Numerical and Analytical Study of the Torsional Stiffness of a Flexible Disks Coupling

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Abstract. This work presents a study of torsional stiffness of flexible disk coupling used in synchronous generators coupled in diesel combustion engines. Stiffness analyzes are important basic systems to identify possible instabilities in torque transmission as a function of number disks. Torsional stiffness is an important parameter for the control of torsional vibrations in rotating machines analytically obtained based on simplified models. As it is a simplified method, the analytical method presents results, where constructive details of the coupling are not considered. This study focused analyzes the torsional stiffness responses considering different numbers of disks, screws pretension and disk material through numerical solution based on Finite Elements Method (FEM). The numerical results induced that the torsional stiffness increases in a non-linear way with the number of disks, being this behavior also identified in the analytical solution. It was also possible to verify that the screws pretension does not affect the torsional stiffness significantly.

Keywords: Torsional stiffness, flexible disk coupling, numerical analysis; theoretical analysis

1 Introduction

Power systems, such as generators, pumps, and turbomachinery with rotating components often incorporate coupled shafts to transmit power throughout the system. There are generally two types of coupling devices: rigid and flexible, according to Mott [1]. Flexible disk couplings are widely used in compressors, gas turbines, and aerospace applications because of their ability to compensate in almost all directions. A flexible coupling is a device used to connect the ends of two shafts, transmit torque, and at the same time, accommodate slight misalignments which develop in service. The flexible disk couplings were invented by M.T. Thomas in 1919. The original design used a pack of thin metal disks, which would flex to account for any misalignments [2]. Mancuso [3] describes about selection factors and design equations of rigid and flexible coupling.

The torsional stiffness of disk coupling has a great influence on shaft torsional vibration, and in many applications low-order torsional resonance may occur due to insufficient stiffness in the disk coupling. Investigation of the factors influencing the torsional stiffness of disk coupling will help to improve the design and control of shaft torsional vibration. Calculation of flexible coupling torsional stiffness is required when analyzing the torsional vibrations of the reciprocating machinery train. While having the lowest torsional stiffness of all the elements of the train, flexible coupling has a significant influence on the natural frequencies of torsional vibration. However, considering structural complexity of coupling, precise definition of its torsional stiffness is quite a difficult task [4]. Many researchers have contributed to the literature on the stiffness and corresponding dynamic behavior of flexible couplings. Buryy et al. [4] present a method to calculate the torsional stiffness of flexible disk based coupling in the study of the response of its finite element model under the action of torque. The results of the theoretical model were confirmed in the experimental measurement of the torsional stiffness of the flexible disk coupling. Zhao et al. [5] builted a 3D finite element (FE) model to estimate the stiffness of a disk coupling, taking the behavior of friction and contact into consideration. Feng et al. [6] calculated the axial stiffness of a trunnion joint and investigated the effects of rotation speed on its axial stiffness. Miaomiao et al. [7] studied a finite element model of the waist type laminated membrane coupling which considers the influence of the contact between laminations. In order to evaluate the performance of the proposed method, four stiffness characteristics of the coupling are analyzed, including torsional, axial, radial and angular stiffness.

Flexible torsional couplings are used primarily to transmit power between rotating components in industrial power systems, including turbomachinery, while allowing for small amounts of misalignment that may otherwise lead to equipment failure. Many researchers have contributed to the literature on turbomachinery applications. Gam [8] presents details of the coupling characteristics calculation. Mancuso et al. [9] present an overview of coupling failure modes and modeling techniques. Corcoran et al.[10] provide a similar overview but focused specifically on gas turbine systems. Szenasi [11] studied the modal form and natural frequencies of complete engine trains and Mondy [12] analyzed the natural frequencies of turbomachinery strings.

The torsional coupling lumped characteristics, such as torsional- and flexural stiffness, as well as natural frequencies of vibration are important for design of the entire power system and, therefore, must be calculated or computed with a high degree of accuracy. Francis et al. [13] compare theoretical, numerical, and experimental methods of characterizing torsional stiffness of a family of metallic disk type flexible couplings. They demonstrate the sensitivity of torsional stiffness to various design parameters and characterization assumptions, including boundary conditions, level of model detail, and material properties of the coupling's components.

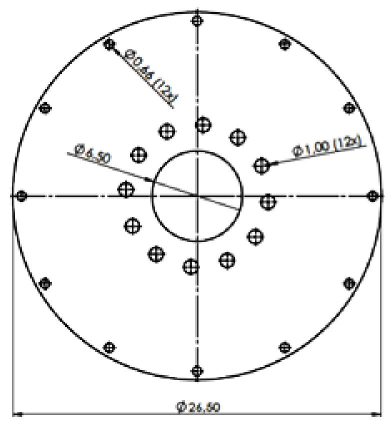
Shaft misalignment and rotor unbalance are major concerns in rotating machinery. In order to understand the dynamic characteristics of these machinery faults, Xu and Marangoni [14] developed a theoretical model of a complete motor-flexible coupling-rotor system capable of describing the mechanical vibration resulting from misalignment and unbalance. Experimental studies were performed on a rotordynamic test apparatus to verify the theoretical model of shaft misalignment and rotor unbalance and the results of the theoretical predictions were in good agreement with the experimental measurements [15]. In Ovalle's [16] thesis, the misalignment of vibrating machines was investigated by simulating parallel and angular misalignments by means of flexible disk package couplings using a finite element program.

This study focused on characterization of torsional stiffness of a metal disk coupling. This work presents a theoretical and numerical analysis of the coupling system formed by flexible coupling with application in diesel generator sets of STAMFORD® Alternators/1840kW/1800 rpm. The operating characteristics as well as the coupling data were provided by a thermoelectric (*Brentech Energia S / A* - Aparecida de Goiânia, Brazil) in order to analyze the influence on the number of coupling disks considering the effects of vibration on synchronous generators.

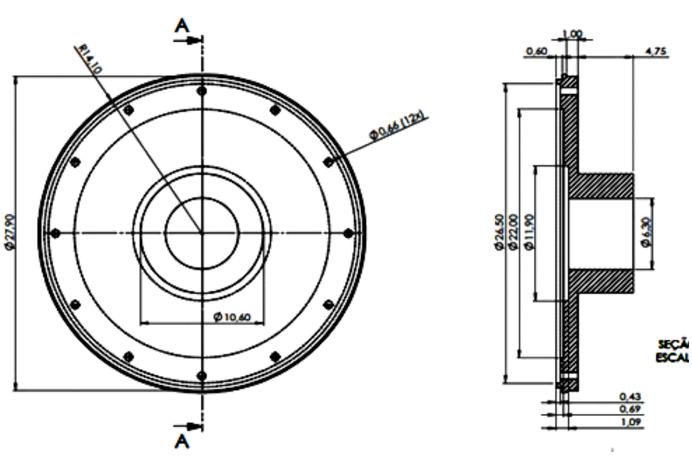
2 Characterization of the coupling

Flexible torsional couplings are used primarily to transmit power between rotating components in industrial power systems, including turbomachinery, while allowing for small amounts of misalignment that may otherwise lead to equipment failure [17]. The studied flexible coupling is a power transmission agent between the diesel engine and the electricity generator, as shown in Fig. 1. The flexible coupling analysed is composed by 6 metal disks, as Fig.1(a). Details of coupling dimensions are shown in the Figure 1,b-d.

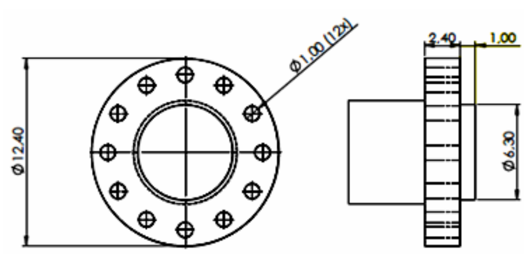
a)Flexible metal disk coupling (from *Brentech Energia S.A.* company)



b) Metal disk drawing, D_{ext}= 26,50 inch (0.67m) , disk thickness =0.05 inch (1.2 mm)



c)Motor flywheel drawing



d)Generator hub drawing

Figure 1. Dimensions of the metal disk coupling (dimensions in inch)

Table 1 presents material properties of coupling assembly components. The mechanical properties given in Tab.1 and the coupling dimensions shown in Fig. 1 are used to determine the numerical and analytical torsional stiffness. It is to be noted that in addition to the axisymmetric assumption, the analytical coupling model omits bolted connections.

Table 1. Mechanical properties of coupling assembly components.

Components	Material	Density (kg/m^3)	Young's modulus (GPa)	Poisson coef.
Motor flywheel	AISI 4140	7850	205	0.29
Generator hub	AISI 4140	7850	205	0.29
Metal disk	AISI 1006	7872	201	0.29

3 Methodology

3.1 Analytical approach

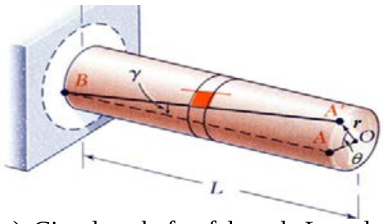
From Hook's Law, the stress-strain relationship of circular shafts in torsion is given by Eq. (1), as Beer et al. [18]

$$\tau = G\gamma \tag{1}$$

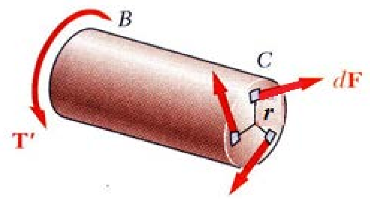
where τ is the shearing stress, *G* is the modulus of rigidity or shear modulus of the material and γ is the shearing strain

We observe from circular shaft subject to torsion as Fig.2(a) that, for small values of γ, the arc length AA' can be expressed as $AA' = \gamma L$. But, on the other hand, the arc length can be expressed $AA' = r\theta$. Therefore, γ is given by $\gamma = r\theta / L$, where the angular rotation of the disk about the axis of the shaft is represented by θ as shown in Fig 2(a). It also represents the shaft's angle of twist. When the shearing strain is maximum at the surface of the shaft, where *r* = r_e, The shearing strain can expressed as $\gamma_{\max} = r_e\,\theta / L$. Eliminating θ from equation of the shearing strain, we obtain

$$\gamma = \frac{r}{r_e}\gamma_{\max} \qquad and \qquad \tau = \frac{r}{r_e}\tau_{\max} \tag{2}$$



(a) Circular shaft of length L and radius r twisted at an angle θ and a shearing strain γ



(b) Torque due the shearing forces dF on the element of area dA

Figure 2. Shaft subject to torque (adapted of the Beer et al. [18])

Denoting by r the perpendicular distance from the force *dF* of the axis of the shaft, and expressing that the sum of the moments of the shearing forces dF about the axis of the shaft is equal in magnitude to the torque T, as Fig.2(b), we write [18]

$$T = \int r\,dF = \int r\left(\tau\,dA\right) \tag{3}$$

Substituting for τ from (2) into (3), we write

$$T = \int r\,\tau\,dA = \frac{\tau_{\max}}{r_e}\int r^2 dA \tag{4}$$

where the integral of Eq. (4) represents the polar moment of inertia, *J*, of the cross section.

The results are known as the elastic torsion formulas,

$$\tau_{\max} = \frac{T\,r_e}{J} \qquad and \qquad \tau = \frac{T\,r}{J} \tag{5}$$

where, from statics, the polar moment of inertia of a circle of radius r is $J = \pi r^4 / 2$ or $J = \pi D^4 / 32$ where *D* is diameter of the shaft. In the case of a hollow circular shaft of inner diameter D_i and outer diameter D_e of the polar moment of inertia is

$$J = \frac{\pi}{32}\left(D_e^{\,4} - D_i^{\,4}\right) \tag{6}$$

Substituting $\gamma = r\theta/L$ and Eq. (6) into Eq. (1), we have that $\tau = G\gamma$ is equal the relation $Tr/J = G\left(r\theta/L\right)$. Therefore, we obtain

$$T = \frac{GJ}{L}\theta \tag{7}$$

If the shaft (or a disk) is displaced by θ from its equilibrium position, the shaft provides a restoring torque of magnitude. Thus the shaft acts as a torsional spring with a torsional spring constant, Rao [19]. Therefore, the torsional stiffness of a hollow circular shaft are given by

$$K_T = \frac{T}{\theta} = \frac{GJ}{L} = \frac{\pi G}{32L}\left(D_e^{\,4} - D_i^{\,4}\right) \tag{8}$$

Each component is assumed to have a shape of a solid cylinder made of isotropic linear elastic material (Flexible Couplings Standard - American Gear Manufacturers Association - AGMA [20]). The torsional stiffness of a given component in a series is calculated by Eq. (8).

3.2 Numerical approach

The finite element method was used in order to numerically obtain the torsional stiffness of the model. Numerical modeling was performed via ANSYS Workbench R17. The flexible coupling non structurated mesh was generated with ~1.500.000 elements, as shown in Fig.3a.

For the analysis of mesh independence, three mesh configurations were considered, defined as: M1, M2 and M3, as shown in Table 2. The torque calculation was defined as a convergence criterion. As can be seen in Fig. 3, the M3 mesh was chosen because it has a small torque variation of only 1.983%, in relation to the M2 mesh, so opted for the mesh with 4707303 elements

Table 2..Mesh independence analyse of a coupling with 3 metal disks.

	element size (mm)	node number	number of elements	torque	error
M1	10	494282	118760	7,81E+05	-
M2	5	1153790	351926	8,08E+05	3.378%
M3	2,5	4707303	1477150	8,25E+05	1.983%

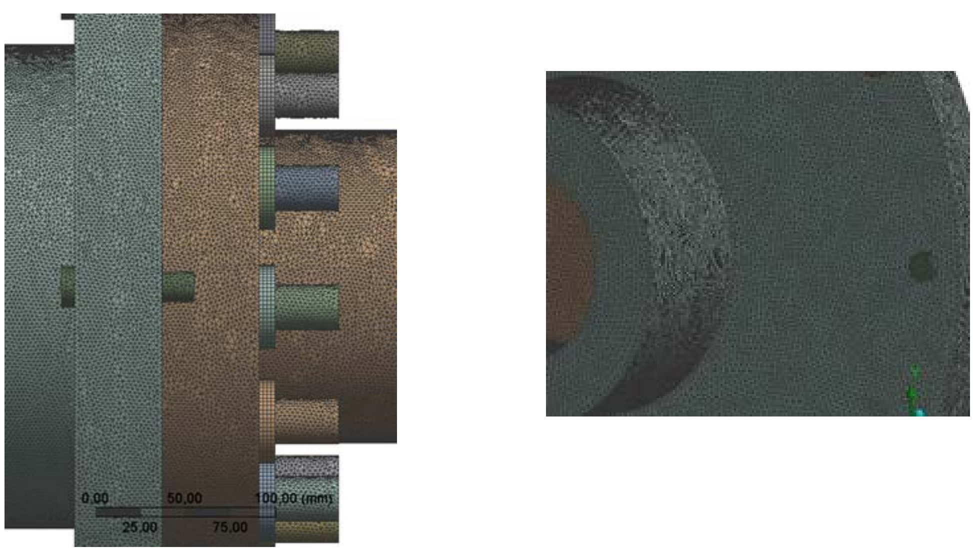


Figure 3. Mesh view: coupling mesh with 1477150 elements

The contacts between the elements of the set were configured as follows: the contact between screws, nuts, steering wheel and hub was simulated using "Bonded" contact. The contact between the disks was defined as Frictional with a coefficient of friction of 0.2. The rest of the contacts were simulated as "No separation, as Fig. 4.

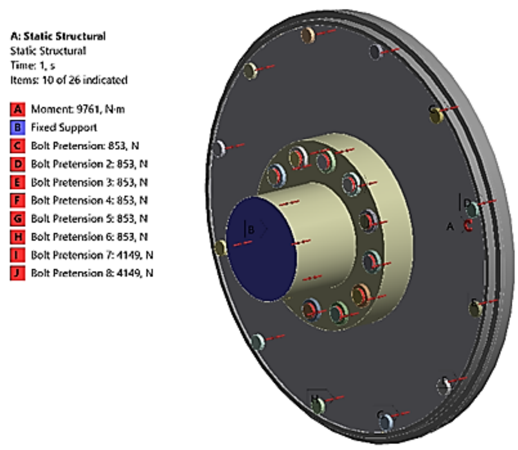


Figure 4: Boundary conditions: moment (torque), fixed support and bolt pretension

The 3D geometry was slightly disfeatured by removing small rounds and chamfers that would not have a critical impact on mode shapes or overall coupling stiffness. The FEM was prepared for a torque application through the flywheel with the opposite end clamped. The coupling modeling was twisted by the angular deformation of 2° on the flywheel face through a "remote displacement" tool. The torque generated by the angular deformation was measured by "Moment Reaction Probe" tool. On the opposite face (hub face) was applied fixed support. Figure 2(b) shows the input conditions. Note that only 10 of the 26 boundary conditions are indicated in the legend.

In order to save computational resources, as well as to avoid lengthy simulation time, small details such as chamfers were eliminated from the model since they do not produce significant influence on the result. Furthermore, as the purpose of the study is to analyze the influence of the number of disks and the tightening torque of the bolts on the torsional stiffness of the coupling, the rod was also simplified. By evaluating the results obtained from each model, it was possible to obtain the torsional stiffness of each model through Eq. (10), considering the numerical integration.

$$K_T = \frac{dT}{d\theta} \simeq \frac{\Delta T}{\Delta \theta} \tag{10}$$

4 Results

4.1 Numerical and analytical results

The complex coupling's geometry is replaced by a simplified coupling assembly model as depicted in Figs. 3(a). The torsional stiffness of the coupling assembly can be computed as a series

$$K_T = \frac{1}{K_{flywheel}} + \frac{1}{K_{6\,discs}} + \frac{1}{K_{hub}} \tag{11}$$

The simplified theoretical coupling model was compared with the numerical calculation of torsional stiffness using the dimensions and geometry from Fig.1 and Table 1, as shown in ANSI/AGMA 9004 standard [19]. The studied flexible coupling is composed by 6 metal disks. Table 3 presents the analytical torsional stiffness results. Torsional stiffness computed using this simplified FEA model is compared with the theoretical results as shown in Table 4. The percentage error of 2,789 % can be attributed to the difference in boundary conditions used in the analytical approach.

Table 3. Analytical torsional stiffness results

	Flywheel	Disks	Hub	K_T (Nm/rad)
Torsional stiffness (Nm/rad)	2.84×10^{8}	2.11×10^{1}	2.76×10	2.51×10^{7}

Table 4. Comparison of analytical and numerical torsional stiffness

Simplified models	Torsional stiffness (N.m/rad)	% error
Analytical model	2.58×10^{7}	**2.789%**
Numerical model (FEM)	2.51×10^{7}	

The torsional stiffness value of a coupling can be affected by several factors, such as geometry, mechanical properties or number of disks, among other parameters. The disk shape, thickness, and quantity can vary across manufacturers but are all relevant as this flexible element impacts the performance of the coupling. These components account for shaft misalignment and transmittal of torque through the system, Francis *et al.* [13]. The specific coupling used in this study has a standard disk assembly size of approximately 6 individual disks. This study investigates the impact study the impact of the variation in the number of disks on the torsional stiffness of the coupling. The number of disks included in the study ranged from 3 to 12 disks in increments of 3. The study the torsional stiffness of the coupling as presented in Table 5 and Fig. 4.

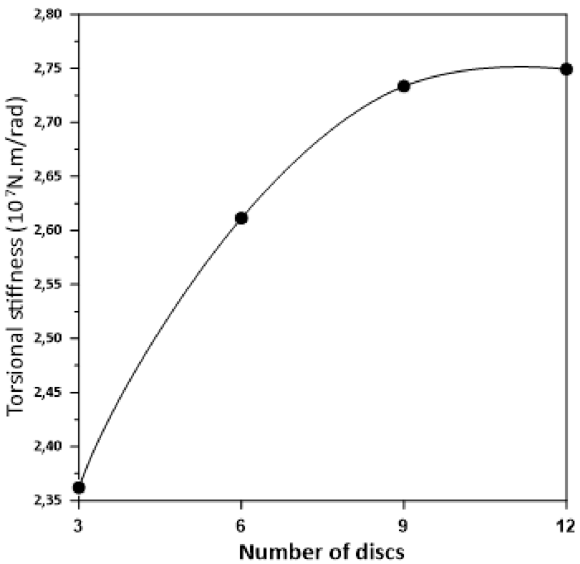


Figure 4. Variation of disks number and torcional stiffness.

Table 5. Study of the influence of the number of disks on torsional stiffness

Number of discs	Torsional stiffness (10^7 N.m/rad)
3	2.3621
6	2.6113
9	2.7334
12	2.7492

Based on Figure.4, it is possible to apply the Richardson extrapolation to identify if with 12 disks there is the asymptotic limit and the new torsional stress value above the 12 disks. This criterion is widely used in numerical analysis to determine the independence of the mesh. Richardson extrapolation is a sequence acceleration method used to improve the rate of convergence of a sequence of estimates of some values as can seen in Richardson and Gaunt [21]. In this study, the Richardson's extrapolation obtained by increasing the number of disks is 2.751540 x10^7.

The pack disks with 3 and 6 disks models were utilized to verify the influence of screw pretension on the torsional stiffness of the coupling. Table 6 shows that pretension have a very low influence on torsional stiffness.

Table 6. Study of the influence of pretension on torsional stiffness

Number of disks	Pretension (N)	Torque (10^5 N.m)	Torsional stiffness (10^7 N.m/rad)	deviation
3	1000	8.2452	2.3621	-
3	10000	8.2453	2.3621	0.001%
3	50000	8.2460	2.3623	0.008%
6	1000	9.1151	2.6113	-
6	10000	9.1152	2.6113	0.001%
6	50000	9.1152	2.6113	0.001%

Table 7. Study of the influence of disk material variation on torsional stiffness.

	Torsional stiffness		
Number of disks	AISI 1006 (N.m/rad)	AISI 1020 (N.m/rad)	Percentage difference
3	2,3799E+07	2,3621E+07	0,749%
6	2,6974E+07	2,3621E+07	3,193%
9	2,7427E+07	2,7334E+07	0,342%
12	2,8346E+07	2,7492E+07	3,012%

The resulting changes in Young's modulus (material property) for differents sets of disks are compared in Tab. 7. Analyzing the Tab. 7, we can observe that a change of material has the largest impact on the coupling torsional stiffness with even numbers of disks.

5. Conclusions

The simplified theoretical coupling model was compared with the numerical calculation of torsional stiffness using the dimensions and geometry from Fig.1 and Table 1. The torsional stiffness obtained by simplified FEA model is compared with the theoretical results as shown in Tab. 4. The percentage error of 2,789 % can be attributed to the difference in boundary conditions used in the analytical approach.

The torsional stiffness value of a coupling can be affected by several factors, such as geometry, mechanical properties or number of disks. This study investigated the impact of variable disk pack sizes on the torsional stiffness of the coupling. Disk pack sizes included in the study ranged from 3 to 12 disks in increments of 3. Based on Figure.5, the Richardson extrapolation was applied to identify the asymptotic limit and the new torsional stress value above the 12 disks. The Richardson extrapolation obtained is 2.751540 $\times 10^{7}$ (N.m/rad).

Disk sets with 3 and 6 disks models were utilized to verify the influence of screw pretension on the torsional stiffness of the coupling. Table 5 shows that pretension have a very low influence on torsional stiffness.

Finally, this study proposed to analyze the sensitivity of torsional stiffness for different disc materials.

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