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ANALYTICAL STUDY ON HORIZONTAL VIBRATION VELOCITY AND ACCELERATION IN SOILS NEAR RAILWAY LINES

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Abstract. Buildings are subject to vibrations caused by the environment, such as those from industry, constructions, vehicle traffic, among others. These vibrations can damage the building over time or cause discomfort for its users. The construction of buildings near railway lines is a reality increasingly present in large and small cities around the world. This work shows an analytical methodology for predicting velocities and accelerations in the soil caused by rail traffic, which affect nearby buildings, people or nearby improvements. Using equations validated in the literature, an analysis of the physical vibrations and noise generated by train traffic will be performed. The objective is to achieve a better understanding of the propagation of vibrations and to make better use of regions close to the railway lines. They are verified for four soil types in order to evaluate their influence on the propagation of the velocity and instantaneous acceleration in the horizontal axis of induced vibration in the medium. Other parameters are also analyzed, such as the distance from the railway, train speed and load, and the type of track. These analyses are performed using a mathematical program. In this way, it is expected to contribute to the standardization and understanding of the behavior and propagation of vibrations caused by the railways.

Keywords: Vibration velocity. Vibration acceleration. Ground vibration. Railroad. Train speed.

1 Introduction

Due to the increasing real estate speculation in the urban perimeters, the construction of automobile and railway traffic lanes near the buildings has become inevitable. The flow of trains induces vibrations in these buildings that can affect their occupants, as well as the structural system and the operation of equipment. Therefore, the lack of planning contributes to the generated vibrations having more pronounced and damaging effects on constructions. Therefore, the authorities and agencies responsible for the supervision and execution of buildings in urban environments try to establish criteria for the reduction of this potential damage.

Railway vibrations mainly caused by the contact between the wheel and the railroad tracks. The contact forces are vertical, longitudinal and transverse, with the vertical load being most responsible for vibrations in buildings located near railroads. The vibrations propagate through the ground as waves, causing the walls, floor and building structure to vibrate. Thus, the train traveling on the tracks is considered the source of vibration, that is, where the dynamic load is generated; the ground constitutes the path where the waves propagate; these waves are transmitted to the building, which is considered the receptor. Therefore, each of these regions has its own properties and parameters that affect the overall vibration levels. Although these regions function as a sequence, it can sometimes be useful to study each one separately.

The objective of this work is to present an analytical methodology for predicting speeds and accelerations of horizontal ground vibrations caused by passing trains. The equations used in this work were proposed by Eason [1], whose study was limited to train speeds below the Rayleigh wave speed.

2 Types of Waves

The vibrations generated at the source propagate through the free field in the form of waves. These waves are divided into body or volume waves and surface waves. Body waves propagate inside the earth and are divided into two types: primary ones, also called P waves, and secondary ones, called S waves. P waves are longitudinal waves that make the ground vibrate parallel to the direction of the wave, being the first to reach the surface, as they have a higher propagation speed. They are known as compression waves and have smaller amplitudes. They provide elastic solid bodies with changes in volume, without changes in shape.

S waves are transverse or shear waves, which means that the ground is displaced perpendicular to the direction of the propagation. They are slower than P waves, being the second to reach the surface and propagate only in solid bodies. They cause changes in shape, with no change in volume, and their amplitude is several times greater than that of P waves, but smaller than surface waves.

The propagation velocity of waves P (c_P) and S (c_S) are calculated by the following expressions

$$\boldsymbol{c}_P = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad \boldsymbol{c}_S = \sqrt{\frac{\mu}{\rho}} \tag{1}$$

where λ is the elasticity constant, μ is the modulus of stiffness (also known as the shear modulus), ρ is the density of the medium. Thus, the velocity of the P and S waves is not constant, but varies directly with the stiffness of the materials (the more rigid, the higher the velocity) and inversely with the density (the denser, the lower the velocity).

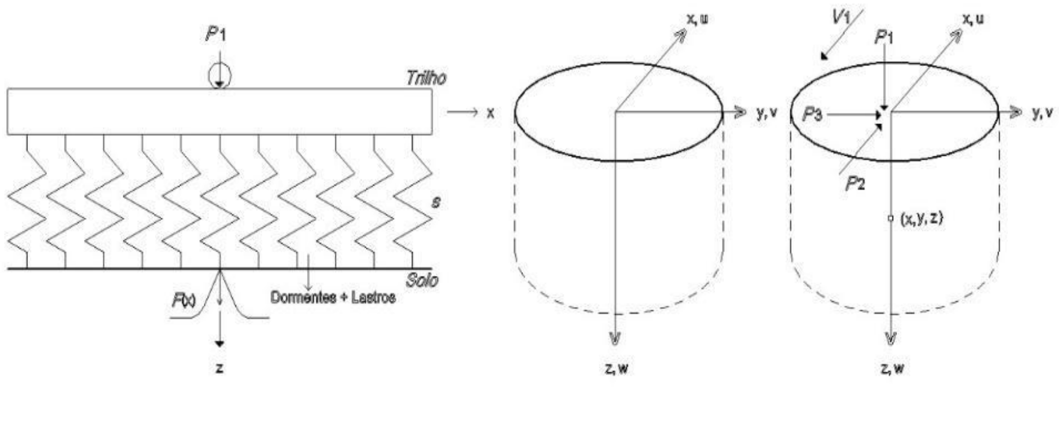
3 Mathematical Formulation

For a moving train, it is inferred that its vertical loads are much greater than its longitudinal and transverse loads. Therefore, in modeling the movement of the train along the track, only the displacements caused by the vertical load in relation to the ground are considered. The railway is considered an infinite Euler-Bernoulli beam (rail) on an elastic foundation (sleepers and ballast), whose stiffness coefficient is *s* (Fig. 1a).

The ground displacements (*u*, *v* e *w*) are evaluated according to the respective Cartesian coordinate axes *x*, *y* and *z*, where the ground surface is defined by the plane $z = 0$ (Fig. 1b), indicating that the positive axis of *z* points to the soil interior. In this case, the soil is considered homogeneous, elastic, isotropic, linear and semi-infinite, subject to a moving load with constant velocity applied to its surface.

Eason [1] used the stress-strain relations (Generalized Hooke’s Law) to deduce the equations of the ground motion. These equations are solved through integral transforms, such as the Fourier transform and its inverse, and through mathematical manipulations, the equations for the field of soil displacements are obtained, due to the point charge, P1, which moves along the rail, in the *x-axis* direction (Fig. 1c).

Considering the wheel load *P1* elastically distributed at $x = 0$ and $z = 0$, then the load distribution function can be written as it follows:



(a) (b) (c)

Figure 1. (a) Model of the railway; (b) Displacements *u*, *v* and *w* of the ground; (c) Load train axis (Source: Author)

$$F(x) = \frac{P_1}{2\alpha} e^{-|x|/\alpha} \left(\cos\left(\frac{|x|}{\alpha} \right) + \sin\left(\frac{|x|}{\alpha} \right) \right). \qquad \alpha = \sqrt[4]{\frac{4EI}{s}}. \tag{2}$$

In the expression above, α is the characteristic length of the railway track, which depends on the modulus of elasticity of the rail material, E, and on its inertia, I. Among the equations mentioned by Carvalho [2], there is the equation that measures the horizontal displacement v, along the y axis, which is the object of this study and is expressed by

$$v = \frac{F \;\; x \;\; r}{4\pi^2 \mu} \int_0^{\pi} \frac{\sin\varphi \cos(\theta - \varphi)}{H} \left\{ \frac{\left(1 - \frac{1}{2} \alpha_2^2 \cos^2 \varphi \right)}{R_1^2} - \frac{\gamma_1 \gamma_2}{R_2^2} \right\} d\varphi \tag{3}$$

$$w = \frac{F \;\; x \;\; z}{4\pi^2 \mu} \int_0^{\pi} \frac{\gamma_1}{H} \left\{ \frac{\gamma_1 \left(1 - \frac{1}{2} \alpha_2^2 \cos^2 \varphi \right)}{R_1^2} - \frac{\gamma_1}{R_2^2} \right\} d\varphi. \tag{4}$$

In the expressions above, z represents the depth of the ground and μ is the shear modulus of the medium through which the wave propagates. The parameters γ_1, γ_2, R_1, R_2 and H are mathematical simplifications, defined as

$$\gamma_1 = \sqrt{1 - \alpha_1^2 \cos^2 \varphi} \qquad \gamma_2 = \sqrt{1 - \alpha_2^2 \cos^2 \varphi}. \tag{5}$$

$$H = \left(1 - \frac{1}{2} \alpha_1^2 \cos^2 \varphi \right)^2 - \gamma_1 \gamma_2. \qquad \alpha_1 = \frac{V_t}{c_P} \qquad \alpha_2 = \frac{V_t}{c_S}. \tag{6}$$

$$R_1 = \sqrt{r^2 \cos^2 \;\; \theta - \varphi \;\; + \gamma_1^2 z^2} \qquad R_2 = \sqrt{r^2 \cos^2 \;\; \theta - \varphi \;\; + \gamma_2^2 z^2}. \tag{7}$$

In these expressions, r and θ represent the location in polar coordinates of the point in space where the displacements are measured. The coefficients α_1 and α_2 are determined from the train speed Vt and the speeds of waves P (c_P) and S (c_S), defined by eq. (1). The eq. (4) can be used to extract numerical solutions at any point inside the ground, but it cannot be solved on the surface, since at $z = 0$, the limit integral is not equal to the integral limit. To calculate the instantaneous ground velocity on the *y-axis*, eq. (3) must be derived in relation to time, and acceleration is obtained with the derivative of the velocity, as it can be observed in the following expressions.

$$\frac{d}{dt} v = v_v = \frac{d}{dt} \left(\frac{F \;\; x \;\; r}{4\pi^2 \mu} \right) \int_0^{\pi} \frac{\sin\varphi \cos(\theta - \varphi)}{H} \left\{ \frac{\left(1 - \frac{1}{2} \alpha_2^2 \cos^2 \varphi \right)}{R_1^2} - \frac{\gamma_1 \gamma_2}{R_2^2} \right\} d\varphi \tag{8}$$

$$\frac{d^2}{dt^2} v = a_v = \frac{d^2}{dt^2} \left(\frac{F \;\; x \;\; r}{4\pi^2 \mu} \right) \int_0^{\pi} \frac{\sin\varphi \cos(\theta - \varphi)}{H} \left\{ \frac{\left(1 - \frac{1}{2} \alpha_2^2 \cos^2 \varphi \right)}{R_1^2} - \frac{\gamma_1 \gamma_2}{R_2^2} \right\} d\varphi \tag{9}$$

3.1 Validation

A computational program based on the formulation of Eason (1965) was developed in the mathematical application MAPLE, in order to evaluate the problems studied here. The formulation was validated by comparing a given situation (eq. 4), with the equations presented in the work developed by De Barros (1994) [3]. The two mathematical formulations consider the soil as homogeneous, elastic, isotropic and semi-infinite. The railway vehicle is represented by a moving point load, at constant speed, applied to its surface. De Barros, in his work, observed apparent mathematical singularities in the mathematical script of Eason's formulation. Due to this mathematical impossibility, Barros chose to recalculate the expressions, evaluating the integrands numerically, so that they behave in the equation in a well-behaved manner, making the solution in a well-defined set. The train is represented by a point mobile load *P1*, equivalent to a mass with value of 10,000.00 kg, with the total mass of the wagon equal to 80 tons distributed in eight points of contact with the track (four wheels on each rail), moving at a speed of 324 km/h which is close to the value of the Rayleigh wave velocity on the ground (331.2 km/h). The geotechnical characteristics of the soil are 50 MPa for the longitudinal modulus of elasticity, 0.25 for Poisson's ratio, and 2000 kg/m³ is the soil density. Measurements are taken for a time interval from -0.05 s to 0.05 s, being the interval *t=0* the point where the largest vertical displacement in the ground occurs, whose value is 3.48 mm. To avoid the mathematical singularities mentioned above, polar coordinates are considered fixed for each analysis situation. At this moment, the mobile load is aligned with the observation point that is one meter deep in the ground. Therefore, it appears that there is good agreement between the two results, especially at the peak point (Figure 2).

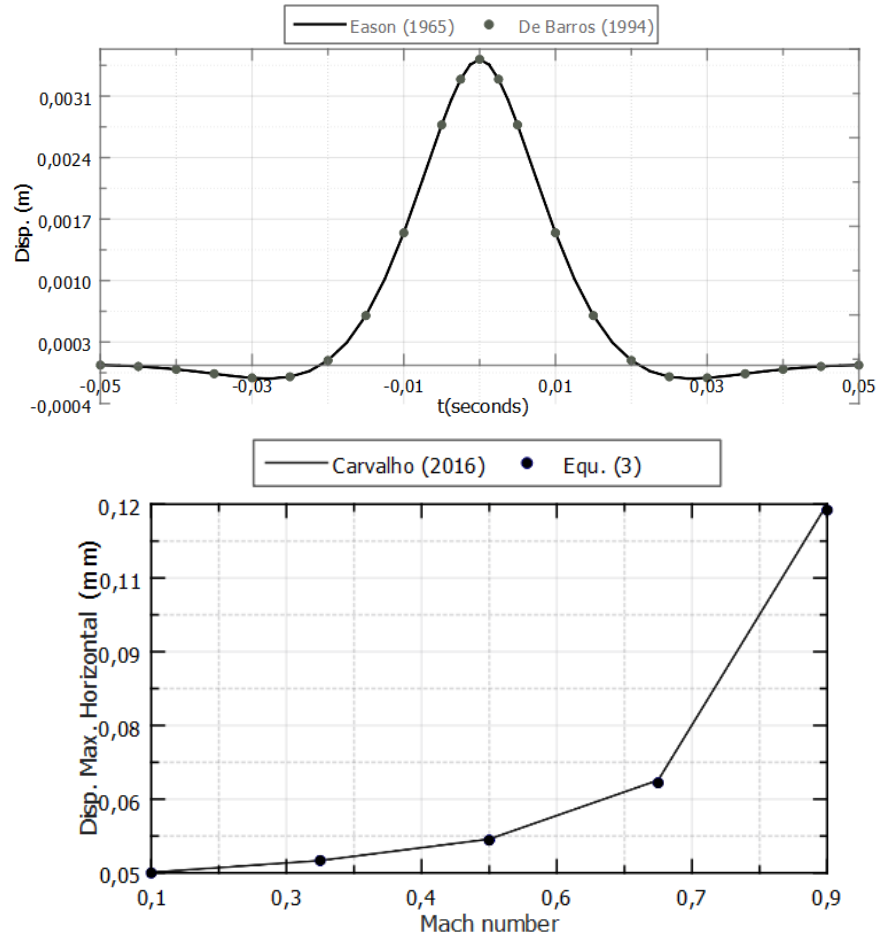


Figure 2. Comparison between Equation (4) and Barros (1994) (*x,y,z = 0,0,1*) / Comparison between Equation (3) and Carvalho (2016) (*x,y,z = 0,1,1*)

For horizontal displacements, the validation is done by analyzing the Mach number at *t=0*, that is, at the time when the displacement on the horizontal axis is maximum, which means that only the maximum values are used to make the graph. The comparison is made between equation (3) in the present work and a graph present in Carvalho's work (2016). Considering the train speed, the Mach number *M2* is defined as a dimensionless measure of speed, corresponding to the ratio between the train speed and the speed of the S waves on the ground *M2=Vt/cs*. It can be observed in Figure 2 that there is almost exact agreement between the results.

4 Case Study

According to FTA [4], train-induced vibration consists of rapidly fluctuating motions with an average motion of zero. There are several different methods that are used to quantify the level of vibration. *Peak Particle Velocity* (PPV) is defined as the maximum instantaneous positive or negative peak of the vibration signal. As in the velocity study, an acceleration graph is drawn for each situation of train speed, ground and track distance. From the analysis of the graph, the peak particle acceleration is defined as the maximum instantaneous positive or negative peak of the vibration signal. The vibration velocity level, *G*, in decibels, is extracted from the work by Xia. *et al*.[5]. The vibration velocity and acceleration level, *Lv* and *G*, in decibels, are defined as shown in the following expression.

$$L_V = 20\log_{10}\left(\frac{v_t}{v_{ref}}\right). \quad G = 20\log_{10}\ a\ + 60 \tag{10}$$

The average speed of the railways varies from 23.40 km/h to 31.07 km/h. Depending on the power of the locomotive and the number of cars hitched, trains can reach a maximum speed of 60 km/h, according to ANTT. For the present study, five types of speed are considered: 20 km/h, 30 km/h, 40 km/h, 50 km/h and 60 km/h.

For the purposes of this study, the value of 100,000 kg will be used. The analysis is carried out at a distance of up to 40 meters from the runway, on the *z* displacement axis at a depth of 10 cm from the ground surface. The type of rail used in the analysis is TR-57, whose value of α (eq. 2) is equal to 0.77.

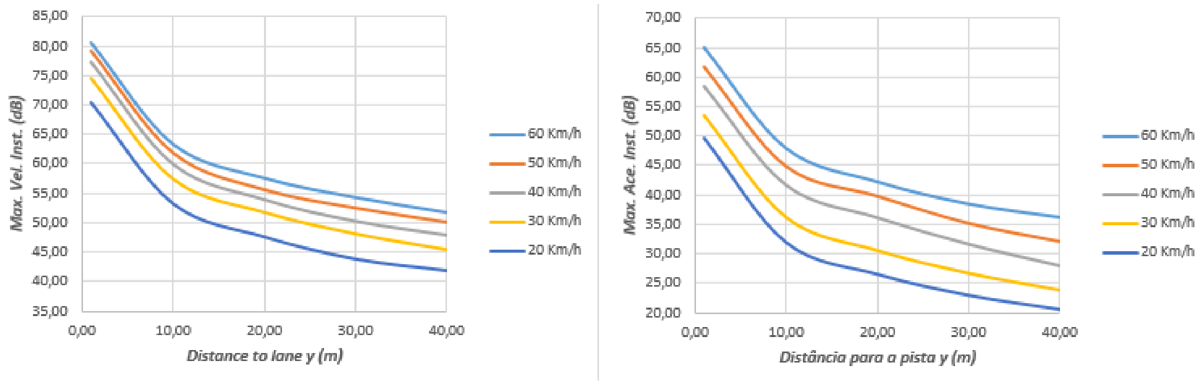
The existing railroads in Brazil pass through the most diverse types of soil. For the purposes of this study, in order to evaluate the influence of the soil type on the final velocity response, a parametric analysis is performed by varying its properties for three types of soil (Table 1). The properties of soil 1 medium coarse sand were extracted from Zou *et al*. [6], those of soil 2 were taken from Kourrossis *et al.* [7], those of soil 3 were taken from Feng *et al*.[8].

Table 1. Soil characteristics for the case study

Type soil	c_R (m/s)	c_S (m/s)	c_P (m/s)	ν	ρ (kg/m³)	E (N/m²)	G (N/m²)
Soil 1	220,98	240,20	416,00	0,25	1960,00	282,71 x 10^6	113,08 x 10^6
Soil 2	157,56	169,84	317,74	0,3	1600,00	120 x 10^6	46,1 x 10^6
Soil 3	26,33	28,29	54,99	0,32	1850,00	39,1 x 10^6	14,81 x 10^6

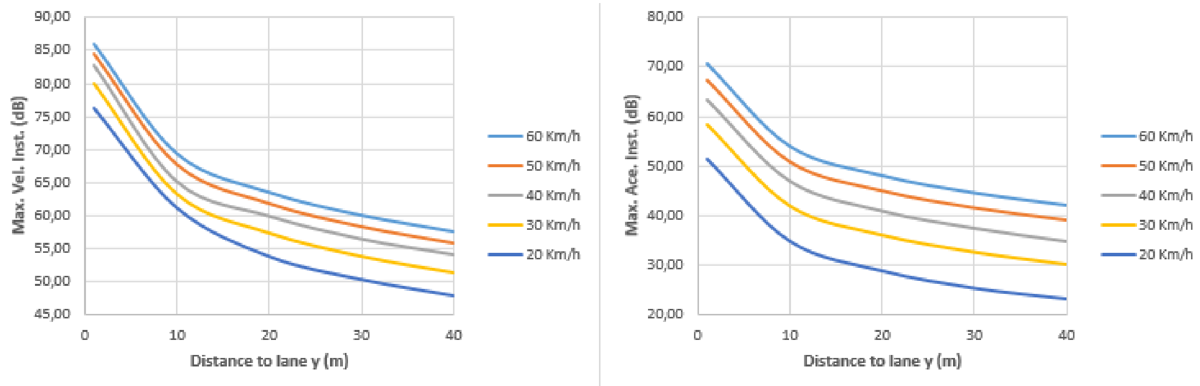
5 Soil Analysis

The vibration velocity was addressed for the *z* axis for other soil types in the analyses made in CILAMCE 2020 by Barroso Filho and Araújo (2020); in this work, both the velocity and the vibration acceleration on the *y* axis will be addressed, that is, the vibration acting on the soil particles horizontally. For the vibration velocity of the soil particles 1, with a wagon load of 100 tons, the velocity values of the soil particles can reach 80 dB only in the vicinity of the railway line, decaying quickly up to 15 meters from the track. After this distance, the decay is more attenuated; for train speeds above 40 km/h, in a range of up to 10 meters from the track, vibration speeds are above 60 dB which, according to the FTA, is the limit to be perceived by soil users. For the vibration acceleration, for all the analyzed train speeds and all the analyzed distances, the maximum value reached is 65 dB, decreasing rapidly as one gets further from the track, a value less than 70 dB that is not perceived by users; the acceleration reaches values less than 35 dB at 40 meters from the track, which are not perceptible.



(a) vibration speed (b) vibration acceleration

Figure 3. Maximum speed (dB) vs. track distance (m) to ground 1



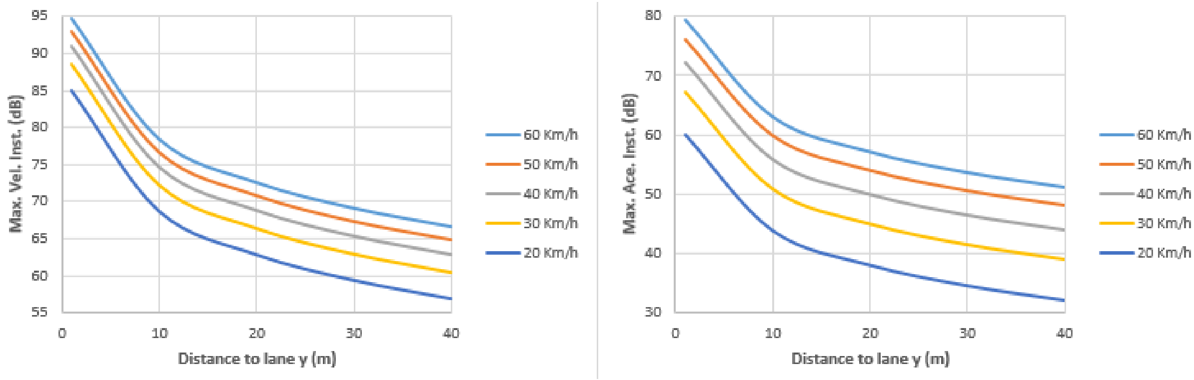
(a) vibration speed (b) vibration acceleration

Figure 4. Maximum speed (dB) vs. track distance (m) to ground 2

For the vibration velocity of soil particles 2, considering the first situation (Fig. 4a), for all analyzed train speeds, it is observed, in relation to the distances from the railway track, that the vibration impact is less than 100 dB, reaching 80 dB only in the vicinity of the railway line, the only effect felt by dwelling users being a momentary annoyance. In the second situation (Fig. 4b), for the vibration acceleration, concerning the analyzed train speeds and distances, the maximum value reached is approximately 70.6 dB, decreasing rapidly as one moves away from the track. This value, slightly higher than 70 dB, which is the limit for perception, acceleration reaches values lower than 40 dB at 40 meters from the track, for all train speeds, where they are not perceptible.

As for the vibration speed of soil particles 3, considering the first situation (Fig. 5a), for all train speeds analyzed in this case, it is observed that, for the distances in relation to the railway track, the vibration impact is less than 100 dB, reaching 90 dB only in the vicinity of the railway line. The effect felt by users of nearby housing was reading and vision difficulties. In the second situation (Fig. 5b), for the vibration acceleration, all the analyzed train speeds and all the analyzed distances related to the case, the maximum value reached is approximately 79.33 dB, decreasing rapidly as one moves away from the track a value greater than 70 dB, which is the limit for perception. The acceleration reaches values less than 48 dB at 40 meters from the track, which are not perceptible.

It is perceived for both the z-axis and the y-axis that the analytical equations used in this work are more sensitive to train speed when analyzing the acceleration than when analyzing speed. From the perspective of ground vibration acceleration, the higher the speed of the rail vehicle, the greater the increase and elevation of the curve of maximum ground acceleration in relation to the distance from the track, when compared with the same analysis done for the maximum ground speed. This finding was made for all types of soils analyzed for any orientation axis.



(a) vibration speed (b) vibration acceleration

Figure 5. Maximum speed (dB) vs. track distance (m) to ground 3

6 Conclusion

From the above, it can be concluded that the analytical expression of Eason [1], for the calculation of the displacement of the ground due to the passage of trains, is satisfactory and represents its behavior very well. As for the case study, the following conclusions can be obtained: (a) the stiffer the soil (higher modulus of elasticity), the lower the velocity and acceleration of the soil horizontal vibration; (b) the higher the velocity of P, S and R waves in the soil, the lower the ground vibration velocity; (c) the decay of accelerations and ground vibration speeds is higher in the range up to 15 meters from the track; after this mark, the decay continues to be more attenuated; (d) when the speed and load of the wagons increase, the velocity and vibration acceleration of the ground also increase; (e) the ground vibration acceleration is more sensitive to train speed increase when compared to ground vibration speed.

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