Finite Element Analyses of mesh-objectivity for Smeared, Damage and Discrete models applied to concrete cracking

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Abstract. This paper will address the issue of mesh objectivity regarding Smeared, Damage and Discrete models applied to concrete cracking in the framework of the Finite Element Method. It will be shown that assigning the same stress-strain relationship with softening for all elements regardless their size and shape will lead to spurious results. This will make the fracture energy decrease as the mesh is refined, sometimes even converging zero which is completely unacceptable. This problem is related to the negative slope of the tangent stiffness tensor, that is, the increase in strain with decreasing stress which will make a small portion of the body experience softening while the rest of it unloads elastically. In order to circumvent this issue, it must be employed a regularization technique, also known as localization limiter. The first localization limiter that this study will adopt is the Crackband technique applied to both Smeared and an isotropic scalar Damage models. It is easily implemented in an existing Finite Element code and its main feature is the assumption of a bandwidth where the crack is supposed to propagate. It will be presented that the Crackband technique is the simplest but crudest approach. Nevertheless, it will assure the convergence of the results but its accuracy hinges on choosing the bandwidth properly. The second localization limiter adopted in this paper is the Nonlocal integral-type technique which recovers the objectivity by taking the weighted average of a variable that controls cracking. This technique will be used in the isotropic scalar Damage model. Finally, it will be discussed the Discrete crack approach through interface elements with vanishingly thickness. This technique won't need any regularization since the constitutive relationship is already written in terms of Stress-Displacement. The three approaches will be employed in the simulation of a notched fiber-reinforced concrete beam and compared with its experimental data. In order to show the feasibility of each methodology, these analyses will debate their convergence properties which is the main focus of the present study and features such as computational effort, crackpath and assessment of input parameters.

Keywords: Finite Element, Mesh-objectivity, Concrete, Smeared, Damage, Discrete.

1 Introduction

Some finite element models for concrete cracking which emerged in the 1960s used to consider crack propagation by means of a stress-strain constitutive equation with softening so that, once reaching tensile strength, nominal stress would gradually decrease until zero. Such an approach was widely used and implemented in commercial software. Some years later, though, the scientific community realized that this approach was completely wrong. That is because softening constitutive equations violate the Drucker Stability Criterion (originally proposed by Rodney Hill). It leads to a phenomenon called *strain localization* in which a vanishingly region undergoes softening while the rest of the body unloads elastically. Numerically, it means that results become more brittle with mesh refinement because strains localize in a row (or surface) of finite elements and consequently convergence to zero energy dissipation is observed. This pathological behaviour will be referred to as *Mesh-objectivity* issue, although there isn't an unified terminology so literature also use the terms *Mesh-dependency*, *Mesh sensitivity* or *Unobjectivity*. Since it is fundamentally a mathematical rather than numerical problem, it will occur for any discretization method (Finite Element, Finite Difference, Meshless, etc) and for any softening constitutive model (Damage, Smeared crack, Microplane, Softening Plasticity, etc). On the other hand, models for which crack is regarded as a displacement discontinuity never present convergence to zero energy dissipation as mesh is refined. Their constitutive equation are already written in terms of stress-displacement so as to ensure a given energy dissipation per unit area (*Fracture Energy* - G_f) but they are still susceptible to numerical difficulties, as will be shown later.

2 Crack models

2.1 Smeared crack model

Smeared crack models date back to Rashid [1] when analyzing concrete vessels and was later improved by several authors. It decomposes the strain as $\varepsilon = \varepsilon_e + \varepsilon_c$ so that the constitutive equation becomes $\sigma = E(\varepsilon - \varepsilon_c)$, which is convenient to couple cracking with other phenomena (creep, plasticity, shrinkage, etc). Variables ε, ε_e and ε_c denote, respectively, the total (observable) strain, elastic strain and crack strain.

2.2 Damage model

The concept of damage was introduced by Kachanov [2]. To date, there are several sophisticated damage formulations taking into account the anisotropic nature of cracking but the scalar isotropic model usually give acceptable results. In this case, the stress-strain law becomes $\sigma = (1-D)E_0\varepsilon = E_s\varepsilon$ where D, E_0 and E_s denote, respectively, the damage, initial stiffness and secant stiffness. So, one might deduce that $D = 1 - E_0/E_s$ which greatly simplifies the computational implementation since the damage law doesn't need to be explicitly assessed.

2.3 Cohesive Zone model

At unaided human eye a crack in concrete is generally viewed as a sharp discontinuity but when examining it more thoroughly one will observed that a finite region of inelastic processes takes place around the major crack which is called the *Fracture Process Zone* (FPZ). In 1976, Hillerborgh *et al.* [3] developed a Cohesive Zone model called *Fictitious Crack Model* (FCM) where the FPZ is lumped into a vanishingly line in 2D or surface in 3D. So crack is modelled considering that cohesive forces exist between opposite sides of a microscopic crack. These forces gradually decreases and vanish at late stages of degradation when the crack is widely open (macroscopic crack).

3 Crack description

3.1 Weak discontinuity

Smeared crack and Damage models naturally describe crack as *weak discontinuities* [4] because cracking is regarded at the constitutive equation level so the displacement field remain continuous and crack is modelled changing the stresses and strains.

3.2 Strong discontinuity

When adopting interface elements or using more advanced discretization techniques such as *Extended Finite Element Method* (XFEM), *Generalized Finite Element Method* (GFEM) or *Partition of Unity Finite Element Method* (PUFEM) crack is modelled as a geometrical entity, that is, a *strong discontinuity* [4] (also known as *discrete crack approach*) since displacements are discontinuous in the opposite sides of the crack. In fact, interface elements do not represent a true strong discontinuity since they have a very small, yet finite, thickness which is translated by the high stiffness prior to cracking. Strong discontinuity is the natural choice when using Cohesive Zone models.

4 Localization limiters

4.1 Crack band

This technique was proposed by Bažant and Oh [5] and is the simplest but crudest approach. Consists in changing the slope of the post-peak curve according to the size of the Crack band. Such region is approximately equal to the size of the finite element but its rigorous assessment is also presented in the literature (cf. Oliver [6] and Jirásek [7]). The Crack band approach is equivalent to spreading the Cohesive Zone model over a finite element.

4.2 Nonlocal integral

Proposed by Pijaudier-Cabot and Bažant [8]. It consists in taking the weighted average of a variable that controls crack. In damage models the nonlocal variable is usually the equivalent strain. For smeared crack models one might consider the crack strain (ε_c). This technique is much more sophisticated than the Crack band since its convergence is always ensured and FPZ does not shrinks into a zone of zero thickness with mesh refinement. Its computational implementation requires searching all finite elements inside the radius of a given element. Nonlocal implementation is far from trivial but still feasible if done properly.

5 Numerical applications

Now, some numerical applications will be presented to assess the Mesh-objectivity (convergence) of the Finite Element models. To do so, three-point bending tests of fiber-reinforced notched concrete beams were selected. The modelling of fiber-reinforced concrete structures has nuances that will not be able to fit this paper but one of them must be remarked.

There are two approaches to model fibers' effect in concrete: consider concrete and fibers as a single homogeneous material or regard them separately. The former approach is straightforward since one usually translate it, for instance, through increasing G_f. The latter approach is much more complicated because it requires generating fibers in random positions inside the structure. In this paper, the former approach was chosen but this is not truly realistic and numerical difficulties arise. The Fracture Energy for the next simulations is about the order of 10^4 J.m^{-2} which is far higher than 10^2 J.m^{-2} for plain concrete.

The experimental data for the following applications were selected from Andrade [9] and consists in prismatic notched specimens of 69.90 mm (width), 68.15 mm (height), 200.00 mm (span), 2.57 mm (notch width) and 9.08 mm (notch height). The damage models with both Crack band and Nonlocal integral were already implemented by Costa [10] and Costa *et al.* [11].

5.1 Smeared crack model with Crack band technique

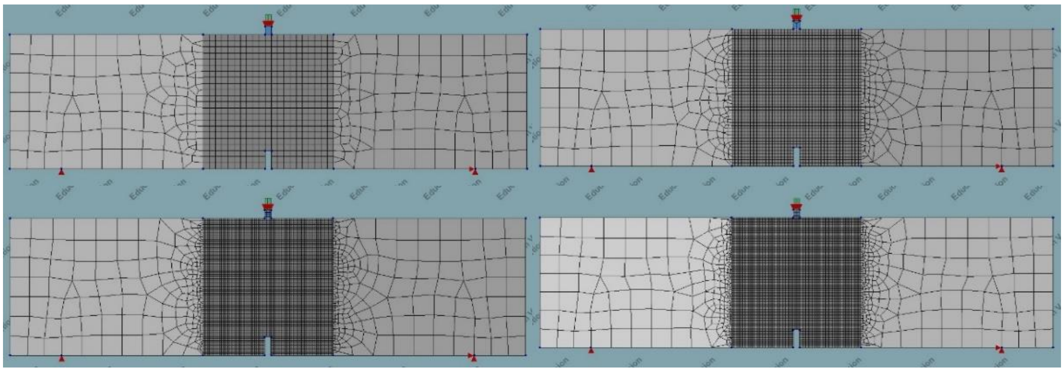


Figure 1. Mesh 1 (top-left), mesh 2 (top-right), mesh 3 (bottom-left) and mesh 4 (bottom-right)

Figure 2. Crack pattern for Smeared crack model with Crack band: mesh 1 (top-left), mesh 2 (top-right), mesh 3 (bottom-left) and mesh 4 (bottom-right)

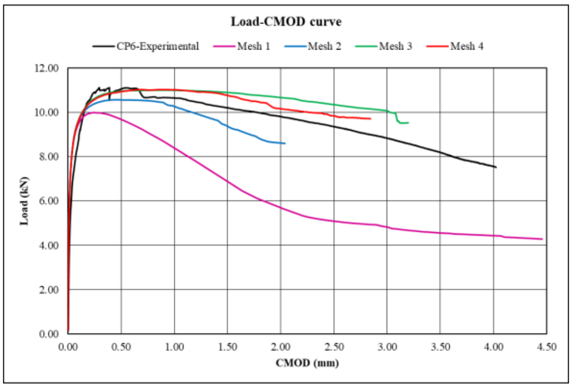


Figure 3. Load-CMOD curve for Smeared crack model with Crack band technique

5.2 Damage model with Crack band technique

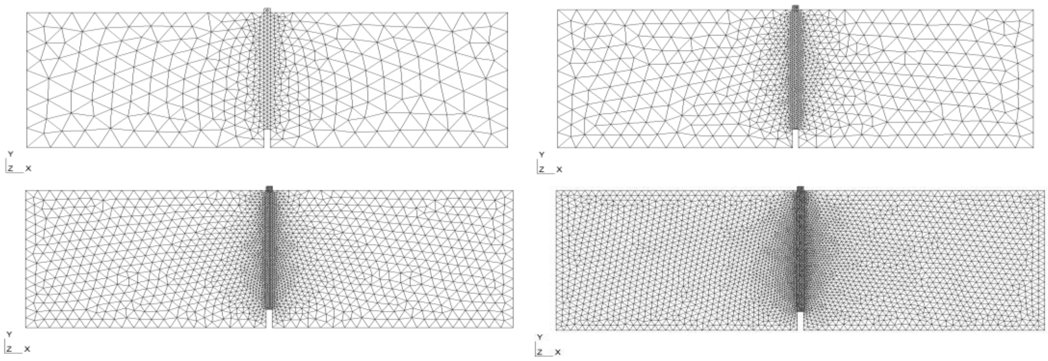


Figure 4. Mesh 1 (top-left), mesh 2 (top-right), mesh 3 (bottom-left) and mesh 4 (bottom-right)

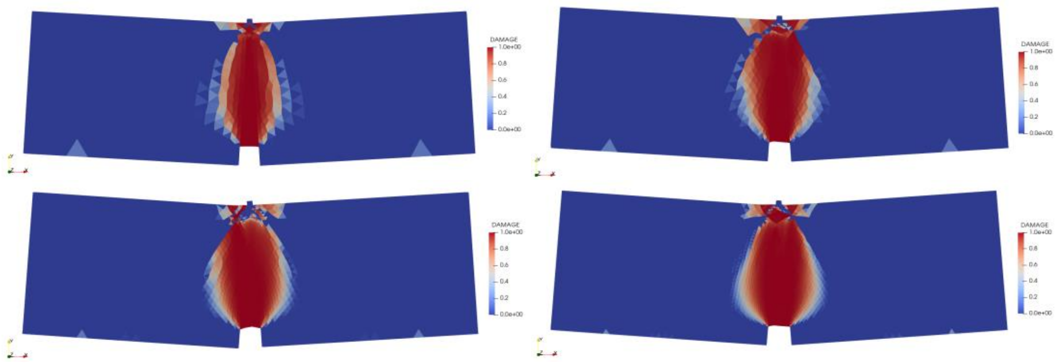


Figure 5. Crack pattern for damage model with Crack band: mesh 1 (top-left), mesh 2 (top-right), mesh 3 (bottom-left) and mesh 4 (bottom-right)

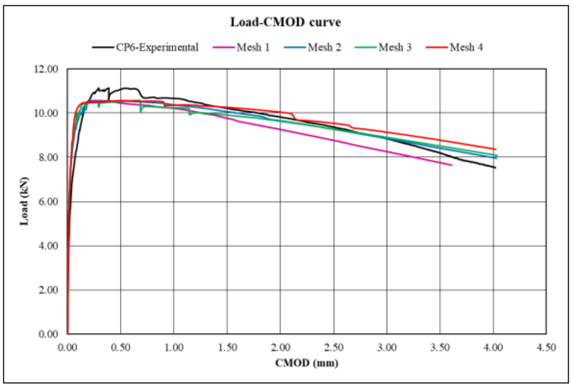
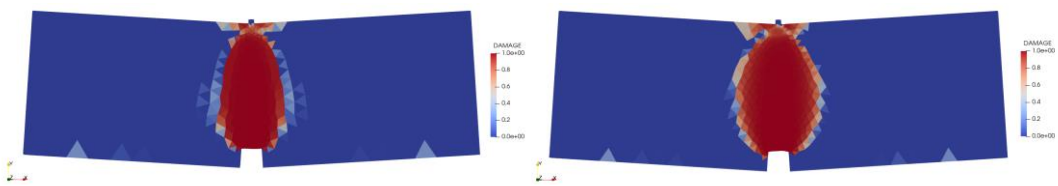


Figure 6. Load-CMOD curve for damage model with Crack band technique

5.3 Damage model with Nonlocal integral



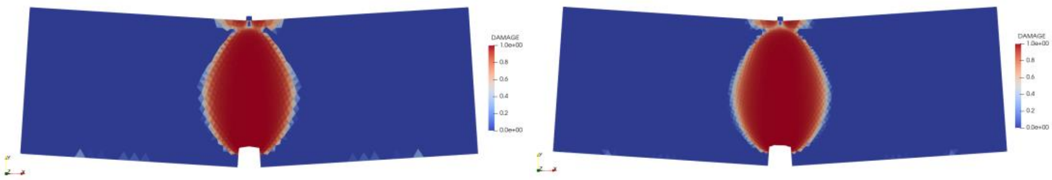


Figure 7. Crack pattern for damage model with Nonlocal integral: mesh 1 (top-left), mesh 2 (top-right), mesh 3 (bottom-left) and mesh 4 (bottom-right)

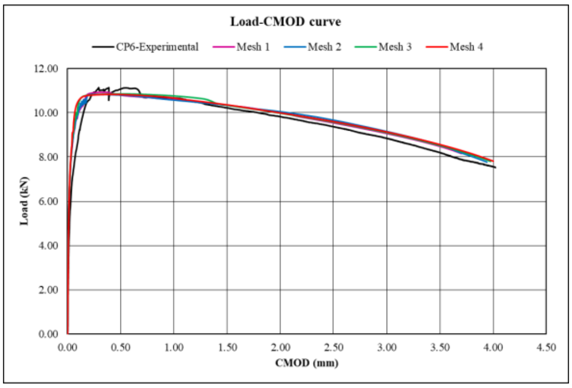


Figure 8. Load-CMOD curve for damage model with Nonlocal integral

5.4 Interface elements with Cohesive Zone model

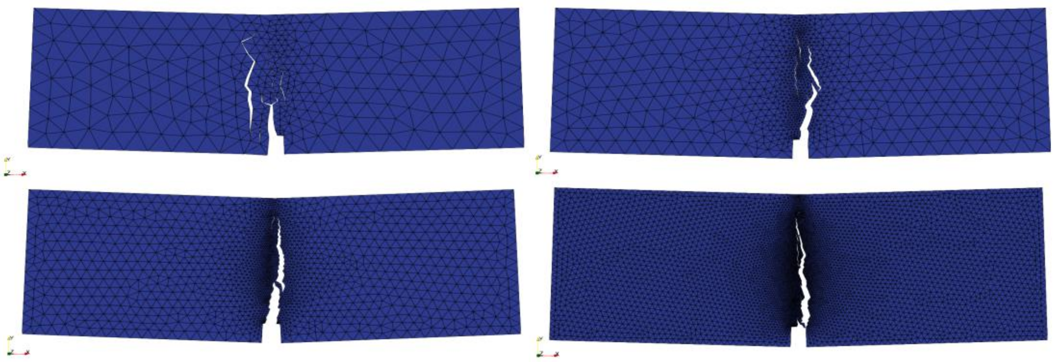


Figure 9. Crack pattern for Interface elements with Cohesive Zone model: mesh 1 (top-left), mesh 2 (top-right), mesh 3 (bottom-left) and mesh 4 (bottom-right)

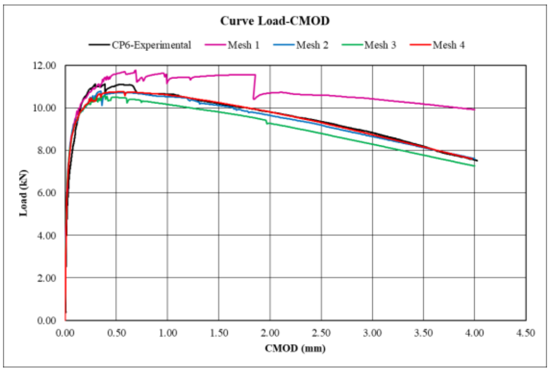


Figure 10. Load-CMOD curve for damage model with Interface elements with Cohesive Zone model

6 Concluding remarks

This paper presented the Mesh-objectivity issue which makes softening constitutive models converge to perfectly brittle cracking as mesh is refined. It was discussed that this problem is related to the softening constitutive equation which requires a regularization technique. On the other hand, Cohesive Zone models do not need such regularization since it naturally ensures a given energy dissipation per unit area. Some definitions regarding concrete crack modeling were presented, namely: Crack models, Crack description and regularization techniques. Experimental data were selected from literature for a three-point bending test of a fiber-reinforced notched concrete beam. Fibers and concrete were regarded as a single homogeneous material which is a very crude approximation for two reasons. First, increasing G_f does not make the structure display multiple and highly tortuous cracks, typical in fibrous concrete and, secondly, high G_f demands rather small load steps because it's necessary to allow a given element start softening so that the others will unload elastically which in turn will localize strains giving rise to a crack. This is not verified in the simulations and is certainly precluding convergence. Curiously, the Nonlocal integral is converging although it's probably converging to much higher G_f. The computational effort for both Smeared crack and damage models with Crack band is negligible requiring at most the assessment of the Crack bandwidth. The Nonlocal model is far harder to implement efficiently due to its computational complexity but convergence is always ensured. Interface Elements using Cohesive Zone model leads to meaningful crack paths but their small thickness (high stiffness) leads to very ill-conditioned matrices.

Acknowledgements. This study was financed in part by the Brazilian Agency Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) —Finance Code 001. The support of Brazilian funding agencies Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ) are also gratefully acknowledged.

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