Some Comments on Signal Processing Analysis in Nonlinear Dynamics and Chaos

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Abstract. Signal processing analysis in nonlinear dynamics applications is the main subject of this article, written for overview comparing different time-frequency analysis methods applied to nonlinear mechanical systems. The theory was carefully exposed and complemented with sample applications on mechanical vibrations and nonlinear dynamics. A particular phenomenon that is also observed in nonlinear systems is the resonant capture and Sommerfeld effect, which occurs due to the interaction between a non-ideal energy source and a mechanical system. Another application is to characterize the chaotic dynamics of mechanical systems using signal processing techniques. In addition, experimental and simulated signals are used that show the methods have high accuracy for the analysis of nonlinear dynamics.

Keywords: Time-frequency analysis, Wigner-Ville distribution, Synchrosqueezing Wavelet transform.

1 Introduction

Time-frequency analysis (TFA) for mechanical vibrations in stationary and non-stationary operations is the main subject of this work, designed to be an introducing to TFA comparing different techniques mechanical signals. The theory will be exposed and complemented with sample applications on mechanical vibrations and nonlinear dynamics. A particular phenomenon that is also observed in non-stationary systems is the Sommerfeld effect [1], which occurs due to the interaction between a non-ideal energy source and a mechanical system. An application through TFA for the characterization of the Sommerfeld effect is presented. Furthermore, this work exhibits an application of frequency-domain techniques for chaos characterize. The techniques presented here are applied in synthetic and experimental signals of mechanical systems.

Applications of TFA techniques to characterize the Sommerfeld effect can be found in [2-4]. The use of signal processing techniques to characterize chaotic dynamics has been widely used, as can be seen in [5-8]. This paper consists of four sections: in this section, Section 1, a short introduction was made regarding the topics covered in this work, in Section 2 the Mathematical background of TFA is presented, as well as its formulation, in Section 3 two case studies are described, and the results obtained from synthetic and experimental signal processing are presented and discussed and in Section 4 are made some final comments regarding the techniques presented in this paper.

2 Mathematical background

2.1 Wavelet Transform

The Wavelet Transform uses a variable window, where the resolutions vary along the time-frequency spectrum, in order to obtain all the information contained in the frequency plane. Equation 1 presents the Continuous Wavelet Transform (CWT).

$$W(a,b) = \int_{-\infty}^{\infty} x(t)\overline{\psi}_{a,b}(t)dt \tag{1}$$

Where,

$$\overline{\psi}_{a,b}(t) = \frac{1}{\sqrt{a}}\overline{\psi}\left(\frac{t-b}{a}\right) \tag{2}$$

In (2) the term $\overline{\psi}(t)$ is the prototype windowing, known as the Mother Wavelet. This analysis determines the correlation of the signal x(t) through translations and scale changes, for a given Mother Wavelet.

A detailed study of the formulation is found in [5].

2.2 Wigner-Ville distribution

In the last few years, alternative time-frequency representations have been studied and the Wigner-Ville distribution (WVD) has received great attention from signal processing researchers. According to [41, 42], the WVD can be derived by generalizing the relationship between the power spectrum and the self-correlation function for nonstationary signals. For a continuous signal x(t), the Wigner-Ville distribution is defined as:

$$WVD_x(t,f) = x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)e^{-j2\pi f_\tau}d\tau \tag{3}$$

An in-depth study of the method is presented in [9].

2.3 Synchrosqueezing Wavelet transform

The Synchrosqueezing Wavelet Transform (SWD) basically consists of three steps. The first step is to calculate the CWT, according to Equation 1. In the second step, a preliminary frequency ω(a, b) s obtained from the oscillatory behavior of W_x(a, b) at a, so that [10]:

$$\omega(a,b) = -i\big(W_x(a,b)\big)^{-1}\frac{\partial}{\partial a}W_x(a,b) \tag{4}$$

In the third step the transformation from the time scale plane to the time-frequency plane is performed. Each value of W_x(a, b) is assigned again to (a, ω_l). Where ω_l denotes the frequency that is closest to the preliminary frequency of the original (discrete) point ω(a, b). Thus [10]:

$$T(a,\omega_1) = (\Delta\omega)^{-1}\sum_{b_k:|\omega(a,b_k)-\omega_l|\leq\Delta\omega/2} Wx(a,b_k)b_k^{-3/2}\Delta b \tag{5}$$

In (6) $\Delta\omega$ denotes the width of each frequency bin $\Delta\omega = \omega_l - \omega_{l-1}$ and equivalent for Δb.

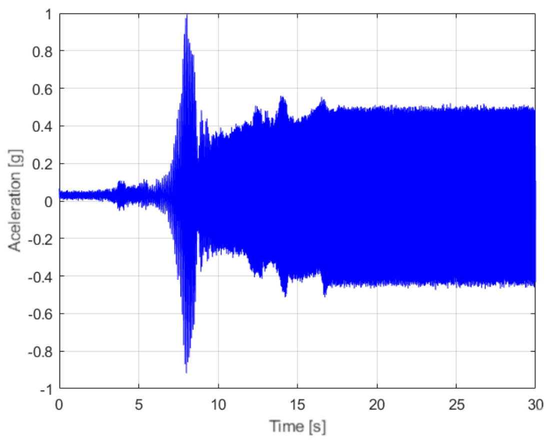
3 Case studies

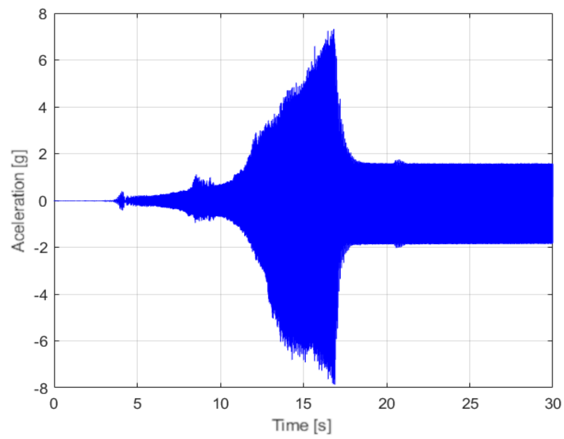
3.1 Sommerfeld Effect analysis (experimental signals).

In this section, the experimental signals presented in Figures 1a and 1b come from a mechanical system, a portal frame, with a 1 degree of freedom (DOF), this system is detailed in [2,11]. The goal of this analysis is to characterize the resonance and Sommerfeld phenomenon, in the time and frequency domains. In this system the Sommerfeld Effect occurs at the second natural frequency of the frame, 45 Hz. Here we use the acceleration signals from a mechanical system, with 1-DOF, which has its experimental and data acquisition procedure carefully described in [4].

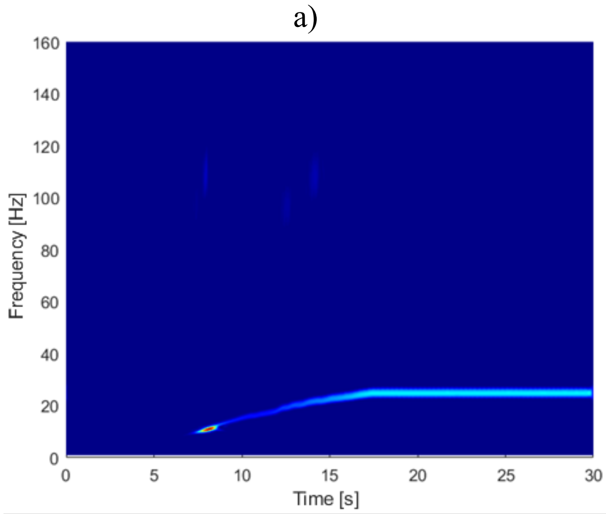
The signals were acquired with a sampling frequency of 1000 Hz and 32768 points, for a total acquisition time of 32,768 s. These signals are transient signals of the motor starting with a small permanent at the end, in which the motor rotation is varied from 0 to 20 Hz in cases where there is no Sommerfeld Effect (Figure 1a) and from 0 to 65 Hz in cases where the Sommerfeld Effect is present (Figure 1b).

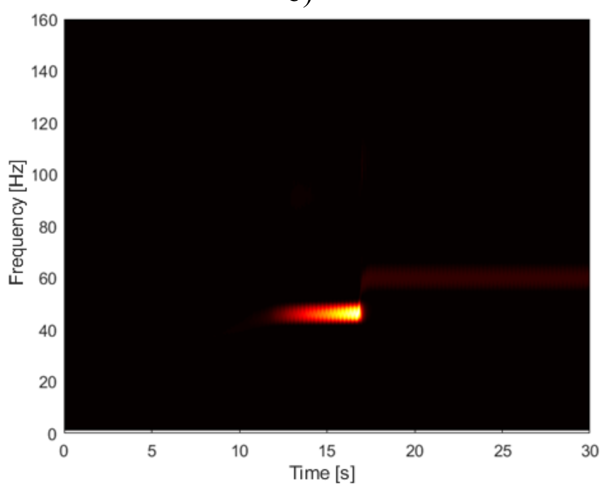
Figure 1 also presents the time-frequency analysis of the system by applying CWT (Figure 1c and 1d) and presents the response in the time domain (Figures 1a and 1b). In applying the CWT (Morlet Wavelet) to the system (Figures 1c and 1b), the resonance frequency is clearly identified, as well as the stationary regime of the system. In Figure 1d it is clearly noted that the system is stuck at the resonance frequency and then the nonlinear jump phenomenon occurs, characteristic of the Sommerfeld effect. The Sommerfeld Effect occurs due to a nonlinear interaction between the motor rotation and the dynamic response of the gantry. The results of the SWT analysis are presented (Figures 1f and 1g). In both cases the resonance phenomenon and Sommerfeld effect are well characterized, with good energy concentration in the time-frequency distribution. The technique is quite adequate for the characterization of the proposed problem.



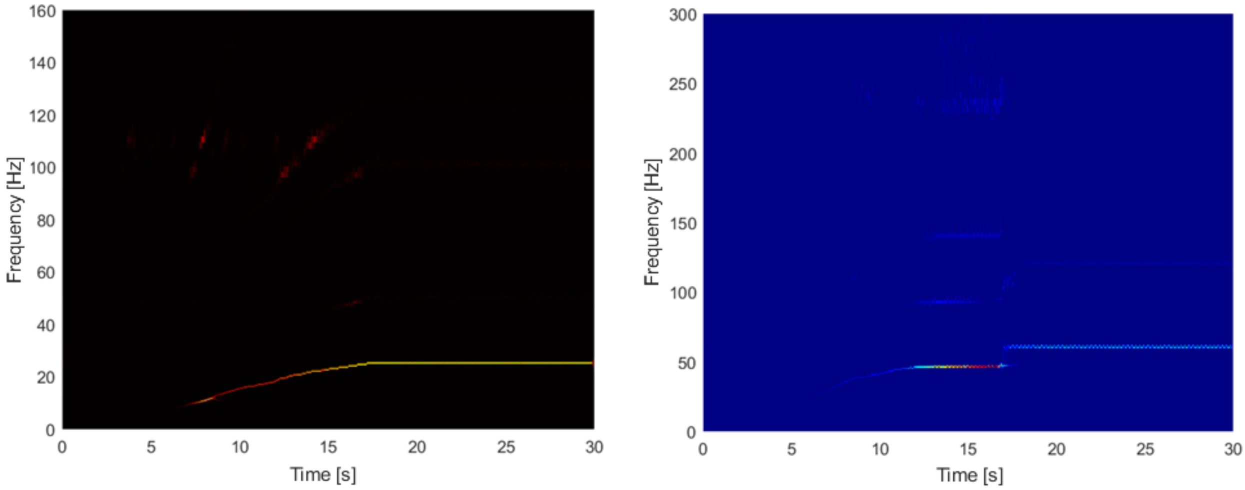


a) b)





c) d)



e) f)

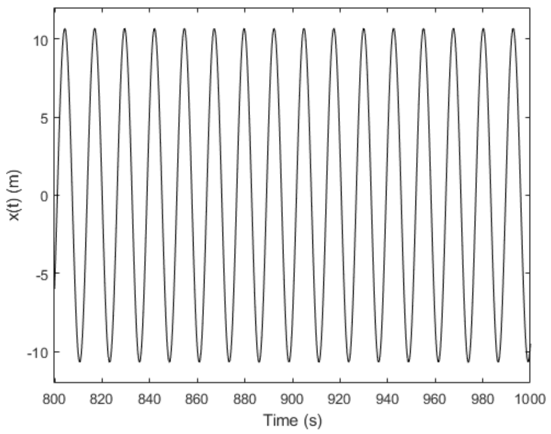
Figure 1. Time-Frequency analysis of experimental signal of a mechanical system with 1-DOF: (a) Response in the time domain without the presence of the Sommerfeld effect , (b) Response in the time domain with the presence of the Sommerfeld effect , (c) CWT - Response in the frequency domain (without the Sommerfeld effect) (d) CWT - Response in the frequency domain (with the Sommerfeld effect), : (e) SWT - Response in the frequency domain (without the Sommerfeld effect), (f) SWT - Response in the frequency domain (without the Sommerfeld effect).

3.2 Chaos analysis in frequency domain (synthetic signals)

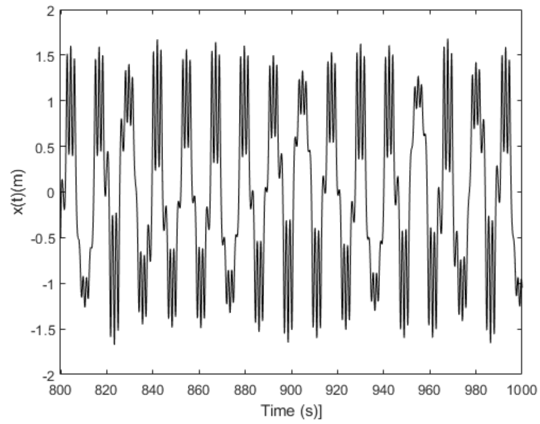
In this section, the synthetic signals shown in Figures 2a (Periodic) and 3a (Chaotic), in the time domain, come from an energy harvesting system, modeled with a Duffing spring and fractional damping. The system and the mathematical model are described in detail in [6]. The goal in this analysis is to characterize the chaotic dynamics of the system in the frequency domain using CWT, WVD, SWT.

In general, when using time-frequency analysis methods to characterize chaos, one looks for a variation in the frequency spectrum. The analysis of the periodic signal is very well characterized with all methods (CWT, WVD, SWT), as well as the natural frequency of the system, as can be seen in Figures 2c, 2e and 3g.

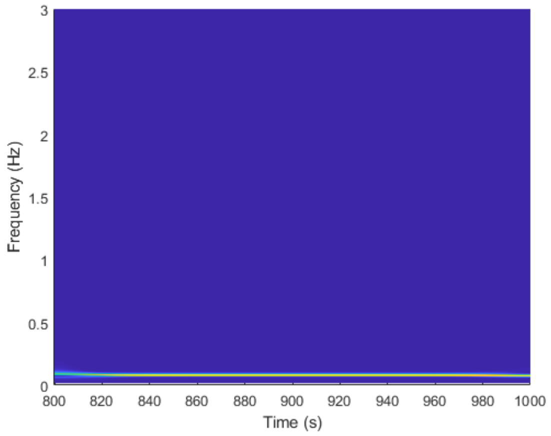
In the analysis of the chaotic signal using CWT (Morlet Wavelet), Figure 2d, we notice the abrupt variation in the frequency spectrum but it is not adapted to characterize the dynamics of the system. Applying the WVD to chaotic signal, it is possible to visualize in Figure 2f. There is a shift in frequencies, explicitly at higher frequencies, seen through the discontinuity of the energy concentration. Therefore, WVD can characterize the frequency shift shown by chaotic signals The analysis with SWT (Figure 2h) it is possible to characterize the duplication period, the dynamics of system (chaos)and the nonlinearity effects of the system. This tool is better suited to characterize the dynamics of the system.


x(t) (m)
Time (s)

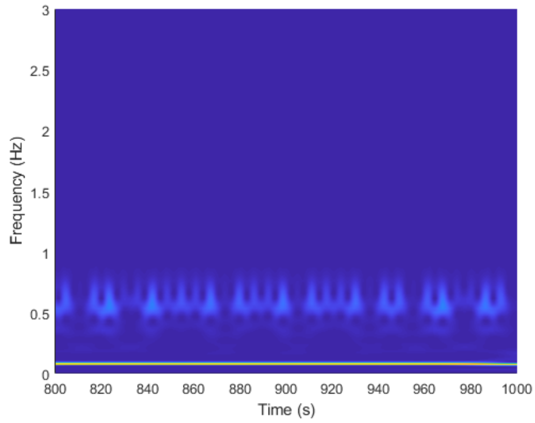
a)


x(t)(m)
Time (s)]

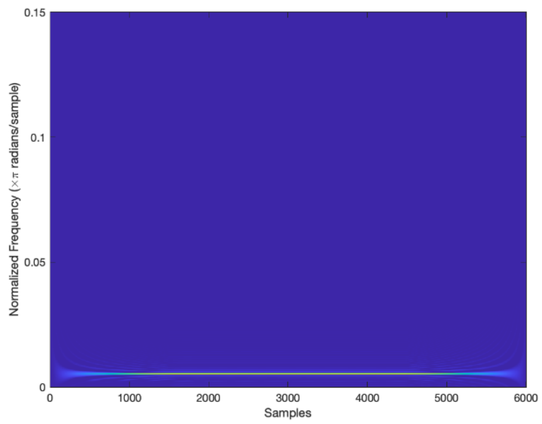
b)


Frequency (Hz)
Time (s)

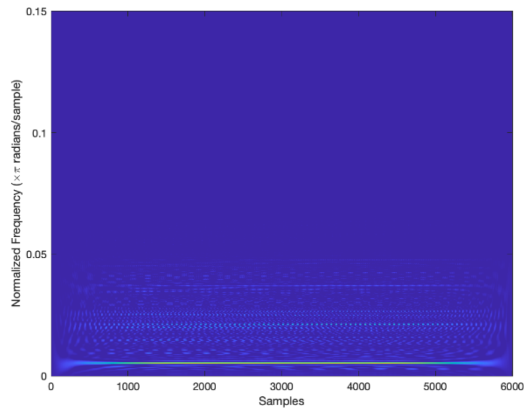
c)


Frequency (Hz)
Time (s)

d)


Normalized Frequency (×π radians/sample)
Samples

e)


Normalized Frequency (×π radians/sample)
Samples

f)

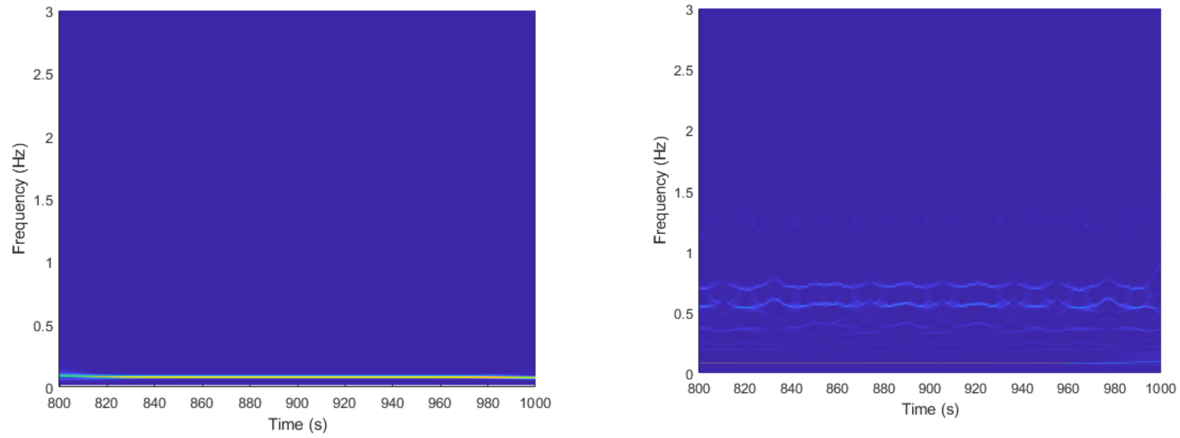


Figure 2. Time-frequency analysis of a 1-DOF mechanical system signal system with Duffing spring and fractional damping (a) Response in time domain of periodic signal, (b) Response in time domain of chaotic signal, (c) CWT - Response in frequency domain of periodic signal, (d) CWT - Response in frequency domain of chaotic signal: (e) WVD - Periodic signal frequency domain response, (f) WVD - Chaotic signal frequency domain response, (g) SWT - Periodic signal frequency domain response, (h) SWT - Periodic signal frequency domain response.

4 Final Remarks

In this study several current non-stationary signal processing techniques were investigated, particularly in the study of mechanical vibrations, evaluating their efficiency in separating frequency components, as in evaluating. In particular the characterization of the Sommerfeld effect and the nonlinear jump.

CWT, present good resolution in the time-frequency plane, but CWT still obfuscates other components when there are high-energy concentrations at some signal frequency. Finally, the results obtained through SWT were excellent, presenting an optimal time- frequency resolution and minimum energy dispersion. Therefore, it is quite adequate for the characterization of the Sommerfeld effect.

About chaos characterization, CWT was able to characterize the abrupt variation in the frequency spectrum. Efficient in analyzing the chaos characterization.

WVD allowed us to verify the energy concentration around the natural frequency and to characterize the chaotic behavior through frequency spectrum discontinuities. However, for energy harvest analysis, it was not efficient to present energy concentration. SWT The synchrosqueezed wavelet transform method is an extension of the wavelet transform that adds empirically decomposed elements and frequency reassignment techniques.

Overall, SWT and WVD presented satisfactory results only in the identification of chaotic dynamics characteristics. Each transformation used, therefore, has its strengths and weaknesses, making them strongly complementary and widely applied in nonlinear signal analysis.

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