Bi-modularity representation proposal of quasi-brittle materials through isotropic degradation constitutive model

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Abstract. Quasi-brittle materials are governed by different stress-strain laws when subjected to tension and compression, so this characteristic is called bi-modularity. The classical Mazars model represents the behavior of bimodular materials through damage variables that comprises degradation in compression and tension. These variables are obtained through an exponential type law based on a single parameter to limit the material elastic domain and a unique equivalent strain. In this scenario, a new approach is presented. The proposed model considers modifications in the damage calculation, employing independent damage evolution laws and equivalent strain for tension and compression. This modification aims a better representation of the realistic behavior of quasi-brittle materials, especially the correlation between the model parameters and the material properties.

Keywords: bi-modularity, quasi-brittle materials, isotropic degradation, damage laws.

1 Introduction

Quasi-brittle materials, such as concrete, rocks, and some ceramic materials, have different behavior when subjected to compression and tension load solicitation. This behavior is called bi-modularity. In concrete, for example, in uniaxial tensile tests, the formation of microcracks perpendicular to the load can be observed due to the opening of microvoids caused by the curing of material process or due to the heterogeneity of the compound. When these microcracks join, mesoscale cracks appear, causing a fast loss of stiffness. In the compression regime, it is observed that the material has a non-linear behavior in the initial stages of loading due to the appearance of microcracks caused by the slipping or detachment of aggregates in the cementitious paste. In advanced stages of loading, due to the Poisson effect, the material cracks in a direction parallel to the loading, so the material strength decreases, and the loss of stiffness increases progressively more slowly.

In this context, the constitutive degradation models, which are based on the mechanics of continuous damage, seeks to represent the non-linear behavior of materials through variables called damage, which is used to characterize the cracking state, relating the progressive degradation of the medium through reduction of material stiffness and strength properties. The Mazars [1] damage model predicts the bimodular behavior of quasi-brittle materials through the coupling of damage variables that represent the degradation caused by compression and tension. Compression and tension damage are calculated from a single exponential-type law.

The purpose of this work is to formulate a model based on the Mazars [1] considering the behavior of material under compression or tension loadings computed independently. So, the proposed model establishes different damage laws for tensile and compression states. It will be seen later that the model has the versatility to assign any law format to represent the damage evolution. This strategy makes the parameterization process of bimodular materials easier. Thus obtaining a well approximate correspondence of the real behavior of quasi-brittle materials.

2 Formulation of isotropic degradation models

From the principles of thermodynamics, damage and permanent deformations are irreversible processes in the analysis. Isotropic degradation models assume that only the elastic properties of the material are affected by damage (D), so the thermodynamic potential ($\rho\psi$) can be written as

$$\rho\psi = \rho\psi_e(\varepsilon_e, D) + \rho\psi_p(\varepsilon_p) \tag{1}$$

where, ε_e are the elastic strains and ε_p are the permanent strains.

For quasi-brittle materials, permanent deformations can be considered negligible, so the portion of thermodynamic potential ($\rho\psi_p(\varepsilon_p)$) referring to permanent deformations can be disregarded for these models. The portion of the elastic thermodynamic potential can be defined by

$$\rho\psi_e(\varepsilon_e, D) = \frac{1}{2}(1-D)\varepsilon_{ij}E^0_{ijkl}\varepsilon_{kl} \tag{2}$$

where, ε_{ij} are the components of the strain tensor and E^0_{ijkl} are the components of the elastic constitutive stiffness tensor.

Stress components are calculated using the following equation

$$\sigma_{ij} = \frac{\partial\rho\psi^e}{\partial\varepsilon_{ij}} \tag{3}$$

From this equation, the total stress-strain relationship for isotropic damage models is given by

$$\sigma_{ij} = (1-D)E^0_{ijkl}\varepsilon_{kl} \tag{4}$$

2.1 Formulation of classical Mazars model

The Mazars [1] model is an isotropic degradation model that represents the mechanical behavior of quasi-brittle materials. This model proposes that the material is damaged in at least one of its principal positive directions. For this, the following equivalent strain is used

$$\tilde{\varepsilon} = \sqrt{<\varepsilon_1>^2_+ + <\varepsilon_2>^2_+ + <\varepsilon_3>^2_+} \tag{5}$$

where, $<\varepsilon_i>_+ = \varepsilon_i$, if $\varepsilon_i > 0$ and $<\varepsilon_i>_+ = 0$, if $\varepsilon_i \leq 0$.

The equivalent strain is a transformation of the multiaxial strain state into an equivalent uniaxial strain. This measure is a simple way to compute isotropic degradation in a nonlinear analysis and can relate the material degradation to a damage variable. The damage criterion is verified through a loading function ($f(\tilde{\varepsilon}, \kappa)$), where the equivalent strain is compared with a historical variable (κ).

$$f(\tilde{\varepsilon}, \kappa) = \tilde{\varepsilon} - \kappa \tag{6}$$

Initially, the historical variable assumes an elastic strain limit value (κ_0), and when the constitutive relationship enters the inelastic domain, this variable always assumes the highest equivalent strain value ever reached during the analysis, so

$$\kappa = \kappa_0 \longrightarrow D = 0 \tag{7a}$$

$$\kappa = max[\tilde{\varepsilon}, \kappa] \longrightarrow D > 0 \tag{7b}$$

The damage variable couples degradations caused by compression and tension (D_c and D_t respectively) and are weighted through variables denoted by α_c and α_t which are functions of strain state, therefore

$$D = \alpha_t D_t + \alpha_c D_c \tag{8}$$

Tension damage and compression damage are obtained by an exponential law, given by

$$D_t = 1 - \frac{(1-A_t)\kappa_0}{\tilde{\varepsilon}} - \frac{A_t}{exp[B_t(\tilde{\varepsilon} - \kappa_0)]} \tag{9a}$$

$$D_c = 1 - \frac{(1-A_c)\kappa_0}{\tilde{\varepsilon}} - \frac{A_c}{exp[B_c(\tilde{\varepsilon} - \kappa_0)]} \tag{9b}$$

The parameters A_t, A_c, B_t and B_c are material dependent and can be obtained through the fitting of stress-strain curves from experimental tests of tension and compression. The weight parameters are defined by

$$\alpha_t = \sum_{i=1}^{3} H_i \frac{\varepsilon_{ti}(\varepsilon_{ti} + \varepsilon_{ci})}{\tilde{\varepsilon}^2} \tag{10a}$$

$$\alpha_c = \sum_{i=1}^{3} H_i \frac{\varepsilon_{ci}(\varepsilon_{ti} + \varepsilon_{ci})}{\tilde{\varepsilon}^2} \tag{10b}$$

where $H_i = 0$ for $\varepsilon_{ti} + \varepsilon_{ci} < 0$ and $H_i = 1$ for $\varepsilon_{ti} + \varepsilon_{ci} \geq 0$.

The tensors $\boldsymbol{\varepsilon_t}$ and $\boldsymbol{\varepsilon_c}$ are obtained through the elastic constitutive law with the positive principal stress tensors ($\boldsymbol{\sigma_+}$) and the tensors of negative principal stresses ($\boldsymbol{\sigma_-}$) respectively, so

$$\boldsymbol{\varepsilon_t} = \boldsymbol{C} : \boldsymbol{\sigma_+} \tag{11a}$$

$$\boldsymbol{\varepsilon_c} = \boldsymbol{C} : \boldsymbol{\sigma_-} \tag{11b}$$

where, $\boldsymbol{C}$ is the elastic constitutive tensor of flexibility.

It is observed that the Mazars [1] model is capable of distinguishing the behavior of the material when subjected to predominant tensile and compression stresses. However, because the damage variable depends on the compression damage and tension damage, the parameters used for the compression regime influence the behavior of the material in the tensile regime, especially for the κ_0 parameter.

Having parameters that are not independent in the model becomes a problem for parameterization and its correspondence with the real behavior of quasi-brittle materials, which is characterized by bi-modularity. Furthermore, having a single format for the damage evolution law to represent the damage caused by tension and compression amplifies the difficulties in the parameterization of model.

2.2 Formulation of proposed model

To adequately represent the bimodular behavior of quasi-brittle materials so that the model has a response close to the correct material behavior, is proposed a modification of the classical Mazars [1] model. This proposal consists of creating independent parameters and variables and specific laws for the predominant state of compression and tension. The presented model adopts the format of the original equivalent strain of the model of Mazars [1] for each regime (compressive and tensile) and has the versatility to assign any type of function best suited to represent the evolution of damage in tension and compression.

The equivalent strains ($\tilde{\varepsilon}_t$) and ($\tilde{\varepsilon}_c$) are related to tension and compression, respectively, and are defined by

$$\tilde{\varepsilon}_t = \sqrt{<\varepsilon_1>_+^2 + <\varepsilon_2>_+^2 + <\varepsilon_3>_+^2} \tag{12}$$

where, $<\varepsilon_i>_+ = \varepsilon_i$, if $\varepsilon_i > 0$ and $<\varepsilon_i>_+ = 0$, if $\varepsilon_i \leq 0$.

$$\tilde{\varepsilon}_c = \sqrt{<\varepsilon_1>_-^2 + <\varepsilon_2>_-^2 + <\varepsilon_3>_-^2} \tag{13}$$

where, $< \varepsilon_i >_- = \varepsilon_i$, if $\varepsilon_i < 0$ and $< \varepsilon_i >_- = 0$, if $\varepsilon_i \geq 0$.

The inclusion of a new equivalent strain for a dominant compression state is a better way to follow the damage variation, because using the equivalent strain described by Equation 13 in a state where compression prevails, is possible to capture the damage caused by compression regardless of the influence of the tension state.

The strength criterion of the model is accompanied by two loading functions $f_t(\tilde{\varepsilon}, \kappa_t)$ and $f_c(\tilde{\varepsilon}, \kappa_c)$ which are related to tension and compression, respectively, given by

$$f_t(\tilde{\varepsilon}, \kappa_t) = \tilde{\varepsilon}_t - \kappa_t \tag{14a}$$

$$f_c(\tilde{\varepsilon}, \kappa_c) = \tilde{\varepsilon}_c - \kappa_c \tag{14b}$$

The historical variables for tension and compression (κ_t and κ_c respectively) are defined by

$$\kappa_i = \kappa_{0i} \longrightarrow D = 0 \tag{15a}$$

$$\kappa_i = max[\tilde{\varepsilon}_i, \kappa_i] \longrightarrow D > 0 \tag{15b}$$

where, $i = c$ for compression and $i = t$ for tension.

In order to prescribe specific parameters and laws for a given domain, is necessary to identify the dominant state (compression or tension). The strategy was adopted by Comi [2] and Comi and Perego [3]. Therefore, the first invariant (I_1) of the strain tensor is a criterion that indicates whether the state is predominantly compression or tension

$$\begin{cases} I_1 \geq 0 \longrightarrow \text{ Predominant Tension} \\ I_1 < 0 \longrightarrow \text{ Predominant Compression} \end{cases} \tag{16}$$

The first invariant of the strain tensor is associated with the solid volume variation, when it assumes a positive value, it indicates that the solid had an increase in its volume, otherwise, it indicates a volume reduction. This volume variation indicates the predominance of the strain state of the analyzed point. This strategy enabled the model to assign appropriate laws for each type of identified solicitation.

In the proposed model, the damage variable is not conditional on coupling tension-compression damages. So, the tension damage and compression damage depends on different evolution laws, each one correspondent to the respective dominant state

$$D_t = F_t(\tilde{\varepsilon}_t) \tag{17a}$$

$$D_c = F_c(\tilde{\varepsilon}_c) \tag{17b}$$

Based on the standard damage models of de Borst and Gutierrez [4], is possible to prescribe evolution laws of any type to represent the damage functions F_t and F_c. Some of these types of functions present in the literature are described below.

The adapted form of Carreira and Chu [5] reproduces a polynomial type law. In this function, the effect of nonlinearity is already observed in initial stages of loading

$$F(\kappa_i) = 1 - \frac{f_i \gamma}{\left[\gamma - 1 + \left(\dfrac{\kappa_i}{\kappa_{0i}}\right)^{\gamma}\right] \tilde{E}_i \kappa_{0i}} \tag{18}$$

where, $\tilde{E}_i$ is the equivalent modulus of elasticity, f_i is the material strength limit. So,

$$\gamma = \frac{1}{1 - \left(\dfrac{f_i}{\kappa_{0i} \tilde{E}_i}\right)} \tag{19}$$

An exponential type equation presented by de Borst and Gutierrez [4] is described by

$$F_i(\kappa_i) = 1 - \frac{\kappa_{0i}}{\kappa_i}\{1 - \alpha_i + \alpha_i exp[-\beta_i(\kappa_i - \kappa_{0i})]\} \tag{20}$$

where, α_i e β_i are material parameters.

The damage function used by Jirásek [6] can be defined through a maximum deformation κ_{fi} that the material can achieve, this format produces a linear type law

$$F_i(\kappa_i) = \frac{\kappa_{fi}}{\kappa_{fi} - \kappa_{0i}}\left(1 - \frac{\kappa_{0i}}{\kappa_i}\right) \tag{21}$$

where, $i = c$ for compression and $i = t$ for tension.

3 Numerical Simulations

The Mazars [1] model and the proposed model were implemented in the INteractive Structural ANalysis Environment (INSANE), a computational platform developed at the Department of Structural Engineering of the Federal University of Minas Gerais.

To evaluate the formulation and implementation of the proposed model, the three-point bending test presentede by Petersson [7] was simulated. The geometric data of the beam are specified in Figure 1.

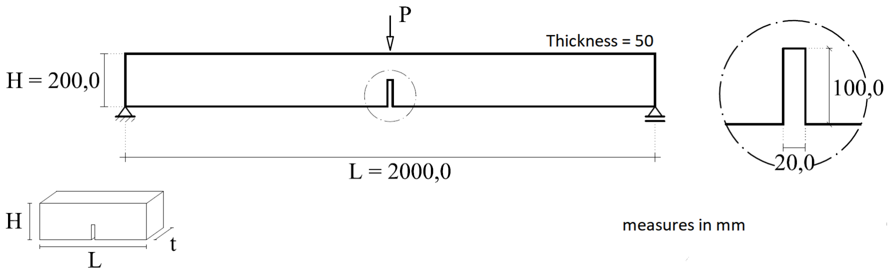


Figure 1. 3-point bending test beam

For this simulation, a mesh composed of quadrilateral elements Q4 was used and considering the symmetry of the structure, only half of the beam was modeled, using a reference load equal to $P/2 = 500N$ (Figure 2).

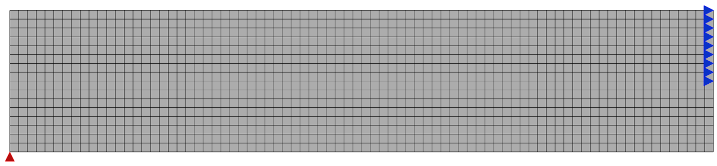


Figure 2. Mesh used to perform the test

The parameters of the concrete were obtained experimentally by Petersson [7] in which the tensile strength (f_t) is in the range of 2.5 MPa to 3.9 MPa and the fracture energy (G_f) between 0.115 N/mm and 0.137 N/mm. Modulus of Young (E) was assumed to be 30000.0 MPa through a theoretical approximation made by the author considering the effects of resonance in beams. The characteristic length (h) obtained was 40 mm.

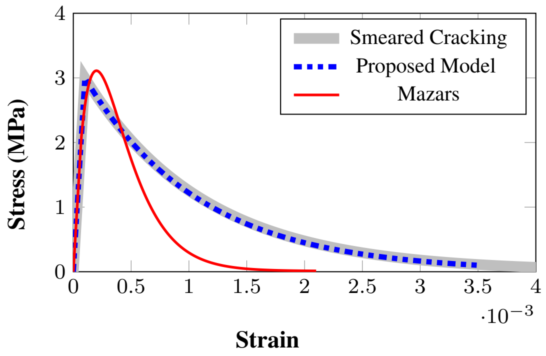
The author does not show data on the Poisson coefficient and the compressive strength of the concrete used, however, for the simulation, normative relationships were used to determine these parameters according to ABNT NBR 6118 [8]. Therefore, the compressive strength (f_c) was defined as 31.6 MPa, the Poisson coefficient (ν) equal to 0.2 and the strain corresponding to the compressive strength (ε_c) equal to 0.002 .

To determine the parameters of the implemented models correspondent to the parameters of the concrete, a smeared cracking model described in Penna [9] was used as reference, once this referred model adopts constitutive laws of Carreira and Chu [5] for compression and Boone and Ingraffea [10] for tension, whose input variables are the actual material parameters. The input data adopted for the smeared cracking model and the data obtained from the parameterization are described in Table 1.

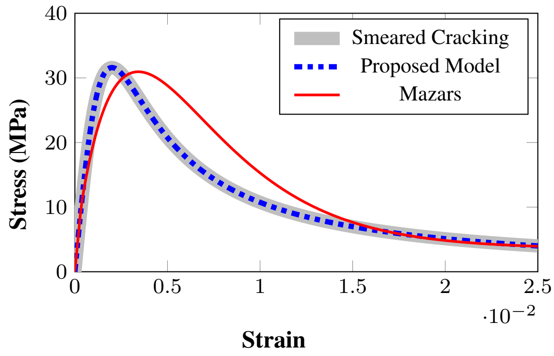
Table 1. Parameters of models

Smeared Cracking Model		**Model of Mazars**		**Proposed Model**	
Law of Ingraffea (tension)	**Law of Carreira (compression)**	**Law of Mazar (tension)**	**Law of Mazar (compression)**	**Exponential Law (tension)**	**Polynomial Law (compression)**
E_0 = 30000 MPa	E_0 = 30000 MPa	E_0 = 30000 MPa	E_0 = 30000 MPa	E_0 = 30000 MPa	E_0 = 30000 MPa
ν = 0.2	ν = 0.2	ν = 0.2	ν = 0.2	ν = 0.2	ν = 0.2
f_t = 3.0 MPa	f_c = 31.6 MPa	A_t = 0.995	A_c = 0.655	α = 0.999	f_c = 31.6 MPa
ε_t = 0.0001	ε_c = 0.002	B_t = 5000	B_c = 1460	β = 1000	κ_{0c} = 0.002
G_f = 0.124 N/ mm		$\kappa_0 = 6.9 \times 10^{-5}$	$\kappa_0 = 6.9 \times 10^{-5}$	κ_{0t} = 0.0001	$\tilde{E}$ = 30000 MPa
h = 40 mm					

After the parameterization of the model (Figure 3(a) and 3(b)), it is possible to verify that the proposed model is able to effectively approximate experimental curves (or model curves, which are defined from experimental data and express in terms of the material properties). This characteristic is due to decoupling of tension and compression variables, aligned with the freedom to adopt a damage function for the model that best represents the material response.



(a)



(b)

Figure 3. Parameterization of model(a) Tension (b) Compression

For this test (Figure 4), the generalized displacement control method was used, with an increment of 0.08 and a convergence tolerance of 1×10^{-4} . Secant equilibrium was used.

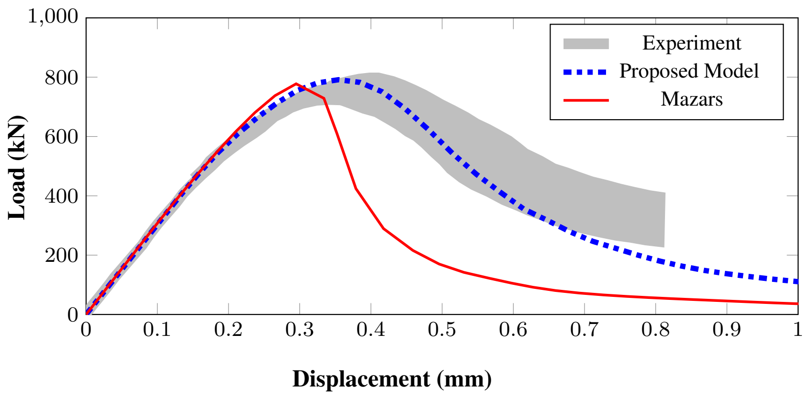


Figure 4. Equilibrium trajectory of the vertical displacement of the load application node

From the results obtained from the models for the test, it is verified that the proposed model presents a good agreement with the experimental result, while the classic model of Mazars [1] presented intense decrease of load capacity in post-peak branch of the curve. This disagreement with the results of the Mazars [1] model is directly linked to the parameterization of the model since the behavior of the material submitted to uniaxial tension behaved more fragile than the other models.

4 Conclusions

In this work, the main characteristics of the quasi-brittle materials of distinct behaviors when subjected to tension or compression are discussed. The Mazars [1] model can reproduce the bimodular behavior of the materials, even having a single type of damage law and a common material parameter to limit the elastic domain in tension and compression behavior. These properties cause difficulties in the parameterization process and the correspondence with the properties of the quasi-brittle material. Therefore, separating the tensile and compression variables and setting the material model with different types of damage functions provides a better approach to the realistic behavior of bimodular materials, like concrete, to the proposed damage model.

The proposed formulation can be extended to other classic isotropic damage models, such as the de Vree et al. [11] model, Lemaitre and Chaboche [12] model, and Simo and Ju [13] model.

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References

[1] J. Mazars. *Application de la mécanique de l'endommagement au comportement non lineaire et à la rupture du béton de structure*. PhD thesis, Universite Paris, Paris, 1984.

[2] C. Comi. A non-local model tension and compression damage mechanims. *European Journal of Mechanics - A/Solids*, 2001.

[3] C. Comi and U. Perego. Fracture energy based bi-dissipative damage model for concrete. *International Journal of Solids and Structures*, 2001.

[4] R. de Borst and M. A. Gutierrez. A unified framework for concrete damage and fracture models including size effects. *International Journal of Fracture*, 1999.

[5] D. J. Carreira and K. Chu. Stress-strain relationship for plain concrete in compression. *ACI Journal*, 1985.

[6] M. Jirásek. Non-local damage mechanics with application to concrete. In *French J Civil Eng*, volume 8, pp. 683–707, 2004.

[7] P. Petersson. *Crack growth and development of fracture zones in plain concrete and similar materials*. PhD thesis, Division of building materials. Lund Institute of Technology, 1981.

[8] ABNT NBR 6118. Projeto de estruturas de concreto - procedimento. *Associação Brasileira de Normas Técnicas*, pp. 238, 2014.

[9] S. S. Penna. *Formulação multipotencial para modelos de degradação elástica: Unificação teórica, proposta de novo modelo, implementação computacional e modelagem de estruturas de concreto*. PhD thesis, Universidade Federal de Minas Gerais, Belo Horizonte, MG, 2011.

[10] T. Boone and A. R. Ingraffea. Simulation of the fracture process at rock interfaces. In *Proceedings of the fourth international conference in Numerical Methods in fracture Mechanics*, pp. 519–531, 1987.

[11] J. H. de Vree, W. A. M. Brekelmans, and van M. A. J. Gils. Comparasion of nonlocal approaches in continuum damage mechanics. *Computers and Structures*, 1995.

[12] J. Lemaitre and J. L. Chaboche. Aspect phénoménologique de la rupture par endommagement. *Journal de Mécanique Appliquée*, 1978.

[13] J. Simo and J. W. Ju. Strain and stress-based continuum damage models - i. *Formulation International Journal of Solids and Structures*, 1987.