Mechanical Performance Analysis of Reinforced Concrete Continuous Beams

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Abstract. Concrete is a material cast from Portland cement and aggregates. Since the stress level around 30% of its compressive strength, a concrete specimen, tested on a single compression, presents nonlinear stress-strain relationship. Steel bars are used to supply its low tensile strength resulting the reinforced concrete that presents complex mechanical performance. For the accomplishment of suitable study of mechanical performance of such a material, it is necessary the adoption of finite element analysis over a nonlinear model, at least, in plane state of stresses. Several models of analysis, such as, the model from the European community's standards, the one from AIC, Torrenfeldt's model, Hognestad's model, the Rashid's Smeared Crack Model, Branson's model, among others, have been proposed to the mechanical behavior of concrete description. The Branson's model is, specially, attractive due to its simplicity, because in its formulation the beam nonlinear behavior is simulated from an equation applicable to a beam structural member, resulting, in this way, computational effort economy. The purpose of this work is to report the nonlinear mechanical performance limit analysis of reinforced concrete continuous beams. To accomplish such a subject, a computational code was developed, based on the finite element approach on the Branson's formulation.

Keywords: Numerical simulation, Finite elements, reinforced concrete, continuous beams.

1 Introduction

Concrete is a solid mass resulting from the hardening of a homogenized mixture of aggregates, Portland cement and water that experiences cracking already in the first days of its synthesis, due to the volumetric contraction experienced in its natural drying process, Wight and McGregor [1].

Due to the concrete heterogeneous nature, in the face of the stresses imposition, the above-mentioned cracking is intensified resulting in a nonlinear mechanical behavior for such a material, which can culminate in instability, Wight and McGregor [1].

The low tensile strength of the highlighted material determines the need to use steel bars to supply this deficiency, resulting in reinforced concrete, Carvalho and Figueiredo [2].

Thus, the mechanical performance analysis of reinforced concrete structural members requires the adoption of nonlinear modeling, which can be effective from the Finite Element Method using on a nonlinear orthotropic model, in Plane State of Stresses, Madureira [3].

The attention to the deformation limits is as fundamental for the dimensioning of the cross-sections of reinforced concrete structural members as the observance of the strength requirements of the involved materials.

The use of nonlinear analysis in its most rigorous meaning in the tasks of calculating displacements of the ordinary structural design represents excessively painful work so that it is necessary to consider the use of simplified alternative procedures, such as the formulation proposed by Branson [4], thus enabling the approximation by beam finite elements and, consequently, computational effort economy.

Preliminary studies over the calculation of displacements of reinforced concrete beams using Branson's model (1968) indicated that its magnitudes obtained in this way are underestimated in comparison with experimental test results published by Burns and Siess [5], justifying, in this way, the effort expenditure in accurate analyses aimed to the appropriate adjustments of such model.

The aim of this paper is to report the mechanical performance numerical simulation of reinforced concrete continuous beams by use a computational program based on beam finite elements applied on the Branson's nonlinear model.

In this way, the obtained results will be compared to those ones obtained from a computational program based on finite element approximation over an orthotropic model and nonlinear constitutive relationships in plane state of stresses for concrete.

2 Modeling

2.1 Branson's model

Branson's model is intended for the calculation of displacements in reinforced concrete beams idealized as reticular bars distinguishing the behavior of the structural member whose critical cross section is on the Stage I from that one referring to the Stage II. Stage I corresponds to that condition in which the critical cross-section still absorbs tensile stresses, and, the condition in which the stretched region of such a section is cracked, characterizes the Stage II, Carvalho and Figueiredo Filho [2].

According to the model highlighted, the boundary between Stage I and Stage II is defined from the bending moment magnitude that would lead to the first cracking arising if the beam is cast in simple concrete, and is given by:

$$M_r = \frac{\alpha f_{ct} I_c}{y_t} \tag{1}$$

since the α parameter correlates Tensile Strength in bending and Direct Tensile Strength, y_t is the distance from the gravity center of the cross section to its stretched edge, I_c is the Inertia Moment of the Gross Section and f_{ct} is the Concrete Tensile Strength defined by:

$$f_{ct} = 0{,}3 f_{ck}^{(2/3)} \tag{2}$$

if the f_{ck} parameter represents the Concrete Characteristic Compressive Strength.

The Bending Stiffness of the cross-section in Stage I must be calculated by:

$$EI = E_{cs} I_c \tag{3}$$

if E_{cs} is the Concrete Elasticity Secant Modulus defined according to the ABNT NBR 6118 [6].

On the other hand, for bending moments in the critical section whose magnitude is greater than M_r, and so, the critical cross section, therefore, in Stage II, the Bending Stiffness of the cross-section should be calculated from:

$$\mathrm{EI} = E_{cs} \left\{ \left(\frac{M_r}{M_a}\right)^3 I_c + \left[1 - \left(\frac{M_r}{M_a}\right)^3 I_{II}\right] \right\} \tag{4}$$

since the M_a parameter represents the bending moment magnitude in the critical section and and I_{II} the Moment of Inertia of the Cracked Critical Section in Stage II.

2.2 Nonlinear orthotropic model

The constitutive matrix in Plane State Stress is defined from the equivalent deformations:

$$\varepsilon_{ei} = \varepsilon_i + \frac{D_{ij}\varepsilon_j}{D_{ii}} \tag{5}$$

since the i and j index represent the main directions, D_{ij} the constitutive matrix elements, and the parameters ε_i and ε_j are the strains along the main directions.

The concrete mechanical performance in compression is simulated from the HOGNESTAD [7] constitutive relationships.

To simulate the concrete tensile mechanical performance a smeared cracked model was adopted, that is suitable to consider the distribution of displacements as a continuous field and thus to dispense topological changing in the finite element mesh, during the calculation procedures.

In the range for which the strains are lower than that corresponding to its uniaxial tensile strength, the concrete mechanical behavior is elastic linear, and for superior magnitudes, it is plastic with softening. The ultimate tensile strain ε_o may be given by the Kwak and Filippou [8] relationship:

$$\varepsilon_o = \frac{2.G_f.ln(3/L_e)}{f_{ct}.(3 - L_e)} \tag{6}$$

For which the L_e parameter is the finite element length represent the dimension and the G_f factor is the fracturing energy per unit area.

The concrete limit stresses are defined from the envelope of stresses proposed by Kupfer and Gerstle [9].

The analysis will be performed according to Desai and Siriwardane [10] in the incremental version :

$$\begin{vmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\tau_{12} \end{vmatrix} = \frac{1}{1-\nu^2} \begin{vmatrix} E_1 & \nu\sqrt{E_1.E_2} & 0 \\ \nu\sqrt{E_1.E_2} & E_2 & 0 \\ 0 & 0 & (1 - \nu^2).G \end{vmatrix} . \begin{vmatrix} d\varepsilon_{11} \\ d\varepsilon_{22} \\ d\gamma_{12} \end{vmatrix} \tag{7}$$

where the"Ei's" are the concrete deformation modules for each of the principal directions. Its transverse stiffness is expressed from:

$$(1 - \nu^2).G = 0.25\left(E_1 + E_2 - 2\nu\sqrt{E_1.E_2}\right) \tag{8}$$

The mass of concrete will be represented by the plane eight nodded elements, the so-called serendipities.

The steel bars mechanical behavior is idealized as a perfectly plastic elastic material and will be simulated from the one-dimensional finite elements.

3 Computational support

The results of support to the analysis of which approach this paper were obtained from the computational code called VIGEFNL drafted according to the FORTRAN automatic language and approximation by beam finite elements of two nodal points and two degrees of freedom by nodal point. The algorithmic pattern of that computational code includes a calculation structure adjusted to the Branson's model (1968) applied to the analysis of continuous beams consisting of reinforced concrete considering its nonlinear mechanical behavior and idealized from reticular bars. Such an implementation strategy culminates in computational effort savings.

The results obtained from the automatic program described above are compared to those ones, obtained by a computational code developed in FORTRAN language and finite elements approximation on a nonlinear orthotropic calculation structure, in plane state of stresses. The algorithmic scheme of this last program considers the nonlinear mechanical behavior of concrete including the formulations of the one-dimensional finite element and the plane finite element described in item 2.2 of this paper.

4 Studied specimens

Continuous beams of reinforced concrete with rectangular cross-section of 0.20 m width and 0.60 m height were analyzed, loaded by uniformly distributed load throughout its length, whose structural idealization is illustrated in Fig. 1, of physical and geometric characteristics detailed in Tab. 1. It was considered class C 30 or C 40 concretes, presenting Poisson’s ratio equal to 0.167. I. The reinforcement consists of CA-50 steel bars, whose Modulus of Elasticity is of 210000.0 MPa was fixed. The study was carried out over 12 cases summarized according to Tab. 1.

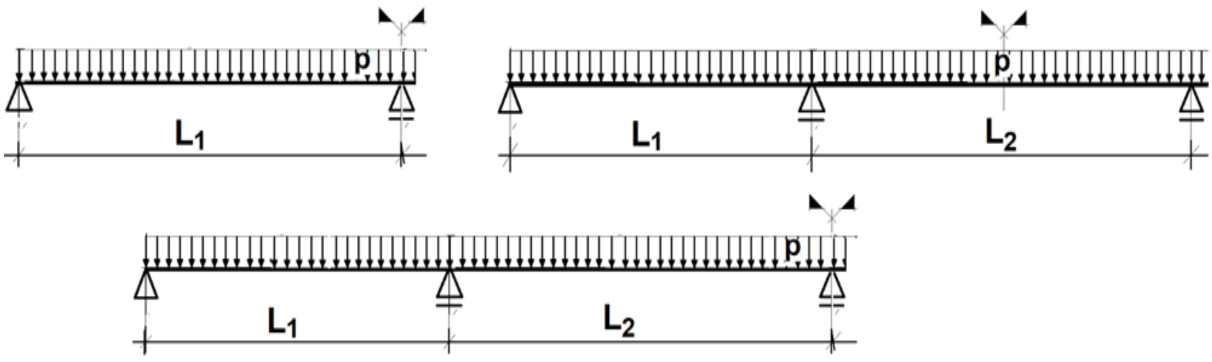

Figure 1. Continuous beams Structural idealization

Table 1 - Studied cases Characterization

Cases	f_{ck} (MPa)	Total of spans	L_1 (m)	L_2 (m)	g (kN/m)	q (kN/m)	A_S cm^2)
10	30	2	4,00	++++	5	13,5	2,14
11	40	2	4,00	++++	7	17	2,93
12	30	2	6,00	6,00	6	14	5,73
13	40	2	6,00	6,00	7	18	7,13
14	30	3	4,00	4,00	6	14	1,95
15	40	3	4,00	4,00	7	18	2,43
16	30	3	6,00	6,00	6	14	4,52
17	40	3	6,00	6,00	7	18	5,63
18	30	4	4,00	4,00	6	14	2,09
19	40	4	4,00	4,00	7	18	2,61
20	30	4	6,00	6,00	6	14	4,25
21	40	4	6,00	6,00	7	18	5,37

5 Results

According to the obtained results, for beams of two spans, the negative bending moments, considering the concrete nonlinear behavior, presented lower magnitudes than the ones corresponding to the material linear elastic condition, Fig. 2, registering a difference of the order of 8.4%, Tab. 2. In the other hand, the positive bending moments, were higher for the material in the nonlinear regimen presenting a difference from 5.2 up to 6.2%.

It should be emphasized, however, that significantly higher differences for the positive bending moments were registered for certain intermediate sections placed between the span center and the support, varying in the range by 15.2% for case number 10, to 24.5% for case number 12, Tab. 2.

For the remaining cases, the trend in terms of the differences mentioned in the previous paragraph was similar, Fig. 3 and 4, resulting in the percentages indicated in Tab. 2.

It should be emphasized, however, that for the beams of three and four spans, cases from 14 to 21, in relation to the lowest of the maximum positive bending moments, which occurs on the second span, the difference was significantly more pronounced than that one recorded for the largest among these maximum bending moments.

Considering the differences reported in the previous paragraphs, specifically, with regard to negative bending moments, if has been observed all the cases analyzed in this paper, the smallest of them was 3.8%, recorded for case number 17, referring to the beam of 3 (three) spans, each span 6.00 m length, cast in C40 concrete, Tab. 2. The largest difference, in turn, was 8.8%, indicated for case number 12, corresponding to the beam of 2 (two) spans, each of them 6.00 m length, cast in C30 concrete.

Regarding the highest positive maximum bending moments, the smallest of these differences, including all the cases analyzed, was 1.6%, Table 2, recorded for case number 14, represented by the beam of 3 (three) spans, each of them 4.00 m length, cast in C30 concrete. The largest difference, in turn, was 6.2%, indicated for case number 12, corresponding to the beam of 2 (two) spans, each of them 6.00 m length, cast in C30 concrete, too.

The smallest of the differences emphasized above, among all the cases analyzed for the positive bending moment in intermediate cross sections placed between the supports and the span center, was by 8.8%, Table 2, recorded for case number 18, represented by the beam of 4 (four) spans, each of them 4.00 m length, cast in C30 concrete. The largest difference, in turn, was 24.5%, indicated for case number 12, corresponding to the beam of 2 (two) spans, each of them 6.00 m length, cast in C30 concrete, too.

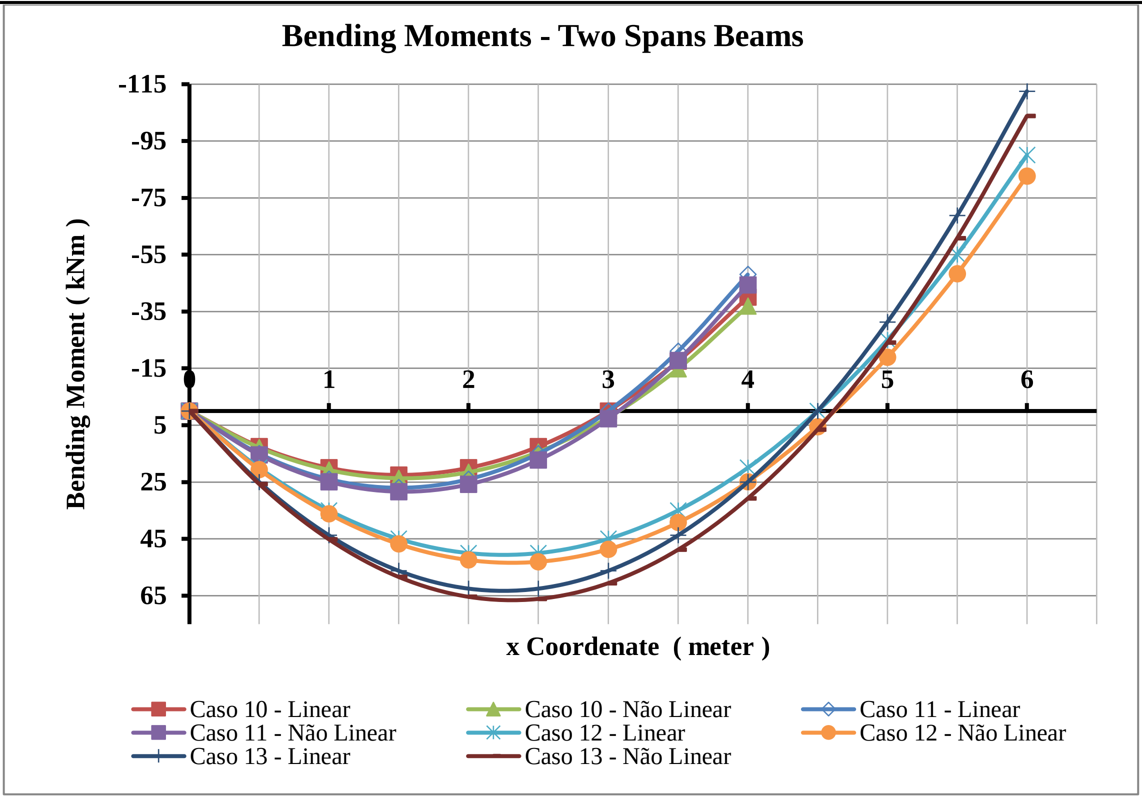

Figure 2. Bending Moments for two spans beams

Table 2. Bending moments

Case	Coord. (meter)	Bending Moment (kNm)			Case	Coord. (meter)	Bending Moment (kNm)		
		Linear	Nonlinear	Dif(%)			Linear	Nonlinear	Dif(%)
10	1.50	22.5	23.7	5.3	**16**	2.50	57.5	58.6	1.9
	2.50	12.5	14.4	15.2		6.00	-72.1	-69.4	3.9
	4.00	-40.0	-36.9	8.4		8.50	15.5	18.0	16.1
						9.00	18.0	20.6	14.4
11	1.50	27.0	28.4	5.2	**17**	2.50	71.9	73.3	1.9
	2.50	15.0	17.3	15.3		6.00	-90.0	-86.7	3.8
	4.00	-48.0	-44.3	8.4		8.50	19.4	22.6	16.5
						9.00	22.5	25.8	14.7
12	2.50	50.0	53.1	6.2	**18**	1.50	24.6	25.4	3.3
	4.00	20.0	24.9	24.5		2.50	16.0	17.4	8.8
	6.00	-90.0	-82.7	8.8		4.00	-34.3	-32.2	6.5
						6.00	11.0	11.7	6.4
13	2.50	62.5	66.1	5.8	**19**	1.50	30.8	31.9	3.6
	4.00	25.0	30.8	23.2		2.50	20.0	21.9	9.5
	6.00	-112.5	-103.8	8.4		4.00	-42.9	-40.0	7.3
						6.00	14.3	14.4	0.7
14	1.50	25.5	25.9	1.6	**20**	2.50	55.4	57.4	3.6
	4.00	-32.0	-30.8	3.9		6.00	-77.1	-72.2	6.8
	5.50	5.5	6.7	21.8		8.50	21.1	24.8	17.5
	6.00	8.0	9.2	15.0		9.00	25.7	29.2	13.6
15	1.50	30.0	30.7	2.3	**21**	2.50	69.2	71.6	3.5
	4.00	-40.0	-38.5	3.9		6.00	-96.4	-90.6	6.4
	5.50	6.9	8.4	21.7		8.50	26.3	30.7	16.7
	6.00	10.0	11.5	15.0		9.00	32.1	36.2	12.8

The smallest of the aforementioned differences found for the smallest of the maximum positive bending moment, including all the cases analyzed in this report, was by 0.7%, Tab. 2, recorded for case number 19, represented by the beam of 4 (four) spans, each span 4.00 m length, cast in C40 concrete. The largest difference, on the other hand, was by 15.0%, reported for cases number 14 and 15, corresponding to beams of 3 (three) spans, each span 4.00 m length, cast in C30 and C40 concrete, respectively.

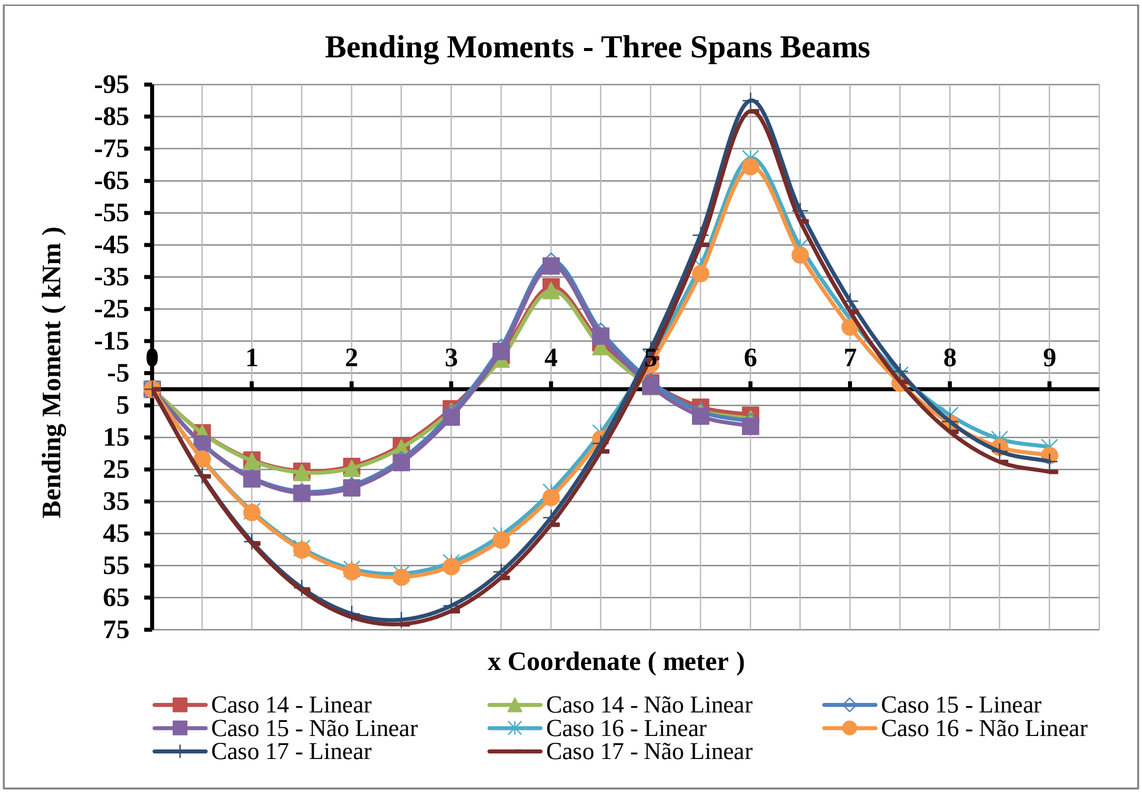

Figure 3. Bending Moments for three spans beams

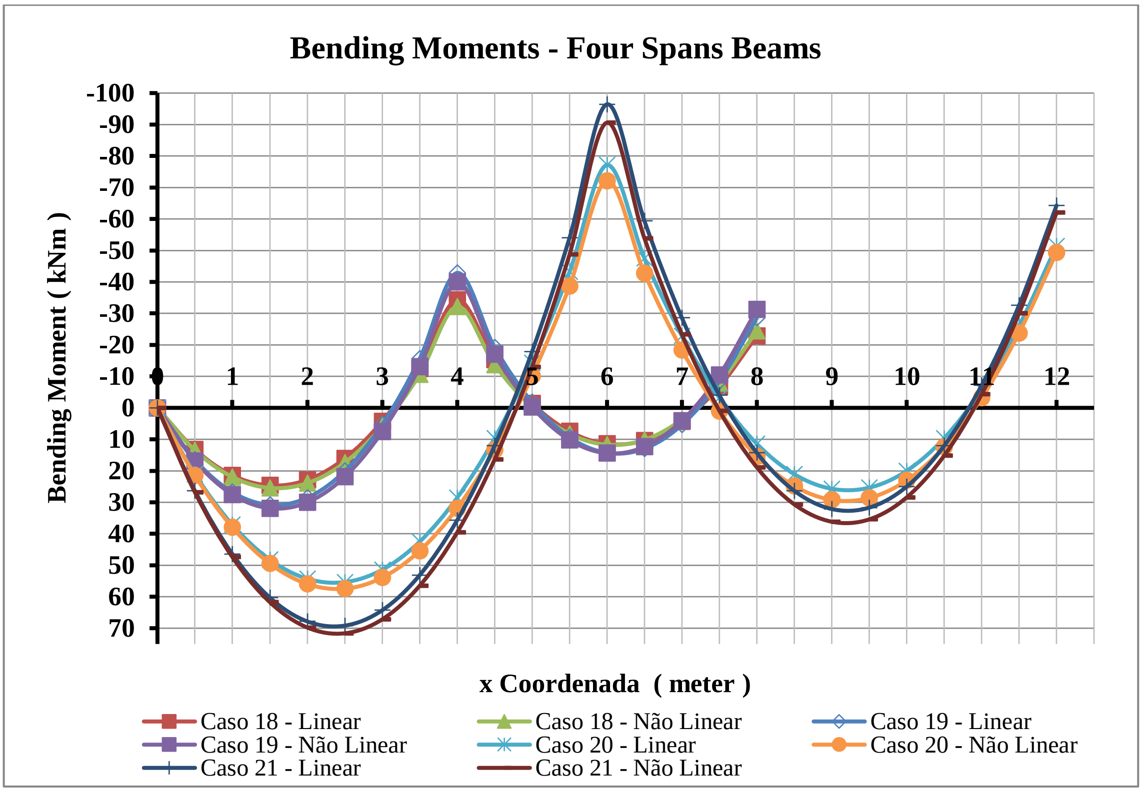

Figure 4. Bending Moments for four spans beams

6 Conclusions

This paper highlights the divergences between elastic linear constitutive modeling and plastic nonlinear modeling in the analysis of continuous beams cast in reinforced concrete.

Computational applications based on finite element approximation on Branson's nonlinear model (1968) were used for the purposes of achieving that subjective.

The results revealed that the negative bending moments referring to the nonlinear modeling presented lower magnitudes than those Arising from the linear version and that the smallest and the largest difference were 3.8% and 8.8%, found for beams of three and two spans, respectively, 6.00 m length.

The numerical simulation indicated higher values for the largest between the positive maximum bending moments obtained through nonlinear modeling and that the smallest and the largest difference were, respectively, 1.6% and 6.2%, recorded for beam of three spans 4.00 m length and for beam of two spans 6.00 m length.

The analysis revealed that, for the cases of beams of three spans 4.00 meters length, the difference in magnitude of the smallest between the maximum positive bending moments reached 15%.

As consequence of the differences in the magnitudes of bending moments reported in the previous paragraph, the consideration of the nonlinear behavior of reinforced concrete results in the needing to adopt positive tensile reinforcements of areas larger than those indicated by their linear modeling.

The verified differences in the positive Bending moments magnitudes, in certain intermediate cross sections between the center of the span and the supports, reached percentages of up to 24.5%, a fact that reveals substantial influence on positive tensile reinforcement distribution along the longitudinal direction of the beam.

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