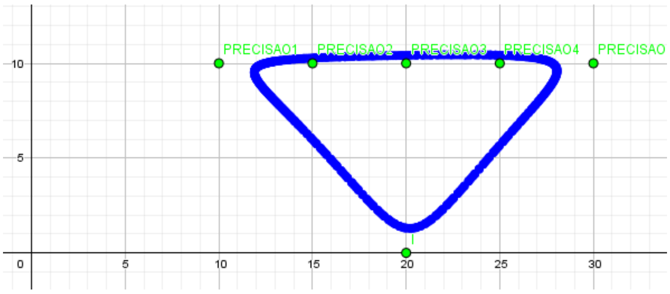

Figure 4. Curve generated by the coupler point of the four-bar mechanism.

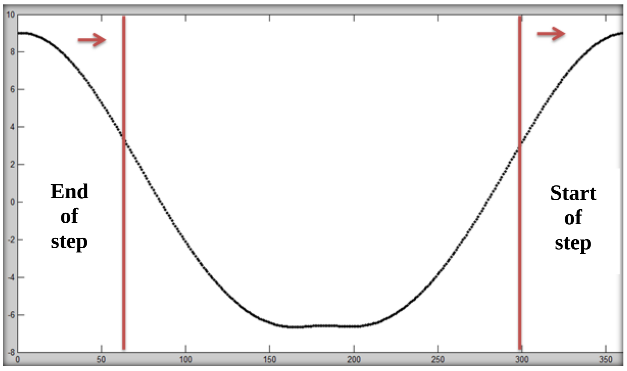

Figure 5. Coupler velocity with respect to the global X-axis with the crank speed of 1 rad/s.

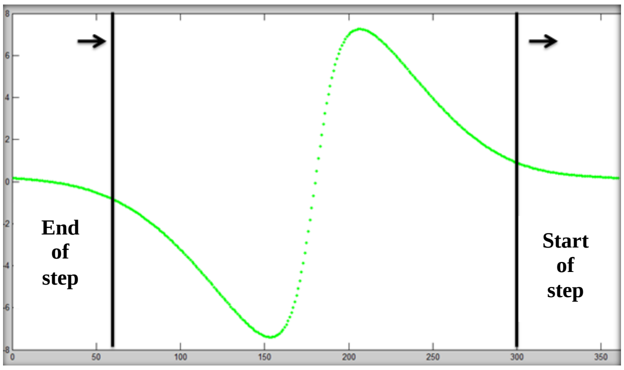

Figure 6. Coupler velocity with respect to the global Y-axis with the crank speed of 1 rad/s.

The residual value of the objective function was Fobj=24.5. Figure 5 and Fig. 6 show the velocities of the foot in relation to the ground. Figure7 shows the final mechanism combined with the pantograph, totalizing 6 links. The dimensions of the pantograph were chosen arbitrarily.

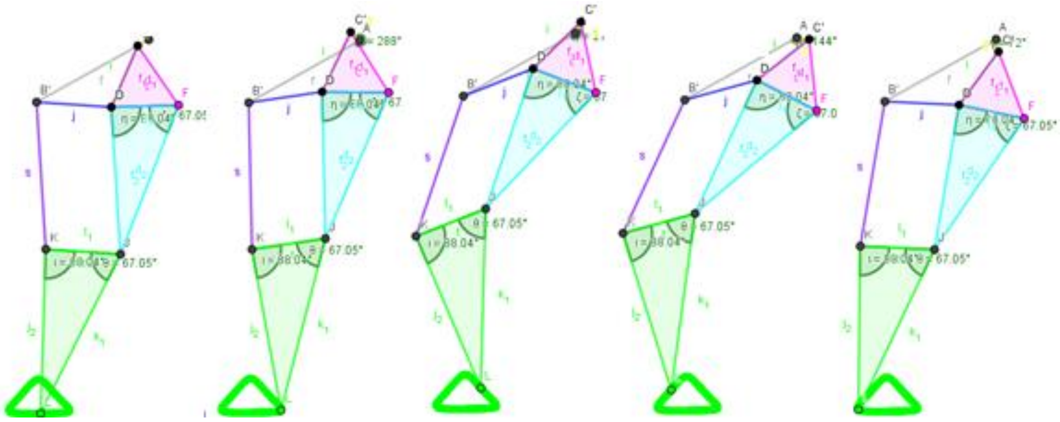

Figure 7. Motion of the leg mechanism in equal spacing of 72º degrees of the crank.

4 Conclusions

The optimization algorithm based on a genetic algorithm with differential evolution implemented in this work is capable of evaluating planar mechanisms to simulate the walking movement. The application to a four-bar mechanism for the curve generation problem using six prescribed points is demonstrated by an algorithm that allows mixing prescribed and non-prescribed time. The obtained mechanism is added to a skew pantograph, forming a six-bar-linkage.

The algorithm allows the fast generation of solutions, and with small errors, if compared to the graphical methodology. However, as a disadvantage, there is the possibility of obtaining solutions that represent local minima, which is common for heuristic methodologies. Among the candidate mechanisms, it was chosen the mechanism that generated a practically symmetrical curve with a rectilinear section, allowing walking in two directions. The solution rendered by the algorithm accounts for the restrictions of length ratio, fixed pivot position, and angle of transmission. The devised algorithm can be applied to other types of kinematic chains.

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