Theoretical analysis of the active human-structure interaction on rectangular plates

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Abstract. When analyzing the effect of human actions on structures, it is common to consider only the dynamic force generated by the crowd (only force models) in the design practice, where the biodynamic properties, such as mass, stiffness and damping, are neglected. However, several studies have shown that human properties influence the dynamic response of structures. On that basis, in this work the human induced vibrations on thin rectangular plates are investigated. For this, a biodynamic model is used to represent the people walk, where the biodynamic properties are coupled with the properties of the plate. The boundary conditions are considered by linear rotational springs on all edges in order to characterize a plate almost clamped. The influence of the biodynamic properties, as walking velocity, mass and damping ratio of a single person in the dynamic response of system is studied. The biodynamic model is also compared with a force-only model and the results are discussed. Obtained results shown that the modeling using the force-only model provides an increase of transversal displacement of plate when compared to the biodynamic model. It was also noted that increased of modal mass and walking velocity cause a reduction of displacements, while increases in pedestrian damping ratio lead to larger vibration amplitudes and the position of amplitude peak point is also modified.

Keywords: Human-Structure Interaction, Biodynamic Models, Rectangular Plates

1 Introduction

In the design practice of floors subjected to human induced vibrations, the modelling of structure with force-only models is recurrent, where a concentrated force is considered to characterize the human behavior, which can be fixed or traveling at constant walking velocity. In analysis using force-only models, the modal parameters of the empty structure are considered and the individual's biodynamic properties, such as stiffness, mass and damping, are neglected. In order to better represent people walking on floors, some researchers have studied a more realistic model by considering a mass jointly with the force-only model, known in literature as mass models. However, mass models still cannot represent all the biodynamic properties of a person and the beneficial effect of the human-structure interaction (HSI) is not noticed, resulting in costly structural designs.

The HSI is the effect of the combination between the dynamic properties of a human body and the structure, where the person acts on the structure and consequently the structure acts on the pedestrian. The passive interaction refers to the standing people positioning on the floor, which can cause changes in the vibration frequency and critical damping of the structure (SHAHABPOOR et al. [1]). On the other hand, the active HSI is when the individual moves over the structure and a dynamic force is added, which is practically null in passive interaction and is related to the activity that the individual can perform, such as walking or running/jumping (GASPAR et al. [2]). In this type of interaction, problems can be modeled using moving models, as studied by Caprani and Ahmadi [3], or by equivalent stationary models, as analyzed by Shahabpoor et al. [1].

Several studies have been dedicated to representing the HSI through biodynamic models, in which spring-mass-damper (SMD) systems with few degrees of freedom are coupled to the structure in order to represent the biodynamic properties of a person acts on the. A SMD model was used to analyze human induced vibrations on the vertical direction of structures was proposed by Toso et al. [4] and Toso and Gomes [5], where the values of the biodynamic parameters were obtained through regression techniques and were given by function of the static mass and step frequency of the person. Further on, Varela, Pfeil and Costa [6] experimentally analyzed an instrumented rigid platform under the action of 53 different people walking individually and noticed that the person's height is also an important factor in determining the dynamic load of human action on the platform.

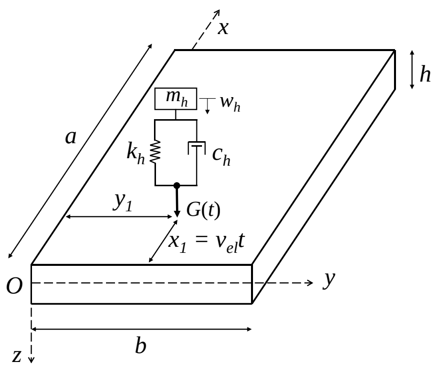
In the work developed by Caprani and Ahmadi [3], the dynamic behavior of a pedestrian bridge considering initially a simple pedestrian and later a crowd, was analyzed. It was noticed that the influence of a simple pedestrian decreases the frequency and increases the damping of the structure, becoming more important when a crowd is considered. It was also noticed that accelerations are smaller when considering a SMD model rather than the use of the force-only to model the structure, showing the beneficial effect of the SMD model for the HSI. Similar results are found by Shahabpoor et. al [1], where the biodynamic properties of a crowd walking on a prestressed concrete slab for different numbers of people are identified.

Gaspar et al. [2] numerically and experimentally compared the dynamic behavior of a vibrating floor during cyclical jumps of an individual at the same point in the structure. In the numerical analyses, initially a force-only model was used and later a biodynamic model to achieve the experimental analysis. From optimization techniques, the individual's biodynamic parameters were obtained. This author noted that the force-only model provided larger acceleration peaks when compared to the SMD model for the near-resonant case. On the other hand, the force-only model comes closer to the experimental response when the excitation frequency moves away from the resonant region.

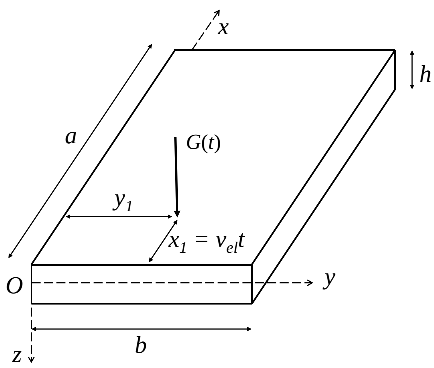
The main objective of this study is to theoretically compare the use of force-only and biodynamic models in modeling rectangular plates subjected to walking people. The Kirchoff non-linear thin elastic plate theory is used to model the plate and the nonlinear Von-Kármán relations are used to describe the deformation relations of plate. The system of non-linear dynamic equilibrium equations is found through the Hamilton principle by application of the Rayleigh-Ritz method and the equations are solved by the fourth-order Runge-Kutta method. The amplitude response of the transverse plate displacement as a function of the pedestrian's step frequency is verified for the force-only and biodynamic models. The influence of some properties of the person, such as walking speed, mass and damping, is also verified.

2 Mathematical Formulation

Consider as elastic rectangular perfect plate with coordinates ($x; y; z$) and displacement fields u, v and w, respectively. The plate has dimensions a and b, thickness h, density ρ, Poisson coefficient ν and Young modulus E. In order to represent the human interaction on the plate, a spring-mass-damper (SMD) model with one degree of freedom and modal mass m_h, equivalent stiffness k_h and damping coefficient c_h is considered, as shown in Fig. 1a.



(a)



(b)

Figure 1. Rectangular plate with (a) SMD and (b) force-only model

The SMD model is located at coordinates (x_1, y_1), then y_1 coordinate is fixed at the center of the plate ($y_1 = b/2$) meanwhile x_1 is variable and is given by the multiplication of the walking velocity v_{el} and time t. In addition to the biodynamic parameters (m_h, k_h and c_h), the application of the pedestrian reaction force $G(t)$ on the plate is also considered. The dynamic behavior of the same plate with a force-only model is also verified. For this, the force $G(t)$ is allocated at the same coordinates (x_1, y_1) as mentioned above and can be seen in Fig. 1b.

In order, to describe the non-linear strain-displacement relations of the mean surface and the changes of curvature the Von-Kármán theory is admitted (Amabili [7]) which is given by:

$$\varepsilon_x = \varepsilon_{x,0} + zk_x, \quad \varepsilon_y = \varepsilon_{y,0} + zk_y, \quad \gamma_{xy} = \gamma_{xy,0} + zk_{xy}$$
$$\varepsilon_{x,0} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_{y,0} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \quad \gamma_{xy,0} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \tag{1}$$
$$k_x = -\frac{\partial^2 w}{\partial x^2}, \quad k_y = -\frac{\partial^2 w}{\partial y^2}, \quad k_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}$$

where ε_x, ε_y e γ_{xy} are the strain components at any point of the plate, and are related to the mean surface by ε_{x0}, ε_{y0} e $\gamma_{xy,0}$ and changes of curvature and torsion at the mean surface are k_x, k_y e k_{xy}.

Neglecting the transverse stresses, the potential energy of the plate is given by:

$$U_P = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^a\int_0^b (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy})\, dx\, dy\, dz \tag{2}$$

Substituting eq (1) in eq (2), the elastic potential energy of the plate can be rewritten as:

$$\begin{aligned} U_P &= \frac{Eh}{2(1-\nu^2)}\int_0^a\int_0^b \left({\varepsilon_{x,0}}^2 + {\varepsilon_{y,0}}^2 + 2\nu\varepsilon_{x,0}\varepsilon_{y,0} + \frac{1-\nu}{2}{\gamma_{xy,0}}^2 \right) dxdy \\ &+ \frac{Eh^3}{2(12(1-\nu^2))}\int_0^a\int_0^b \left({k_x}^2 + {k_y}^2 + 2\nu k_x k_y + \frac{1-\nu}{2}{k_{xy}}^2 \right) dxdy \end{aligned} \tag{3}$$

The pedestrian stiffness k_h is considered by the potential energy of the SMD model U_H and can be written as:

$$\begin{aligned} U_H &= \frac{1}{2}k_h\left(w_h - w\big|_{x=v_{el}t,\, y=y_1}\right)^2 \\ &= \frac{1}{2}k_h\left(w_h^2 - 2w_h\left(\int_0^a\int_0^b \delta(x-v_{el}t)\,\delta(y-y_1)\, w\, dy\, dx\right) + \int_0^a\int_0^b \delta(x-v_{el}t)\,\delta(y-y_1)\, w^2\, dy\, dx\right) \end{aligned} \tag{4}$$

where w_h is the displacement of the SMD model (see Fig. 1b), which refers to the displacement of the mass center of a human body; w is the transverse displacement of the plate and δ is the Dirac delta function.

The plate is considered simply-supported with rotational springs applied in all boundaries to describe an almost clamped effect. The potential energy due to the rotational springs U_M is given by eq. (5). In this way, the total potential energy of the coupled system (plate plus SMD model) results in sum: $U_P + U_H + U_M$.

$$U_M = \frac{1}{2}\int_0^b k_r\left(\left(\left(\frac{\partial w}{\partial x}\right)_{x=0}\right)^2 + \left(\left(\frac{\partial w}{\partial x}\right)_{x=a}\right)^2\right)dy + \frac{1}{2}\int_0^a k_r\left(\left(\left(\frac{\partial w}{\partial y}\right)_{x=0}\right)^2 + \left(\left(\frac{\partial w}{\partial y}\right)_{x=b}\right)^2\right)dx \tag{5}$$

To satisfy all boundary conditions, the following u, v and w fields displacements are adopted:

$$\begin{aligned} u(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} u_{m,n}(t)\,\sin\left(\frac{2m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \\ v(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} v_{m,n}(t)\,\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{2n\pi y}{b}\right) \\ w(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} w_{m,n}(t)\,\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \end{aligned} \tag{6}$$

where m e n are respectively the half-wave numbers in x and y directions; M and N are the number of terms used in each field displacement and $u_{m,n}(t)$, $v_{m,n}(t)$ e $w_{m,n}(t)$ are the unknown amplitudes.

Thus, the vector of generalized amplitudes is given by: $\mathbf{q} = [\, u_{m,n}(t),\, v_{m,n}(t),\, w_{m,n}(t)\,]^T$, wherein its dimension is given by N_q, which is the number of degrees of freedom considering all the field displacement (u, v and w) and excluding the degree of freedom of SMD model.

The kinetic energy of the coupled system T is given by the sum of the kinetic energy of the plate T_P with the kinetic energy of the pedestrian T_H, as described by eq (7):

$$T = T_P + T_H \\ = \frac{1}{2}\rho h \int_0^b \int_0^a \left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2\right) dx dy + \frac{1}{2} m_h \, \dot{w}_h^2 \tag{7}$$

where ρ is the density in kg/m^3, h is the plate thickness, m_h is the mass of the pedestrian and the over-dot indicates the time derivative.

The nonconservative damping forces of plate are assumed to be of viscous type and using the Rayleigh dissipation function (Amabili [7]) can be written as:

$$F_P = \frac{1}{2} c_P \int_0^a \int_0^b \left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2\right) dy\, dx, \quad \text{with } c_p = 2\xi \rho h \omega_n \tag{8}$$

where c_p, ξ, ρ, h and $\omega_{1,1}$ are respectively the damping coefficient, the viscous damping ratio, the density, the thickness and the natural frequency of plate, which is found in the first mode of vibration (m = 1, n = 1).

Similarly, the Rayleigh dissipation function for SMD model F_H is given by eq (9) and the nonconservative damping forces of coupled system is given by the sum: $F_P + F_H$.

$$F_H = \frac{1}{2} c_h \left(\dot{w}_h - \dot{w}\big|_{x=v_{el}t,\, y=y_1}\right)^2 \\ = \frac{1}{2} c_h \left(\dot{w}_h^2 - 2\dot{w}_h \left(\int_0^a \int_0^b \delta(x - v_{el}t)\, \delta(y - y_1)\, \dot{w}\, dy\, dx\right) + \int_0^a \int_0^b \delta(x - v_{el}t)\, \delta(y - y_1)\, \dot{w}^2\, dy\, dx\right) \tag{9}$$

where the human damping coefficient is given by: $c_h = 2\, \xi_h\, m_h\, \omega_h$, in which ξ_h is the viscous damping ratio, m_h is the mass and ω_h is the natural frequency of pedestrian.

The pedestrian vertical force $G(t)$ due the successive footfalls depending on the weight of the person G, the dynamic load factor α_i, the step frequency f_s and the phase angle φ_i can be represented by a Fourier series (ISO [9]; ROSS [10]; VARELA, PFEIL and COSTA [6]) for N_h harmonics. Ross [10] considers that only the three first harmonics (N_h = 3) are enough to describe the human load and this consideration will be used in this work. In this work the biodynamic model is considered as a force $G(t)$. Therefore, the work done by the human load W is given by:

$$W = \int_0^a \int_0^b G(t)\, \delta(x - v_{el}t)\, \delta(y - y_1)\, w\, dy\, dx, \quad \text{with: } G(t) = G + \sum_{i=1}^{N_h} G\, \alpha_i \sin\left(2 i \pi\, f_s\, t - \varphi_i\right) \tag{10}$$

The Rayleigh-Ritz method together with the Hamilton (Amabili [7]) principle are used to obtain the set of nonlinear dynamics equations which are in turn solved by the Runge-Kutta method given by:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} = Q_j, \quad \text{with: } Q_j = \frac{\partial W}{\partial q_j} - \frac{\partial F}{\partial \dot{q}_j} \tag{11}$$

where Q_i is the generalized force obtained by differentiation of the Rayleigh dissipation function and of the virtual work done by external forces for $1 \le j \le N_q$. To consider only the force-only model, U_H, F_H and T_H are neglected.

Therefore, the nonlinear system of motion equations of plate with the force-only model can be written as:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \left(\mathbf{K} + \mathbf{K}_2(q) + \mathbf{K}_3(q,q)\right)\mathbf{q} = \mathbf{F} \tag{12}$$

where $\mathbf{M}$ is the diagonal mass matrix of plate with dimension $N_q \times N_q$; $\mathbf{C}$ is the damping matrix of plate; $\mathbf{K}$, $\mathbf{K}_2(q)$ and $\mathbf{K}_3(q, q)$ are the linear quadratic and cubic stiffness matrix of plate, respectively; $\mathbf{F}$ is the load vector and $\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$ and $\mathbf{q}$ represent the acceleration, velocity and displacement vectors, respectively.

On the other hand, to analyze the plate with the SMD model, all the terms shown are taken into account and one degree of freedom is added to the system of eq (12). Thus, the damping matrix $\mathbf{C}$ and the stiffness matrix $\mathbf{K}$ of plate now are obtained coupled with the SMD degree of freedom. Moreover, a new equation which represents the dynamic equilibrium of the SMD model arise, given by:

$$m_h\, \ddot{w}_h + c_h \left(\dot{w}_h - \sum_{i=1}^{N_q} \dot{q}_i\, \psi_i(x_1, y_1)\right) + k_h \left(w_h - \sum_{i=1}^{N_q} q_i\, \psi_i(x_1, y_1)\right) = 0 \tag{13}$$

in which ψ_i is the mode shape associated to ith generalized coordinate of plate and applied at points $x_1 = v_{el}\, t$ and $y_1 = b/2$.

3 Numerical Results

For this analysis, an equivalent concrete rectangular plate with the following physical and geometrical properties: a = 15 m, b = 13 m, h = 0.10 m, E = 21.74 GPa, ρ = 2549.3 kg/m^3, ξ = 0.02 and ν = 0.2 was chosen so that its natural frequency was resonant with the step frequency, which is adopted f_s = 2,0 Hz. The rotational springs have the following equivalent stiffness: $k_r = 10^6$ N/rad in order to consider the almost clamped condition. According to Varela, Pfeil and Costa [6] the DLF and phase angle to force $G(t)$ can be given by following values: α_1 = 0.556, α_2 = 0.130, α_3 = 0.06, φ_1 = 0.0 rad, φ_2 = 1.494 rad and φ_3 = 1.560 rad. The pedestrian weight is considered G = 738.5 N, the walking velocity is v_{el} = 1.25 m/s and the biodynamic properties of SMD are: m_h = 73.85 kg, ω_h = 2.2 H_z, and ξ_h = 0.3, which implies in, k_h = 14111 N/m and c_h = 612.5 $N\ s/m$ (Caprani and Ahmadi [3]). To model the plate, 39 degrees of freedom in the field displacements were considered using the following generalized coordinates (Amabili [7]; Dias [10]): $u_{2,1}$, $u_{2,3}$, $u_{2,5}$, $u_{2,7}$, $u_{4,1}$, $u_{4,3}$, $u_{4,5}$, $u_{4,7}$, $u_{6,1}$, $u_{6,3}$, $u_{6,5}$, $u_{8,1}$, $u_{8,3}$, $v_{1,2}$, $v_{1,4}$, $v_{1,6}$, $v_{1,8}$, $v_{3,2}$, $v_{3,4}$, $v_{3,6}$, $v_{3,8}$, $v_{5,2}$, $v_{5,4}$, $v_{5,6}$, $v_{7,2}$, $v_{7,4}$, $w_{1,1}$, $w_{1,3}$, $w_{1,5}$, $w_{1,7}$, $w_{3,1}$, $w_{3,3}$, $w_{3,5}$, $w_{3,7}$, $w_{5,1}$, $w_{5,3}$, $w_{5,5}$, $w_{7,1}$, $w_{7,3}$.

Now, the resonance curves of the driven amplitude ($w_{1,1}$) for varying the step frequency of walking people are displayed in Fig. 2 and were obtained for values from 1.8 to 2.2 Hz and normalized with the natural frequency of the plate ($\omega_{1,1}$), for both the force-only and biodynamic (SMD) models.

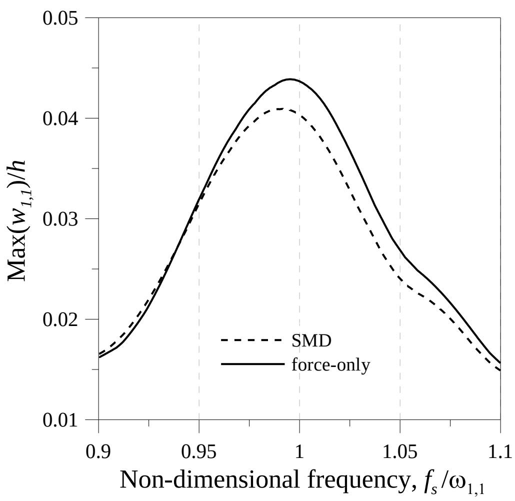
As can be observed in Fig. 2a, as the step frequency is increased, the SMD model (dotted lines) provides smaller transversal amplitudes when compared to the force-only model (continuous line). It is also observed that the peak of the SMD occurs at the left of the resonance ($f_s/\omega_{1,1}$ = 1) point, this is due to a small reduction in the natural frequency of the plate in the presence of the pedestrian. These pheomena is also noticed by Shahabpoor et al. [1] and Gaspar et al. [2] and Gonzaga, Pfeil and Varela [11], showing the effect of HSI consideration and the influence of pedestrian in changing the modal properties of the plate. Another point observed in Fig. 2a is that before the resonance region ($f_s/\omega_{1,1}$ < 0.94), the amplitude given by the SMD model is greater in relation to the force-only model.

Now a parametric analysis is performed to study the influence of walking velocity, modal mass and damping ratio of pedestrian on the response of the plate as observed in Fig. 2b, 2c and 2d.

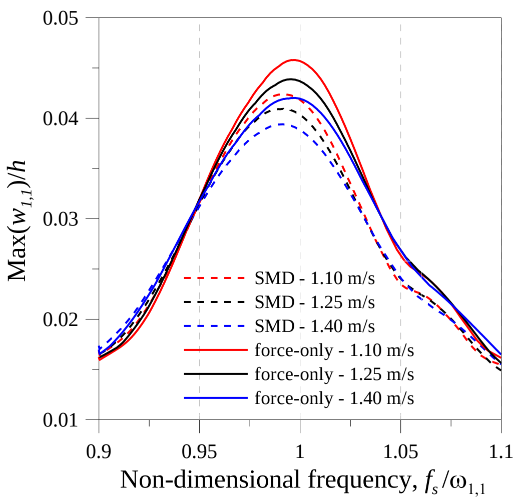
Figure 2b depicts the resonance curves for increasing values of walking velocity, it is possible to observe that as the walking velocity is increased, there is a reduction of the amplitude of displacement showing the pedestrian velocity is inversely proportional to the vibration amplitude.

Figure 2c shows that as the damping ratio of pedestrian increases, the amplitude of vibration also increases and the peak amplitude tends to approach the resonance point ($f_s/\omega_{1,1}$ = 1). This occurrence can be explained by the fact that the damping ratio of the pedestrian is related to the dissipation of energy from the person body movements. As the values of ξ_h increases, the movement restriction of the SMD model also increase and the response of the plate is closer to the response provided by the force-only model.

Finally, in Fig. 2d it is noted that increasing the pedestrian mass, the maximum vibration amplitude in the resonance region decreases. This fact calls attention to the possibility of crowd induced vibrations and their beneficial action on the plate vibration amplitude. However, as it moves away from the resonance region ($f_s/\omega_{1,1}$ > 1), the influence of the modal mass is practically insignificant.



(a)



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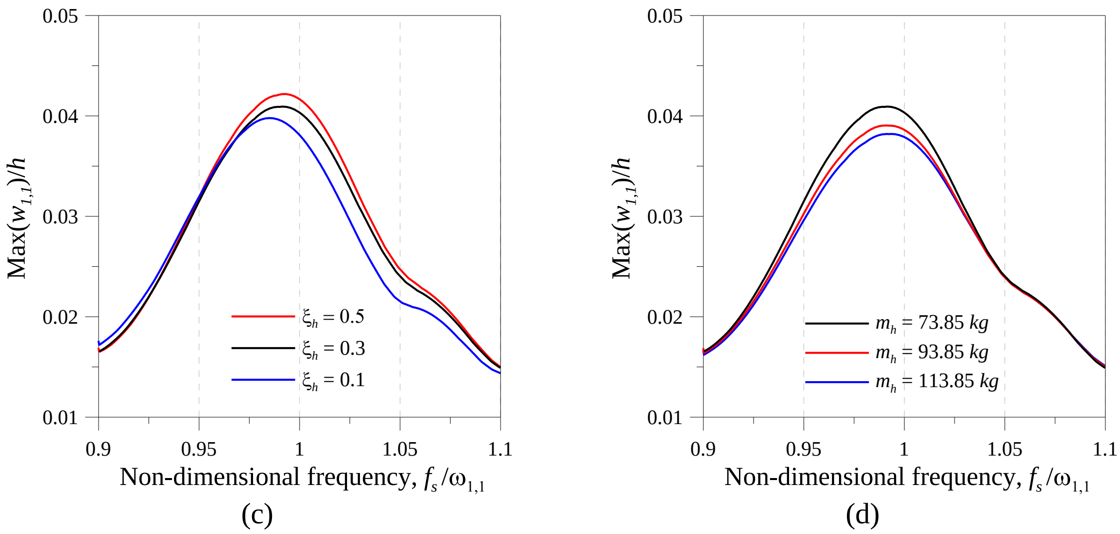

Figure 2. Resonance curves with the (a) comparison between SMD and force-only models for the initial parameters; (b) comparison between SMD and force-only models for the variation of walking velocity v_{el}; (c) SMD model for the variation of damping ratio ξ_h and (d) SMD model for the variation of modal mass m_h

Now, the influence of the plate damping ratio (ξ) on the human-structure interaction is studied. For this, the plate displacement values were obtained for both the SMD model and the force-only model as damping factor ξ values varied from 0.02 (black color), 0.03 (red color) and 0.04 (blue color), as can be seen in Fig. 3a. In each variation performed here, all other coupled system parameters kept their initial values.

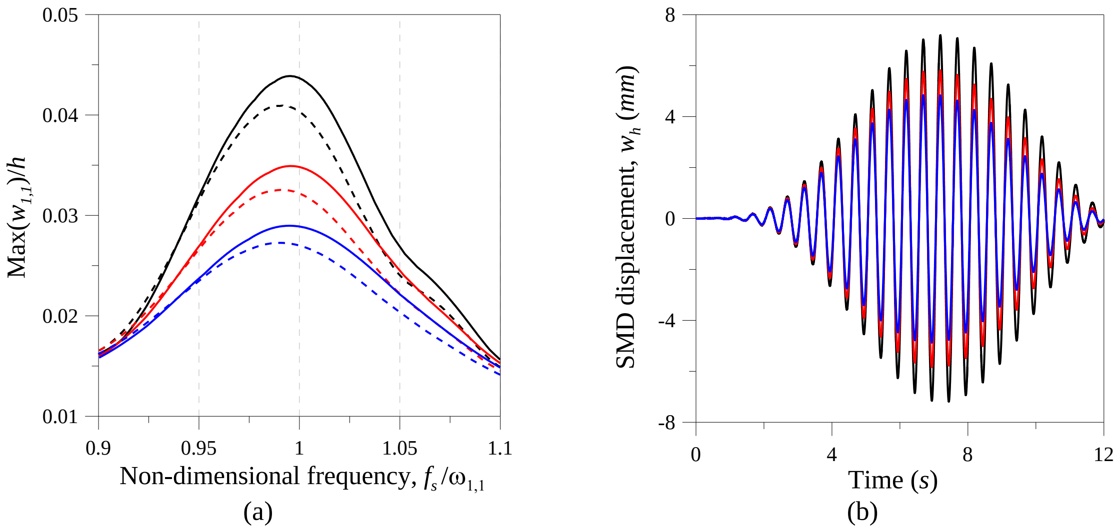

Figure 3. (a) Resonance curves with the variation of plate damping ratio. The black, red and blue lines represent respectively $\xi = 0.02$, $\xi = 0.03$ and $\xi = 0.04$. The continuous lines represent the force-only model and the dotted lines represent the SMD model. (b) Time-history of the SMD displacement w_h for $f_s/\omega_{1,1} = 1$ and $\xi = 0.02$ (black line), $\xi = 0.03$ (red line) and $\xi = 0.04$ (blue line)

From Fig. 3a it can be seen that with the increase of the damping ratio of the plate, the displacement amplitudes decrease and the resonance region tends to become larger. It is also noticed that as the plate damping increases, the divergence between responses provided by the SMD model and the force-only model decreases, i.e., for higher plate damping values, the response of the SMD model is closer the force-only model. Figure 3b explains this behavior by the fact that when reducing the plate vibration amplitude, the displacement of pedestrian mass center also reduces, tending to a locking of SMD which can lose its favorable effect on the HSI. Figure 3b also indicates that the greatest displacement of the pedestrian's center of mass occur at an instant of time after crossing the center of the plate (7.5 sec).

4 Concluding Remarks

The vibration amplitude of a thin rectangular plate is theoretically studied considering a biodynamic model of SMD type with one degree of freedom traveling with a constant velocity on plate, which is compared with a force-only model. Obtained results shown that the amplitude of vibration is smaller and the position of amplitude peak point can be modified when the biodynamic model is considered instead of the force-only model. It was also observed that increased modal mass and walking velocity causes a reduction of displacements, while increases in pedestrian damping ratio lead to greater amplitudes of vibration, showing a sensitivity of the system to these biodynamic parameters which should have their values carefully studied to correctly modeling the HSI effect. The effect of the plate damping ratio on the HSI was also evaluated. The findings shown that the displacement of center of mass of the pedestrian (w_h) is affected by changes in the plate damping, which makes the response of the SMD model approach the obtained response by force-only model for high plate damping values.

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