Numerical analysis of soil-pile interaction problems using 8-node hexahedral finite and infinite elements

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Abstract. A numerical analysis to study three-dimensional soil-pile interaction problems is presented in this paper. The soil mass and the pile are spatially discretized using eight-node hexahedral isoparametric elements with underintegration techniques and hourglass control to suppress volumetric and shear locking. Load transfer between the soil and the pile is performed by a three-dimensional contact algorithm based on the penalty method, where the classical Coulomb's law is adopted as constitutive relation for friction. A corotational approach at element level is adopted to deal with physically and geometrically nonlinear analysis. For dynamic analysis, the Newmark's method is employed for time integration and infinite elements are used at the domain boundary to avoid the reflection of energy waves in the region of interest, providing a quiet boundary. Numerical examples are performed and compared with numerical results obtained by other authors in order to verify the present algorithm.

Keywords: soil-pile interaction, finite element method, reduced integration, contact mechanics.

1 Introduction

The overall response of a structure is directly affected by the interaction between structure, foundation and soil, where the load transference between the soil and the foundation is known as soil-structure interaction. It is notorious that the dynamic response of a foundation resting on a rigid base is significantly different when compared with a flexible soil surrounding the foundation. Effects of soil-structure interaction generally increase the natural period of the structure, as the system stiffness is reduced (Jia [1]). Menglin et al. [2] mentions that one of the effects of considering the soil in the analysis is the increasing of damping in the system due to the radiation of stress waves and due to hysteretic losses in the soil. Both phenomena are difficult to incorporate into numerical analysis, especially hysteretic losses. A simple solution adopted here for the first phenomenon is to employ standard viscous boundaries instead of fixed boundary condition, as formulated by Lysmer and Kuhlemeyer [3]. Another solution is the use of Kelvin elements (spring and dashpot) at the boundary, which was used by Sarkar and Maheshwari [4]. On the other hand, material damping is considered to be hysteretic in nature and is more complex, being simulated here to be proportional to the stiffness matrix (see Sarkar [5]).

Note that simulating three-dimensional dynamic problems of soil-pile interaction, based on the Finite Element Method (FEM), a contact algorithm is required to deal with the load transfer between the two media. In this context, Laursen and Simo [6] presented a finite element treatment for large deformation and frictional contact problems between deformable bodies. A similar approach is showed by Wriggers [7] using a five-node contact element for three-dimensional problems, which was employed in the present work using the penalty formulation. In the penalty method, a penalty parameter is employed to approximately satisfy the impenetrability restriction for two contacting bodies and, therefore, small interpenetrations are allowed between bodies in contact. A detailed review about contact algorithms can be found in Bourago e Kukudzhanov [8].

A reduced-integration finite element formulation is the logical choice for three-dimensional nonlinear problems, which requires a high computational cost, as in the present case. Despite their attractiveness, elements with one-point quadrature require an efficient technique for controlling the so-called hourglass modes. In this context, Duarte Filho and Awruch [9] extended the Hu and Nagy [10] hexahedral element formulation to geometrically nonlinear static and dynamic problems. After that, the numerical model was extended to other problems, such as, highly nonlinear dynamic applications using the Generalized-α method (Braun and Awruch [11]), elastoplastic analysis of soil problems with the Modified Cam-Clay model (Braun and Awruch [12, 13]), composite materials analysis ([Andrade et al. [14]), fluid-structure interaction simulations (Braun and Awruch [15]) and contact problems (Visintainer et al. [16]). Previous works demonstrate the good performance of the present formulation, which was employed here to suppress shear locking instabilities that piles could suffer under bending deformation.

Thus, the main objective of the present work is to investigate the performance of the infinite element to absorb the energy waves when it is applied to dynamic soil-pile interaction problems and compare its efficiency with results obtained by other authors. Soil and single pile are spatially discretized using eight-node hexahedral elements with reduced integration and an efficient stabilization technique. The interface between soil and pile is considered using a three-dimensional contact formulation based on the penalty method and the nonlinearity, inherent of contact problems, is taken into account using a corotational formulation.

2 Numerical model description

The principle of virtual work, which is equivalent to the weak form of the momentum equation, referred to a generic element *e*, with domain Ω_e, may be expressed as (see Belytschko et al. [17] and Wriggers [7]):

$$\int_{\Omega_e} \delta\boldsymbol{u}^T \rho \ddot{\boldsymbol{u}}\, \mathrm{d}\Omega_\mathrm{e} + \int_{\Omega_e} \delta\boldsymbol{u}^T \chi \dot{\boldsymbol{u}}\, \mathrm{d}\Omega_\mathrm{e} + \int_{\Omega_e} \delta\boldsymbol{\varepsilon}^T \boldsymbol{\sigma}\, \mathrm{d}\Omega_\mathrm{e} = \int_{\Omega_e} \delta\boldsymbol{u}^T \boldsymbol{b}\, \mathrm{d}\Omega_\mathrm{e} + \int_{\Gamma_e} \delta\boldsymbol{u}^T \bar{\boldsymbol{t}}\, \mathrm{d}\Gamma_\mathrm{e} + \int_{\Gamma_c} \delta\boldsymbol{g}^T \boldsymbol{t}\, \mathrm{d}\Gamma_c, \tag{1}$$

where $\delta\boldsymbol{u}$ is the displacement field variation, ρ is the material specific mass, $\boldsymbol{u}, \dot{\boldsymbol{u}}$ and $\ddot{\boldsymbol{u}}$ correspond to displacement, velocity and acceleration vectors, respectively, χ is the damping coefficient, $\boldsymbol{\sigma}$ is the vector with element stress tensor components, $\delta\boldsymbol{\varepsilon}$ is a vector with components of the virtual strain tensor due to $\delta\boldsymbol{u}$; **b** is the body force vector and $\bar{\boldsymbol{t}}$ is the prescribed traction vector applied on Γ_e. Note that the final term is due to the contact contribution and it corresponds to the virtual work done by the contact tractions $\boldsymbol{t}$ related to the virtual displacement field $\delta\boldsymbol{g}$ on the contact boundary Γ_c.

Spatial coordinates, displacements, velocities and accelerations are approximated by the nodal values and the shape functions **N** of the eight-node hexahedral finite element. Introducing this approximation into Eq. (1), the well-known equation of motion at element level is obtained as:

$$\boldsymbol{M}^{(e)}\ddot{\boldsymbol{U}}^{(e)} + \boldsymbol{D}^{(e)}\dot{\boldsymbol{U}}^{(e)} + \boldsymbol{K}^{(e)}\boldsymbol{U}^{(e)} = \boldsymbol{P}^{(e)} + \boldsymbol{P}_c^{(e)}, \tag{2}$$

where **M**, **D** and **K** are the mass, damping and stiffness matrices, respectively; **P** is the external force vector and $\mathbf{P}_c$ is the contact force vector. The strain tensor is obtained using the strain-displacement relation $\boldsymbol{\varepsilon} = \bar{\boldsymbol{B}}\boldsymbol{U}$, where $\bar{\mathbf{B}}$ is the gradient matrix.

Previous matrices are evaluated here using the eight-node hexahedral element formulation with one-point quadrature. Despite its advantage in terms of computational cost when compared with the use of full integration, the so-called hourglass modes may occur and an efficient stabilization technique must be employed to stabilize the element formulation (see Duarte Filho and Awruch [9] for further information).

In the present work, the contact formulation is built using the algorithm proposed by Wriggers [7]. The penalty method is adopted here using the “node-to-surface” formulation, where a five-node contact element is defined by the four target nodes (hexahedral element face) and the slave node. In this way, small interpenetrations are allowed between bodies in contact and the non-penetrability condition is loosened.

Note that the problem initially lies in determining the projection of the slave node on the target surface, which is a face of the hexahedral finite element. Therefore, the first step to determine the normal gap consists of a “global search” to find which target elements are candidate to get in touch with de slave node. After that, a “local search” is carried out to evaluate the slave node projection on the target surface using an iterative process. If the contact is not detected, a new search is performed for other contact candidates.

With the contact defined, Eq. (2) can be employed considering that the contact forces are obtained using the penalty formulation. Furthermore, the system of nonlinear equations presented in Eq. (1) is solved using the Newton-Raphson method, where linearization procedures must be carried out considering an incremental-iterative approach. In this case, a contact stiffness matrix is obtained, which may be found in Visintainer et al. [16]. A detailed description on linearization of the contact virtual work may be found in Wriggers [7].

Note that the contact formulation is inherently nonlinear, since the contact surface is not known at first. Therefore, a corotational reference system is employed here to analyze nonlinear problems, where strain and stress tensors are evaluated locally. Considering that the finite element description is fine enough, it is possible to decompose the motion of a continuous medium into a rigid body motion followed by a pure deformation portion. Consequently, the pure deformation part will be small when compared to the element dimensions and the strain hypothesis can be considered properly. In order to maintain objectivity of the stress updates in the corotational system, stress rate measures are performed in this work using the Truesdell rate tensor. More details on the corotational reference system may be found in Braun and Awruch [11].

In geometrically nonlinear problems, the dynamic equilibrium must be iteratively satisfied using the incremental approach, where the stiffness matrix and the internal force vector are considered as functions of the current configuration. The equilibrium equation in an incremental form is described by the following expression (Mondkar and Powell [18]):

$$\boldsymbol{M}\Delta\ddot{\boldsymbol{U}} + \boldsymbol{D}\Delta\dot{\boldsymbol{U}} + \boldsymbol{K}_t\Delta\boldsymbol{U} = \boldsymbol{P}_{t+\Delta t} - \left[\boldsymbol{f}^{\text{int}}(\boldsymbol{U}) + \boldsymbol{M}\ddot{\boldsymbol{U}} + \boldsymbol{D}\dot{\boldsymbol{U}}\right]_t, \tag{3}$$

where $\Delta\ddot{\mathbf{U}}$, $\Delta\dot{\mathbf{U}}$ and $\Delta\mathbf{U}$ are, respectively, vectors containing incremental values of acceleration, velocity and displacement; $\mathbf{K}_t$ is the tangent stiffness matrix, which also contains the contact stiffness matrix, at time t, $\mathbf{f}^{\text{int}}(\mathbf{U})$ is the internal force vector and $\mathbf{P}_{t+\Delta t}$ is the force vector containing external and contact forces at time $t+\Delta t$.

The dynamic analysis is carried out here applying the Newmark's implicit method on Eq. (3) for time integration (see Mondkar and Powell [18] and Bathe [19] for the summarized algorithm).

It is also known that the energy in dynamic analysis is trapped inside the mesh due to reflection of waves when fixed boundary condition is employed. A simple solution for this problem is to use a large domain enough to avoid the reflection of waves in the region of interest, but this solution presents a high computational cost due to the high number of elements and nodes required. Another solution to this problem is to employ boundaries that absorb the energy, which are placed with finite elements that provide a "quite boundary".

Thus, in the present work, a simple standard viscous boundary is employed here, which is known as infinite element. This element has the advantage of being easily incorporated into the finite element algorithm already implemented but its drawback is that the absorption of waves cannot be perfect over a whole range of incident angles, having the best absorption when the incident wave is perpendicular.

Due to its simplicity, this element is presented into some finite element commercial software such as ANSYS and ABAQUS. In static analysis the infinite element provides stiffness based on Zienkiewicz et al. [20] formulation, while in dynamic analysis it provides a quiet boundary based on Lysmer and Kuhlemeyer [3] work, having the property to be independent of frequency. Thus, the solid infinite element developed here is similar to the INFIN257 (ANSYS) and CIN3D8 (ABAQUS) elements.

3 Numerical applications

Two examples considering a single pile, surrounded by a three-dimensional soil layer, under horizontal and vertical harmonic loads are presented bellow in order to evaluate the algorithm implemented. The excitation frequency is given in terms of the dimensionless frequency ($a_0 = \omega d/V_s$, where ω is the excitation frequency, d is the pile diameter and V_s is the shear wave velocity of soil). The time step is assumed to be equal to T/30, where T is the period of excitation. Results are presented in terms of dynamic stiffness of the soil-pile system, where the real (k) and imaginary (k') parts are obtained by computing the complex displacements at the load application point, which represent, respectively, the stiffness and the damping of the system.

3.1 Horizontal loading

This example was presented by Kaynia and Kausel [21] and consists of a single floating pile under dynamic horizontal loading. It is assumed that soil and pile have elastic behavior and the interface is considered as perfectly bonded, i.e., there is an equivalence between the soil-pile interface nodes and no contact algorithm is needed. The following material properties are used for the pile: Young's modulus E_p = 20 GPa, Poisson coefficient ν_p = 0.25 and specific mass ρ_p = 2300 kg/m³. The soil properties are: Young's modulus E_s = 20 MPa, Poisson coefficient ν_s = 0.40 and specific mass ρ_s = 1610 kg/m³. These material properties keep the ratios employed by Kaynia and Kausel [21]. A circular pile with length equal to 7.5 m and 0.5 m in diameter is embedded in a 11.25 m deep soil layer. Material damping ratio equal to 5% is considered for the soil elements, being assumed to be proportional to the stiffness matrix, while no damping is considered for the pile.

Computational domain is presented in Fig. 1, where only one half of the spatial domain is modeled with 7200 hexahedral finite elements, 240 infinite elements and 8608 nodes. All the degree of freedoms are fixed at the bottom of the computational domain and nodes at the symmetry face are constrained in the normal direction. Also, nodes at the pile head are considered fixed in the axial direction, but free to move in the radial directions.

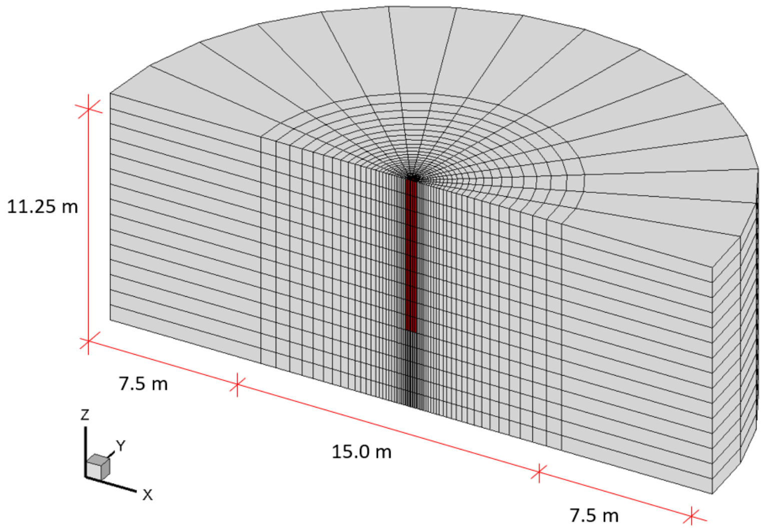

Figure 1. Geometry and mesh configuration for the horizontal loading problem

A horizontal harmonic load with amplitude equal to 25 kN is applied at the pile head. Note that due symmetry, only half of the previous load amplitude is applied in the numerical model. A linear geometric analysis with 500 load steps is performed and the dimensionless frequency ranged from 0.1 to 1.0.

Figure 2 shows the normalized real and imaginary parts of the horizontal dynamic stiffness computed, which are compared with values reported by Kaynia and Kausel [21] and Sarkar [5]. Results are normalized here with respect to the horizontal static stiffness (k_{static}).

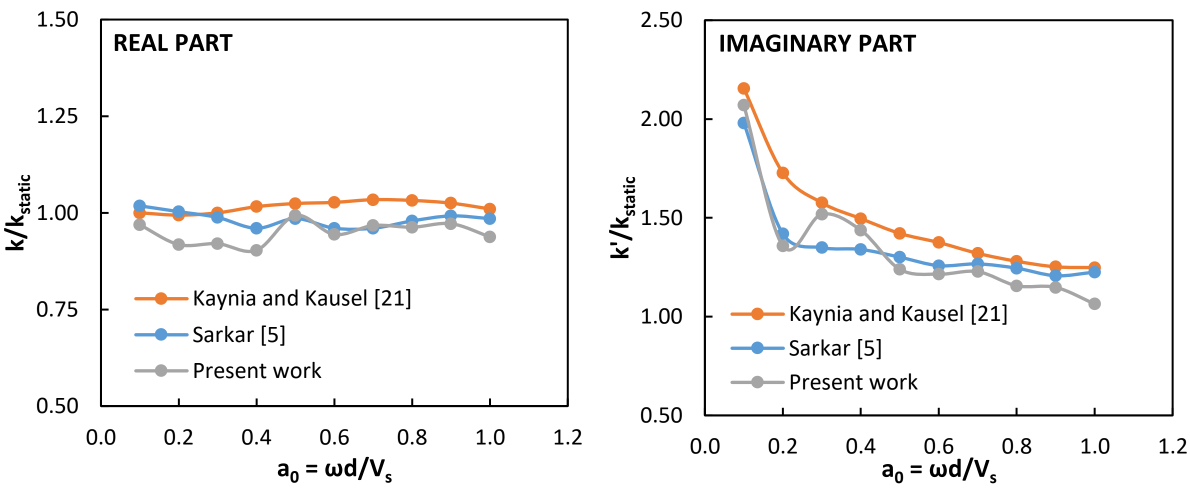

Figure 2. Dynamic stiffness for the horizontal loading problem

It is possible to notice that both real and imaginary parts of stiffness are in agreement with results presented by Kaynia and Kausel [21] and Sarkar [5]. Slight differences can be explained by the different solution methodologies employed by both authors. Kaynia and Kausel [21] assumed that the soil medium is a layered half-space, while Sarkar [5] assumed the presence of a bedrock, as well as in the present work. These assumptions affect the imaginary part and other deviations with respect to Sarkar [5], in the damping response, may be related to the use of Kelvin elements in the lateral radiation boundary by the author.

In order to evaluate the importance of infinite elements in the mesh, another numerical model without infinite elements is tested and the corresponding horizontal time histories at pile head for $a_0 = 0.5$ are presented in Fig. 3. Nodes that previously were at the interface between the finite and infinite elements are now fixed in all directions. It is observed that the energy was trapped in the model with finite elements only, which was not able to obtain a steady state response, as observed in the response of the model that contains infinite elements.

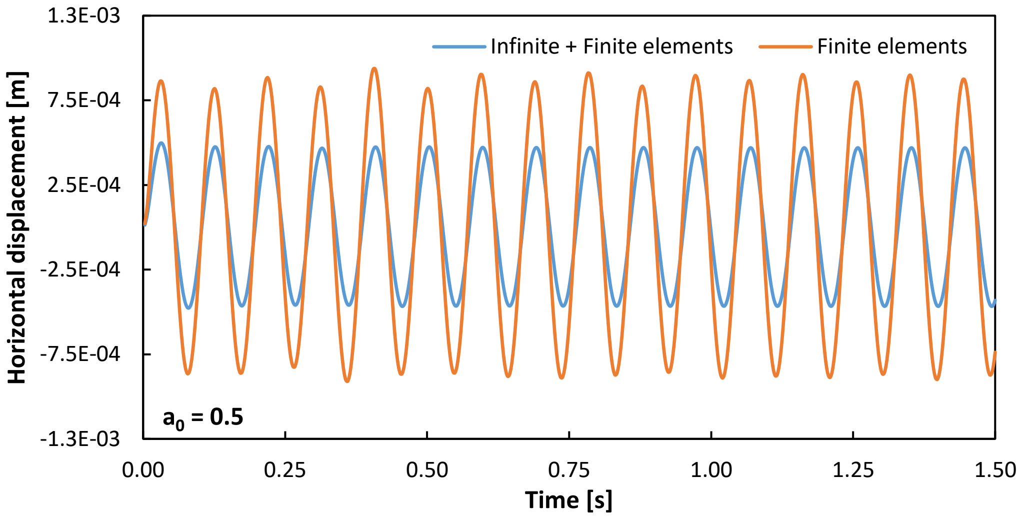

Figure 3. Horizontal displacement response at pile head for $a_0 = 0.5$

3.2 Vertical loading

The present example seeks to evaluate the soil-pile model with contact elements using the problem presented by Nogami and Konagai [22] and consists of a single end-bearing pile under vertical harmonic excitation. The circular pile is 10 m long (L) and 0.4 m in diameter (d). Due to symmetry, one-fourth of the problem is modeled with 4800 finite elements, 160 infinite elements and 6195 nodes, as shown in Fig. 4. Nodes located on the base of the computational domain are fixed in all directions, while nodes at the symmetry plane are constrained in the normal direction.

Keeping the same ratio employed by Nogami and Konagai [22], it is assumed that pile and soil have elastic behavior with Young's modulus equal to E_p = 20 GPa and E_s = 24 MPa, respectively, Poisson coefficient equal to ν_p = 0.3 and ν_s = 0.4 and specific mass equal to ρ_p = 2300 kg/m³ and ρ_s = 868 kg/m.

A vertical harmonic load with amplitude equal to P_{max} is applied at the pile head and a simple linear variation of maximum shear stress, from τ_f at the pile head to $2\tau_f$ at the pile tip, is considered at the soil-pile interface. The contact is assumed with friction coefficient μ = 0.7, normal penalty parameter $k_N = 10^9$ N/m and tangential penalty parameter $k_T = 5 \times 10^5$ N/m. A nonlinear geometric analysis with 480 load steps is performed and the dimensionless frequency ranged from 0.2 to 1.0.

The real and imaginary part of the vertical dynamic stiffness computed for two different values of ratio $P_{max}/\tau_f L^2$ (i.e., 0 and 1), are shown in Fig. 5. Note that $P_{max}/\tau_f L^2 = 0$ corresponds to the perfectly bonded case between soil and pile.

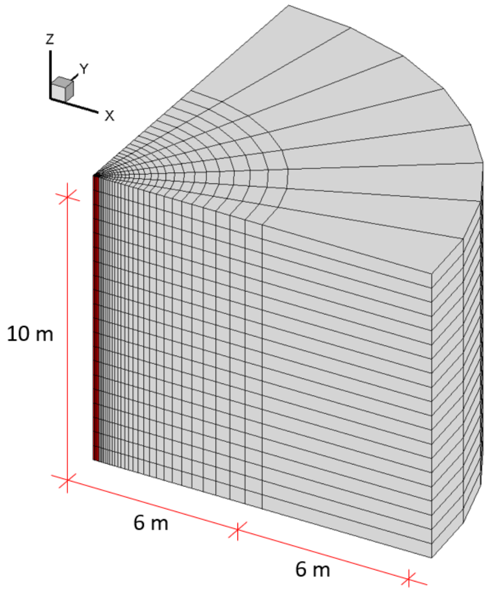

Figure 4. Geometry and mesh configuration for the vertical loading problem

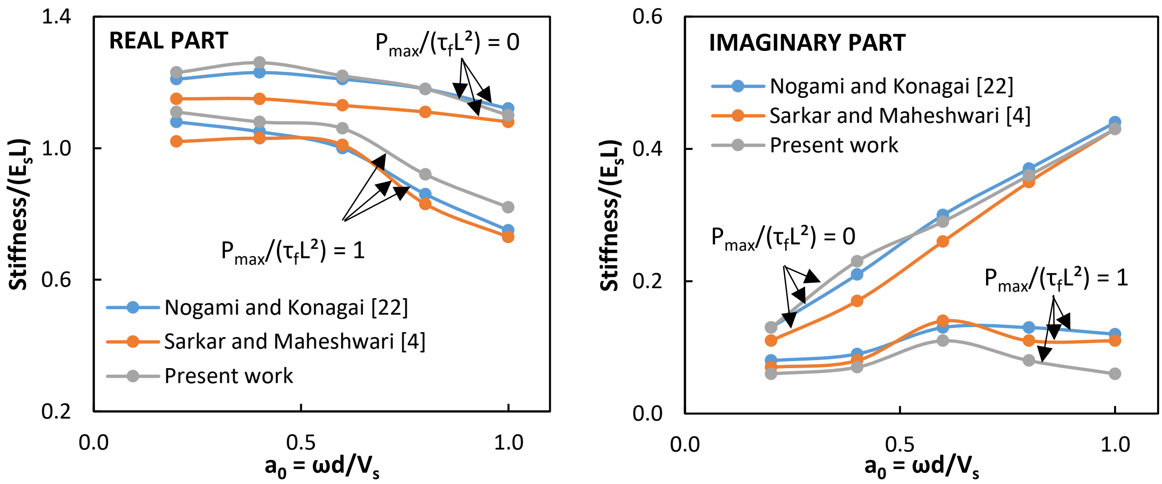

Figure 5. Dynamic stiffness for the vertical loading problem

One can observe a good agreement among results obtained here and those presented by Nogami and Konagai [22] and Sarkar and Maheshwari [4]. It is important to mention that Nogami and Konagai [22] employed Winkler formulation for modeling the problem and Sarkar and Maheshwari [4] used a three-dimensional finite element model. One can see also that both the real and imaginary parts are reduced significantly due the sliding between the soil and pile, especially for the higher frequency range.

4 Conclusions

Numerical simulations employing hexahedral finite elements with reduced integration combined with infinite elements were performed in the present work to evaluate their capacity to absorb energy waves in soil-pile problems during dynamic analysis. It was observed that the infinite elements were fundamental to simulate properly the radiation damping and avoid trapping energy inside the computational domain. Results with standard finite elements only presented an incorrect and unstable response, while the mesh containing infinite elements showed a steady state response. Therefore, although its simplicity, the infinite element formulation proposed here presented a good efficiency and the numerical results showed, in general, a good agreement with those reported by other authors with few deviations. Results considering the contact interface between the pile and the soil showed that the dynamic stiffness is significantly decreased with the sliding between the soil and pile, especially for higher frequency range.

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