The effect of geometry on the dynamic instability of clamped-free cylindrical shells

Zenón José Guzmán Nuñez Del Prado[1] and Tairine R. A. Carneiro[1]

[1]*School of Civil and Environmental Engineering, Federal University of Goiás*
Av. Universitária, 1488, St. Universitário,74605-220, Goiânia, GO, Brazil
zenon@ufg.br - tairine.carneiro@hotmail.com

Abstract. In this work, the influence of geometry on the dynamic instability of clamped-free cylindrical shells subjected to lateral harmonic loads is studied. For this, to model the shell the Koiter – Sanders is considered, and the Rayleigh-Ritz method is applied to obtain a set of non-linear dynamic equations which are solved in turn by the fourth order Runge-Kutta method. A detailed study is performed to evaluate the correct nonlinear coupling of field displacements. To study the dynamic instability, a model with eighteen degrees of freedom is considered and the resonance curves are obtained for three different shell geometries. It is possible to observe that, depending on the geometry ratios, the shell will display softening, hardening, chaotic or quasi-periodic oscillations.

Keywords: cylindrical shells, nonlinear vibrations, geometry effect.

1 Introduction

Cylindrical shells have been largely applied in several engineering branches due to its high capability to strength external and internal forces. In literature, there are many works related to linear and nonlinear analysis of simply supported cylindrical shells but, for clamped-free boundary conditions, the number of works is very scarce. Linear vibrations of clamped-free cylindrical shell have been studied in previous works [1-3] but, for nonlinear vibrations there is a reduced number of works, among them, it can be mentioned the work of Chiba [4,5] who studied experimentally the nonlinear free vibrations of polyester shells and the work of Kurylov and Amabili [6] who, using the Chebyshev polynomials to expand the displacement fields, studied the forced nonlinear vibrations of clamped-free cylindrical shells. The main difficulty in all studies is to find the correct expansions to discretize the fields displacements for nonlinear analysis.

In this work the free and forced nonlinear vibrations of clamped-free elastic cylindrical shells subjected to lateral harmonic load are studied. To model the shell, the Koiter-Sanders nonlinear theory is applied to obtain the strain energy. The field displacements are described by double trigonometric series in both longitudinal and circumferential directions and natural boundary conditions are satisfied. The Rayleigh-Ritz method is applied to obtain a set of nonlinear dynamic equilibrium equations which are, in turn, solved using the fourth order Runge-Kutta method. A convergence study is developed to obtain the field displacement expansion which describe softening behavior of the frequency-amplitude relation involving 18 to 30 dof. Numerical results are compared with literature and a parametric study is developed to study the influence of geometry on the resonance curves.

2 Mathematical Formulation

Consider a perfect clamped-free elastic cylindrical shell with radius R, thickness h, length L, density ρ, Young modulus E and Poisson coefficient ν and subjected to a lateral harmonic load $F(t) = q_r\delta(R\theta - R\bar{\theta})\delta(x - \bar{x})\cos(\Omega t)$ with amplitude q_r acting at point $\bar{\theta}$ and $\bar{x}$ as seen in Fig. 1. The field displacements of the middle surface of the shell are in the axial $u(x,\theta,t)$, radial $v(x,\theta,t)$ and lateral $w(x,\theta,t)$ directions, respectively where x and θ are the axial and radial coordinates and t is the time.

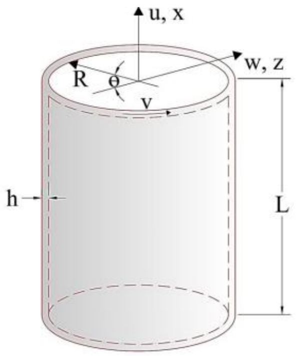

Figure 1. Geometry and coordinates of cylindrical shell.

The strain relations of the cylindrical shell can be written as $\varepsilon_x = \varepsilon_{x,0} + zk_x, \varepsilon_\theta = \varepsilon_{\theta,0} + zk_\theta, \gamma_{x\theta} = \gamma_{x\theta,0} + zk_{x\theta}$ where z is the distance of the arbitrary point of the shell from the middle surface.

The middle surface strain–displacement relations, changes of curvature and torsion according to Sanders–Koiter nonlinear shell theory are given by:

$$\varepsilon_{x,0} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{8}\left(\frac{\partial v}{\partial \mathrm{x}} - \frac{\partial u}{R\,\partial\theta}\right)^2 \tag{1}$$

$$\varepsilon_{\theta,0} = \frac{\partial v}{R\,\partial\theta} + \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{R\,\partial\theta} - \frac{v}{R}\right)^2 + \frac{1}{8}\left(\frac{\partial u}{R\,\partial\theta} - \frac{\partial v}{\partial \mathrm{x}}\right)^2 \tag{2}$$

$$\gamma_{x\theta,0} = \frac{\partial u}{R\,\partial\theta} + \frac{\partial v}{L\,\partial\eta} + \frac{\partial w}{L\,\partial\eta}\left(\frac{\partial w}{R\,\partial\theta} - \frac{v}{R}\right) \tag{3}$$

$$k_x = -\frac{\partial^2 w}{\partial x^2}, \qquad k_\theta = \frac{\partial v}{R^2\,\partial\theta} - \frac{\partial^2 w}{R^2\,\partial\theta^2}, \qquad k_{x\theta} = -2\frac{\partial^2 w}{R\,\partial x\,\partial\theta} + \frac{1}{2R}\left(3\frac{\partial v}{\partial x} - \frac{\partial u}{R\,\partial\theta}\right) \tag{4}$$

The elastic strain energy, neglecting σ_z for plane stress, is given by [2]

$$\mathrm{U} = \frac{1}{2}R\int_0^{2\pi}\int_0^{\mathrm{L}}\int_{-h/2}^{h/2}(\sigma_x\varepsilon_x + \sigma_\theta\varepsilon_\theta + \tau_{x\theta}\gamma_{x\theta})\mathrm{d}x(1 + z/R)\mathrm{d}\theta\mathrm{d}z \tag{5}$$

where stress-strain relations are given by:

$$\sigma_x = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_\theta), \;\; \sigma_\theta = \frac{E}{1-v^2}(\varepsilon_\theta + v\varepsilon_x), \qquad \tau_{x\theta} = \frac{E}{2(1+v)}\gamma_{x\theta} \tag{6}$$

The kinetic energy, considering only translational inertia, is given by:

$$T = \frac{1}{2}\rho_S hR\int_0^{2\pi}\int_0^{\mathrm{L}}(\dot{u}^2 + \dot{v}^2 + \dot{w}^2)\mathrm{d}x\mathrm{d}\theta \tag{7}$$

The work of external forces is:

$$W = R\int_0^{2\pi}\int_0^{\mathrm{L}}(q_x u + q_\theta v + q_r w)\mathrm{d}x\mathrm{d}\theta \tag{8}$$

where q_x, q_θ and q_r are the distributed external loads in lateral, circumferential and radial directions.

Finally, the work of nonconservative forces is:

$$F = \frac{1}{2}cR\int_0^{2\pi}\int_0^{\mathrm{L}}(\dot{u}^2 + \dot{v}^2 + \dot{w}^2)\mathrm{d}x\mathrm{d}\theta \tag{9}$$

where c is the viscous damping coefficient.

The field displacements can be expanded as:

$$w(x,\theta,t)=\sum_{m=1}^{M}\sum_{n=1}^{N} h\,\beta(m,x)\left[q_{m,n,d}(t)\cos(n\theta)+q_{m,n,c}(t)sin(n\theta)\right]$$
$$u(x,\theta,t)=\sum_{m=1}^{M}\sum_{n=1}^{N} h\,\beta_1(m,x)\left[q_{m,n,d}(t)\cos(n\theta)+q_{m,n,c}(t)sin(n\theta)\right] \quad (10)$$
$$v(x,\theta,t)=\sum_{m=1}^{M}\sum_{n=0}^{N} h\,\beta(m,x)\left[q_{m,n,d}(t)\,sin(n\theta)+q_{m,n,c}(t)cos(n\theta)\right]$$

With

$$\beta(m,x)=1-cos\left[\frac{1}{2}\frac{(2m-1)\pi x}{L}\right],\ \ \beta_1(m,x)=sin\left[\frac{1}{2}\frac{(2m-1)\pi x}{L}\right] \quad (11)$$

where m is the axial half-wave number, n is the circumferential wavenumber, subscript d refers to driven mode and subscript c refers to companion mode.

The boundary conditions of a clamped-free cylindrical shell are given by:

$$u=v=w=\frac{\partial w}{\partial x}=0 \text{ at } x=0 \text{ and } N_x=N_{x\theta}+\frac{M_{x\theta}}{R}=M_x=Q_x+\frac{\partial M_{x\theta}}{R\,\partial\theta}=0 \text{ at } x=L \quad (12)$$

Substituting the field displacements of Eq. (10) in Eq. (14), the set of nonlinear equations of dynamic equilibrium can be obtained.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)+\frac{\partial \mathrm{U}}{\partial q_i}=-\frac{\partial F}{\partial \dot{q}_i}+\frac{\partial W}{\partial q_i},\qquad i=1,\dots \text{ dof} \quad (13)$$

where $q_i=\left[u_{1,0},u_{2,0},\dots,u_{1,n,d},u_{2,n,d},\dots,v_{1,0},v_{2,0},\dots,v_{1,n,d},v_{2,n,d},\dots,w_{1,0},w_{2,0},\dots,w_{1,n,d},w_{2,n,d},\dots\right]$.

3 Numerical Results.

Consider a clamped-free cylindrical shell with Young's modulus E=4.65e9 N/m^2, density ρ= 1400 kg/m³ and Poisson coefficient ν= 0.38. Three shell geometries were selected to study its influence on the nonlinear vibrations and Table 1 displays these geometries and geometric ratios. First, in order to obtain the natural frequencies of each geometry, a linear free vibrations analysis was performed considering the following field displacements (without companion mode) $u=(1,0)+(2,0)+(3,0)+\cdots+(1,n)+(2,n)+(3,n)+\cdots$, $v=(1,0)+(2,0)+(3,0)+\cdots+(1,n)+(2,n)+(3,n)+\cdots$ and $w=(1,0)+(2,0)+(3,0)+\cdots+(1,n)+(2,n)+(3,n)+\cdots$. Table 1 shows also the obtained natural frequencies and both axial and circumferential wavenumber of each geometry. Shell geometry 01 can be compared with previous results from Kurylov and Amabili [6] who obtained (m= 1, n= 7) with ω_o = 177.8 rad/sec which means that selected field displacements agree with literature.

Table 1. Selected shell geometries.

Geometry	*L* (m)	*R* (m)	*h* (mm)	*L/R*	*R/h*	ω_o (rad/sec)	*(m,n)*
01	0.48	0.24	0.254	2.0	944	179.165	(1,7)
02	0.72	0.24	0.300	3.0	800	130.728	(1,6)
03	0.24	0.24	1.200	1.0	200	757.200	(1,7)

For the nonlinear analysis of the shell, the correct field displacements showing the coupling and modal interaction displaying softening behavior should be considered. For this, thirteen field displacements without considering the companion mode were selected and the frequency-amplitude relation of each model was obtained, these field displacements are showed in Table 2.

Figure 2 depicts the normalized frequency-amplitude relations of Geometry 1. As can be observed, depending on the field displacements, the shell will display hardening or softening behavior and as well know, cylindrical shells show softening behavior. Models named 18, 20, 24 and 28 dof display hardening behavior and models named 18a, 20a, 21, 22a, 24a, 26, 25 and 30 dof display softening behavior then, for this work, the smallest model 18a will be considered in the nonlinear vibration analysis.

Table 2. Selected field displacements

Model	Modal expansion
18 dof:	$u = w = (1,n),(1,2n),(3,n),(3,2n),3\ axi$ and $v = (1,n),(1,2n),(3,n),(3,2n);$
20 dof:	$u = w = (1,n),(1,2n),(3,n),(3,2n),4\ axi$ and $v = (1,n),(1,2n),(3,n),(3,2n);$
22 dof:	$u = w = (1,n),(1,2n),(3,n),(3,2n),5\ axi$ and $v = (1,n),(1,2n),(3,n),(3,2n);$
24 dof:	$u = w = (1,n),(1,2n),(3,n),(3,2n),6\ axi$ and $v = (1,n),(1,2n),(3,n),(3,2n);$
28 dof:	$u = w = (1,n),(1,2n),(3,n),(3,2n),8\ axi$ and $v = (1,n),(1,2n),(3,n),(3,2n);$
24a dof:	$u = w = (1,n),(1,2n),(2,n),(3,n),(3,2n),3\ axi$ and $v = (1,n),(1,2n),(2,n),(2,2n),(3,n),(3,2n);$
26 dof:	$u = w = (1,n),(1,2n),(2,n),(3,n),(3,2n),4\ axi$ and $v = (1,n),(1,2n),(2,n),(2,2n),(3,n),(3,2n);$
27 dof:	$u = w = (1,n),(1,2n),(2,n),(2,2n),(3,n),(3,2n),(4,n),3\ axi;$ and $v = (1,n),(1,2n),(2,n),(2,2n),(3,n),(3,2n),(4,n);$
18a dof:	$u = w = (1,n),(1,2n),(2,n),(2,2n),3\ axi$ and $v = (1,n),(1,2n),(2,n),(2,2n);$
20a dof:	$u = w = (1,n),(1,2n),(2,n),(2,2n),4\ axi$ and $v = (1,n),(1,2n),(2,n),(2,2n);$
22a dof:	$u = w = (1,n),(1,2n),(2,n),(2,2n),5\ axi$ and $v = (1,n),(1,2n),(2,n),(2,2n);$
21 dof:	$u = w = (1,n),(1,2n),(1,3n),(2,n),(2,2n),3\ axi$ and $v = (1,n),(1,2n),(1,3n),(2,n),(2,2n);$
30 dof:	$u = w = (1,n),(1,2n),(2,n),(2,2n),(3,n),(3,2n),(4,n),(4,2n),3\ axi$ and $v = (1,n),(1,2n),(2,n),(2,2n),(3,n),(3,2n),(4,n),(4,2n)$

where *axi* represents the axisymmetric modes i.e. (1,0), (3,0), (5,0), (7,0), (9,0), etc.

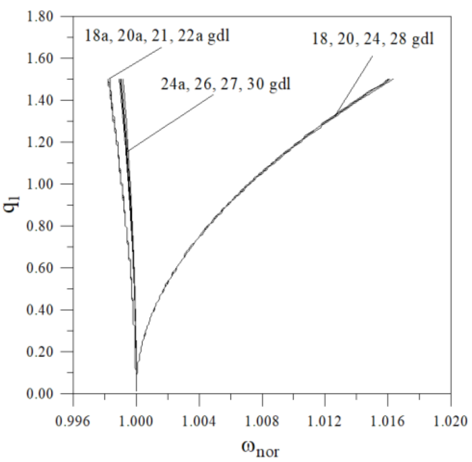

Figure 2. Frequency-amplitude relations for selected field displacement expansions.

Now, the nonlinear forced vibration analysis will be considered, for this, as previously indicated the shell is subjected to a concentrated lateral harmonic radial load $F(t) = q_r\delta(R\theta - R\bar{\theta})\delta(x - \bar{x})\cos(\Omega t)$ acting at point $\bar{\theta} = 0$ and $\bar{x} = L/2$, the damping factor is $\xi = 0.001$. The resonance curves for varying frequency of lateral load will be plotted which were obtained using the force brute method and $\Omega_1 = \Omega/\omega_o$. First, Figure 3 displays the resonance curve fr increasing levels of load amplitude of Geometry 1 and compared with results from Kurylov and Amabili [6] and Chiba [4], as can be seen, the resonance curves show stronger softening behavior that curves obtained by Kurylov and Amabili [6] and are closer to experimental results obtained by Chiba [4]. The difference between results from this work and Chiba can be due to the necessity to consider more terms in modal expansion or initial geometric imperfections.

Figure 4 displays the resonance curves of Geometry 1, which is a taller shell, for increasing values of amplitude of radial load (q_r). As can be observed for q_r = 0.002 N the shell displays almost linear behavior but, when the amplitude of the load is increased to q_r = 0.003 N, the shell starts to show softening behavior with a small jump at dynamic instability point. If q_r = 0.004 N the shell shows softening behavior with a path for large 1T oscillations and after, a region with coexistence of chaotic and stable 1T vibrations. Also, close to Ω_1 = 1.005 there is a point where chaotic oscillations occur. When q_r = 0.005 N the shell shows softening behavior with large vibrations as well as a large region of coexistence of both periodic and chaotic vibrations and another region with chaotic oscillations at $\Omega_1 = 1.002$. Now, when the load goes to $q_r = 0.007$ N and $q_r = 0.008$ N there is a complete changing of behavior because there is small softening but a jump to a path with 1T periodic oscillations as well as the coexistence of small and chaotic oscillations and a window with chaotic oscillations at $\Omega_1 = 1.0025$.

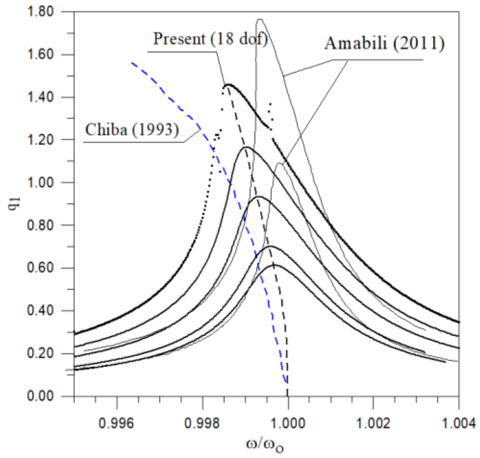

Figure 3. Comparison of resonance curves.

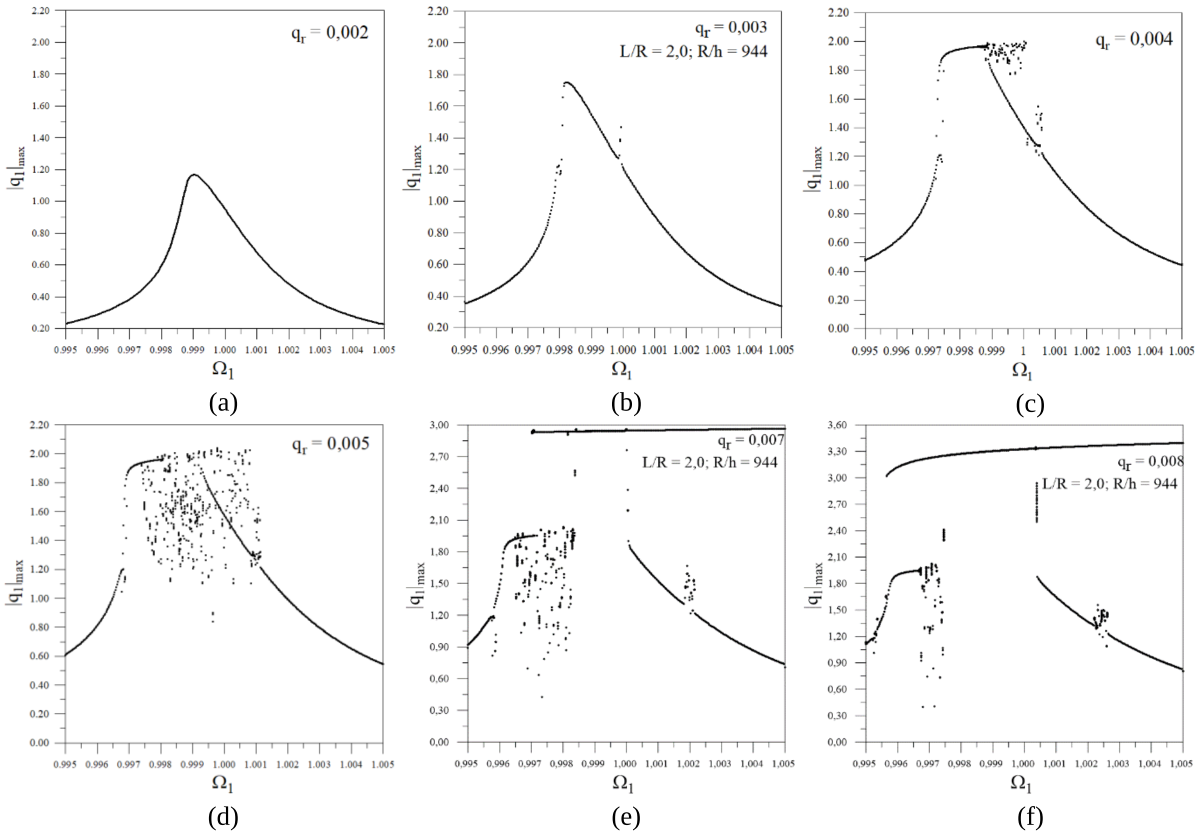

Figure 4. Resonance curves of Geometry 1 without companion mode: (a) q_r= 0.002 N; (b) q_r= 0.003 N; (c) q_r= 0.004 N; (d) q_r= 0.005 N; (e) q_r= 0.007 N; (f) q_r= 0.008 N.

Now, for Geometry 2, Figure 5 depicts the resonance curves for increasing values of lateral load amplitude. In this case the effect of geometric relations become significant because for $q_r = 0.002$ N and $q_r = 0.003$ N the shell display linear behavior but for $q_r = 0.004$ N it starts to display softening behavior. For $q_r = 0.005$ N there the shell shows softening behavior with large amplitude oscillations and no chaotic vibrations and, if $q_r = 0.007$ N and $q_r = 0.008$ N and the shell shows also softening behavior but also a nonlinear path with hardening large amplitude vibrations.

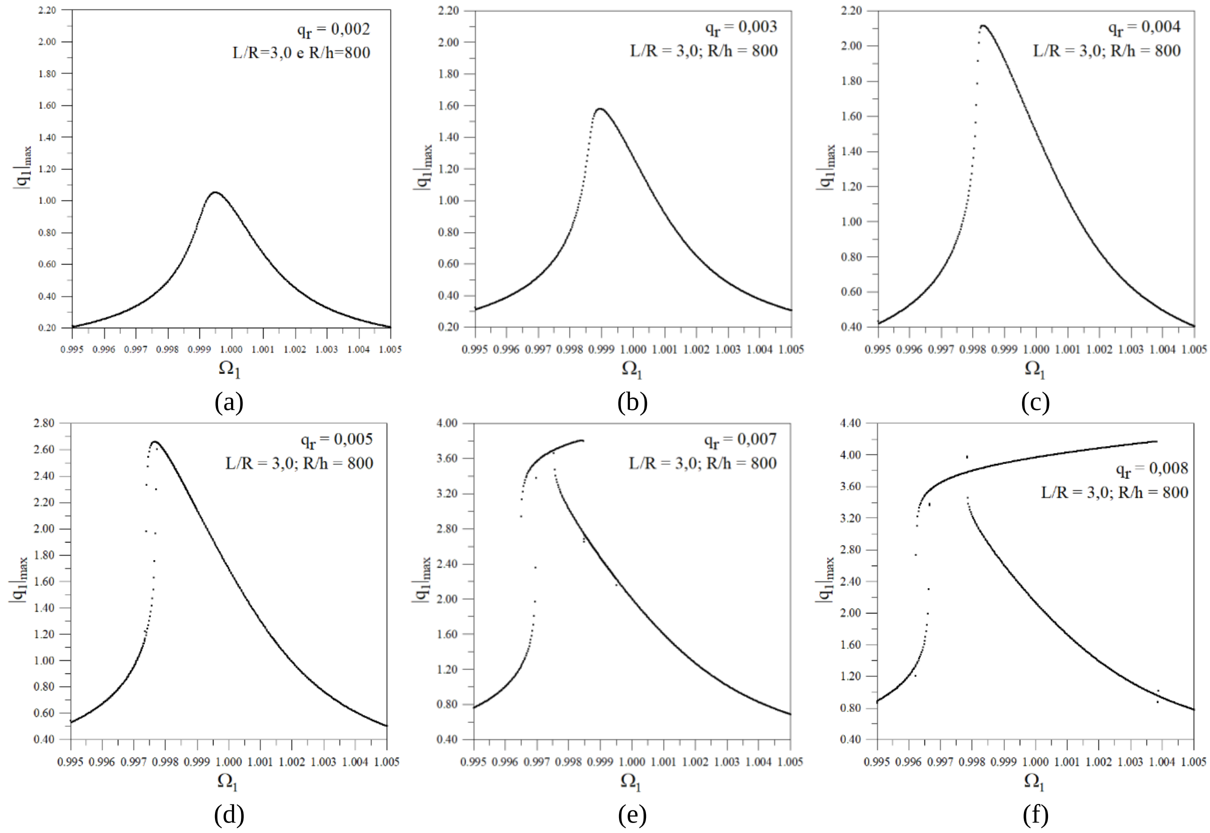

Figure 5. Resonance curves of Geometry 2 without companion mode: (a) q_r= 0.002 N; (b) q_r= 0.003 N; (c) q_r= 0.004 N; (d) q_r= 0.005 N; (e) q_r= 0.007 N; (f) q_r= 0.008 N.

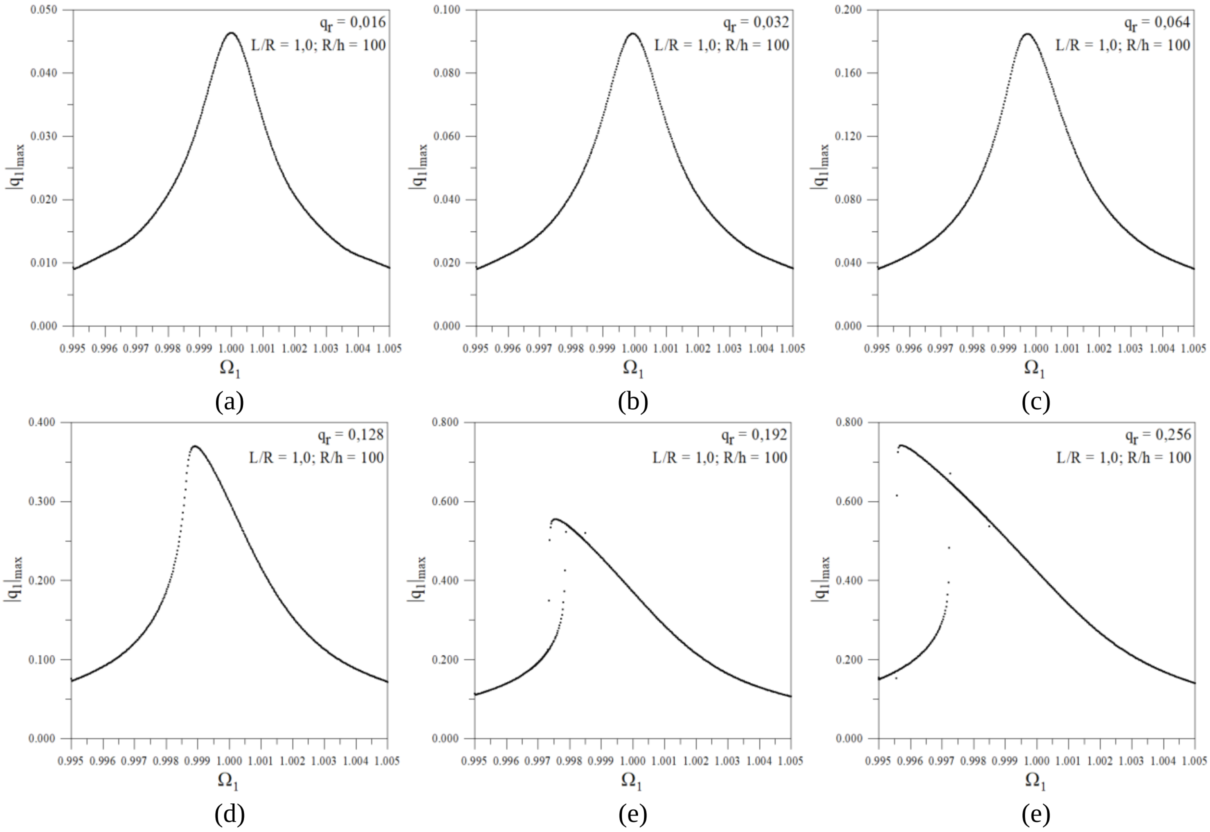

Figure 6. Resonance curves of Geometry 3 without companion mode: (a) q_r= 0.016 N; (b) q_r= 0.032 N; (c) q_r= 0.064 N; (d) q_r= 0.128 N; (e) q_r= 0.192 N; (f) q_r= 0.256 N.

Finally, Figure 6 depicts the resonance curve of Geometry 3 for increasing values of lateral load amplitude. As can be observed, the geometry relations play and important role in the nonlinear response because in this case, for q_r = 0.016 N, q_r = 0.032 N and q_r = 0.064 N the shell displays linear behavior with small amplitude oscillations. For q_r = 0.007 N the shell starts to display softening behavior and for q_r = 0.192 N and q_r = 0.256 N the softening behavior is more evident, but the amplitudes of vibrations are smaller if compared with amplitudes of previous geometry cases.

There is a strong influence of geometry relations on the nonlinear vibrations of clamped-free cylindrical shells because, depending on the rations the shell will display complex nonlinear oscillations with both small and large amplitude vibrations as well as hardening nonlinear paths with large amplitude vibrations.

4 Concluding Remarks.

In this work the free and forced nonlinear vibrations of clamped-free elastic cylindrical shells subjected to lateral harmonic load are studied. To model the shell, the Koiter-Sanders nonlinear theory is applied to obtain the strain energy and the field displacements were described by double trigonometric series in both longitudinal and circumferential directions where natural boundary conditions are satisfied.

Three shell geometries (Geometry 1: L/R=2.0 and R/h=944), (Geometry 2: L/R=3.0 and R/h=800), (Geometry 1: L/R=1.0 and R/h=200) were selected to study the influence of geometry ratios on the nonlinear dynamic response, for this, thirteen field displacements were selected, and a convergence study was developed to find the field displacement that generate softening behavior in the frequency-amplitude relations. A series with 18 dof was selected and the resonance curves for increasing values of lateral load were obtained.

The influence of geometry relations became evident because, depending on its geometry the shell will display linear or softening behavior with small or large amplitude oscillations. For Geometry 1, the shell displays softening behavior and a window with the coexistence of chaotic and periodic oscillations as well as a hardening path with large amplitude vibrations. For Geometry 2, the shell displays softening behavior with a hardening path with large amplitude oscillations but no chaotic motions and for Geometry 3, the shell depicts only softening behavior. These resonance curves show the strong influence of geometry ratios on the nonlinear dynamic response of the clamped-free cylindrical shells.

References

[1] G. B. Warburton and J. Higgs. “Natural frequencies of thin cantilever shells”. Journal of Sound and Vibration, vol. 11, pp. 335-338, 1970.
[2] C. Sharma. “Free vibrations of clamped-free circular cylinders”. Thin-Walled Structures, vol. 2, pp. 175–193, 1984.
[3] H. Tottenham and K. Shimizu. “Analysis of the free vibration of cantilever cylindrical thin elastic shells by the matrix progression method”. International Journal of Mechanical Sciences, vol. 14, pp. 293–310, 1972.
[4] M. Chiba. “Non-linear hydroelastic vibration of a cantilever cylindrical tank—I. Experiment (empty case)”. International Journal of Non-Linear Mechanics, vol. 28, pp. 591–599, 1993.
[5] M. Chiba. “Non-linear hydroelastic vibration of a cantilever cylindrical tank. II Experiment (liquid-filled case)”. International Journal of Non-Linear Mechanics, vol. 28, pp. 601–612, 1993.
[6] Ye. Kurylov and M. Amabili. “Nonlinear Vibrations of Clamped-Free Circular Cylindrical Shells”. Journal of Sound and Vibration, vol. 330, pp. 5363-5381, 2011.