Study of the relationship between fracture energy and concrete hydration degree through a finite element model

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Abstract. The fracture energy of concrete is an important physical parameter that describes its behavior under stress, and it is especially important for modeling concrete structures. As a consequence of the concrete stiffening process, the fracture energy will depend, among other factors, on the degree of hydration of the material. The development of a model that describes this process is important so that concrete behaviors can be predicted at different ages. A form of modeling widely used is the Finite Element Method (FEM), which consists of a numerical model to approximately determinate several physical behaviors. The inverse analysis through FEM starts from the knowledge of some given parameters, from the constitutive laws of a material and from physical results, to correlate a numerical model with similar result, in such a way that it is possible to determine the unknown parameters. The objective of this work is to obtain the relationship between the degree of hydration and the fracture energy of a concrete using the inverse analysis through its numerical modeling. Starting from the results of concrete bending tests, its modulus of elasticity and compression strength, fracture energy was obtained with reverse analysis through the FEM, using the DIANA software. The analysis was repeated for different degrees of hydration, thus establishing a correction between hydration of the concrete and fracture energy.

Keywords: Concrete; Fracture Energy; Hydration Degree; Finite elements.

1. Introduction

The numerical modeling of concrete is of great importance to ensure the safety and usefulness of a concrete structure, especially those with large volumes prone to thermal and shrinkage effects [1]. In this context, the study of concrete at early ages is essential to predict its behavior before hardening. Despite this, there are not many studies that relate the properties of concrete at early ages with its degree of hydration [2], thus providing more accurate data for this modeling. This work aims to fill this lack of knowledge, continuing the work of MOITA [3].

The relationship between the fracture energy and the degree of hydration of concrete were obtained from the inverse analysis made with numerical modeling. Starting from the results of concrete bending tests, its modulus of elasticity and compression strength, fracture energy was obtained with reverse analysis through the FEM, using the DIANA software.

The repetition of this analysis for different degrees of hydration led to a correlation between these two important parameters, in order to allow for more accurate numerical modeling of early age concrete in the future.

2. Experimental data

The inverse analysis of concrete in different degrees of hydration requires as input some characteristic data of the material such as compressive strength and modulus of elasticity. The analysis is verified using the force-displacement curve of the 4-point bending test. These data were obtained for different ages using a curing chamber coupled to an adiabatic calorimeter.

The adiabatic process is achieved by a concrete body isolated and submerged in a water bath in constant movement, both with a temperature measured by a thermometer [4]. At the beginning of the process, both have the same or close temperature, with the increase in the temperature of the concrete due to exothermic reactions, the bath has its temperature increased by electrical resistances connected to a controller that captures temperatures inside and outside the body.

A curing chamber, in which specimens of the same concrete are then submerged in water, is then connected to the calorimeter, and maintaining the same temperature as it is using a similar system of controllers and resistors. The specimens were removed from the curing chamber for bending tests in 4 ready and their temperature was noted to calculate the degree of hydration, as indicated by Fairbairn [1].

$$\propto(t) = \frac{\Delta T_{ad}(t)}{\Delta T_{ad,max}}$$

In equation (1), $\propto(t)$ is the degree of hydration at a time t, $\Delta T_{ad}(t)$ is the adiabatic temperature at a time t, $\Delta T_{ad,max}$ is the maximum adiabatic temperature rise corresponding to the end of the calorimetric test when variations of $\Delta T_{ad}(t)$ are no more measured within the sensibility of the calorimeter.

The flexion test was performed with 4 points in beams of 50x50x230 mm, 16 cm apart on the lower face and 6 cm on the upper face. The test was carried out on a Shimadzu machine model AG-X with a maximum load of 100 kN. The speed in the tests was 0.3 mm/min. Displacement was measured from an LVDT in the center of the underside of the specimen.

The other physical characteristics (compressive strength and modulus of elasticity) of the material were obtained from subsequent tests carried out by Moita [3] and presented in the figures below.

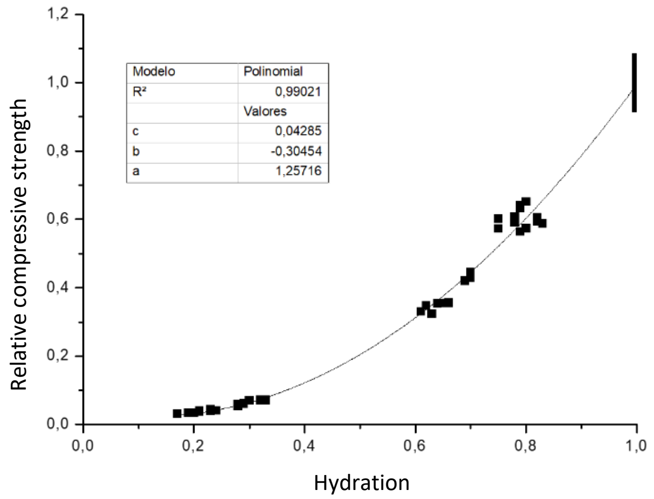


Figure 1. Relation between degree of hydration and relative compressive strength

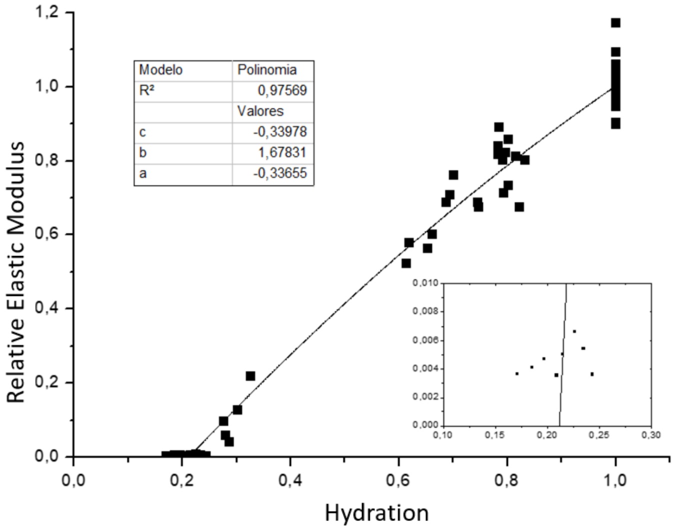


Figure 2. Relation between degree of hydration and relative elastic modulus

3. Inverse analysis through finite element model

The finite element model allows to simulate the behavior of the concrete beam used at the experimental test with considerable precision. For achieving this, it is necessary to suplly the model with real values of the physical parameters of the concrete and then extract a result, which can be a graph that describes its deformation under specified forces. The expected result is a very similar curve to the experimental curve.

The inverse analysis consists in the procedure for determine the value of unknown parameter by comparing the results of model with the result of an experimental test. It consists of changing the input value of the unknown parameter and comparing the results until you have the same effect, as shown by Figure 3.

Starting from some parameters provided by the experimental test, as the compression strenght (fc), tension strenght (ft) and modulus of elasticity (E), considering the physical behavior (constitutive law of the material), and estimating an initial value for fracture energy (Gf), the model effect is a displacement curve of the tensioned beam. This curve is compared with a same type of curve obtained from the real experimental test. The change of the fracture energy value in the model is performed repetitively until the resulting curves match.

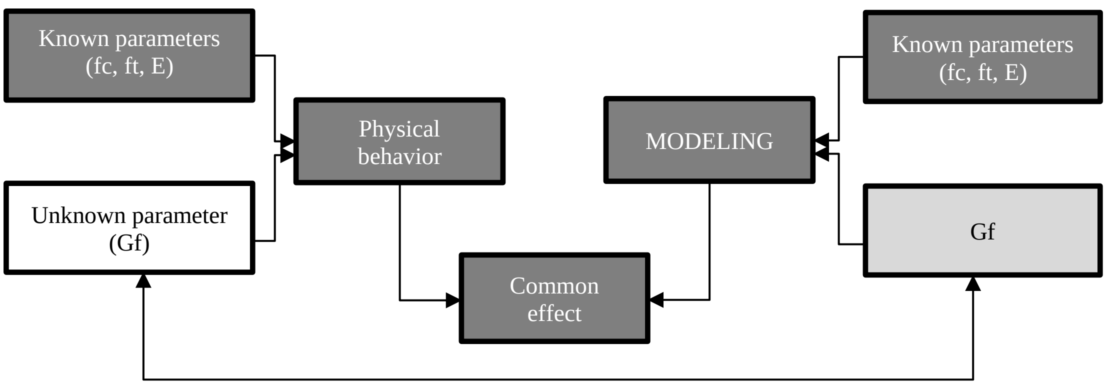
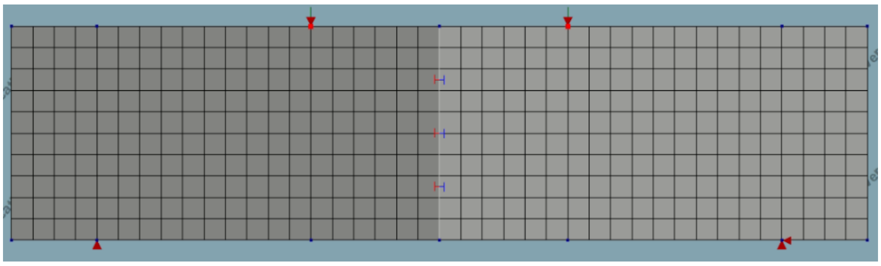


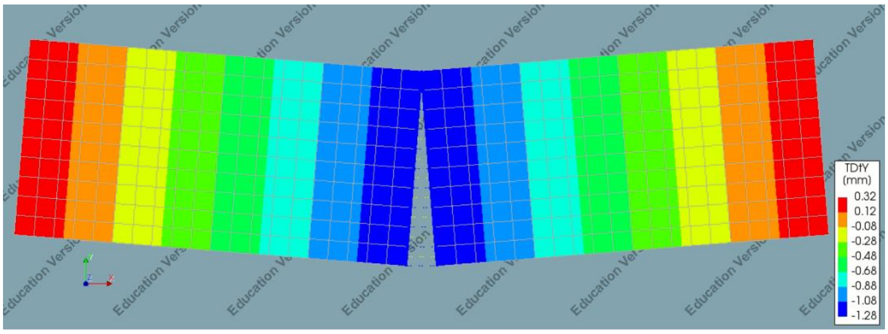
Figure 3 - Inverse analysis procedure

For the finite element modeling, it was used the DIANA FEA software [5]. For obtaining the numerical value of the beam displacement caused by the discrete crack opening during the experimental test described on the previous section, a rectangular mesh with a central interface element was used for modeling the beam. The material parameters were configured according to the inverse analysis procedure, as shown by Figure 4.

Because of the interface element, the phenomenon is concentrated in this midle section of the beam, exactly on the interface element, and the measurement can be easier done by reading the total displacement of the central node.



(a)



(b)

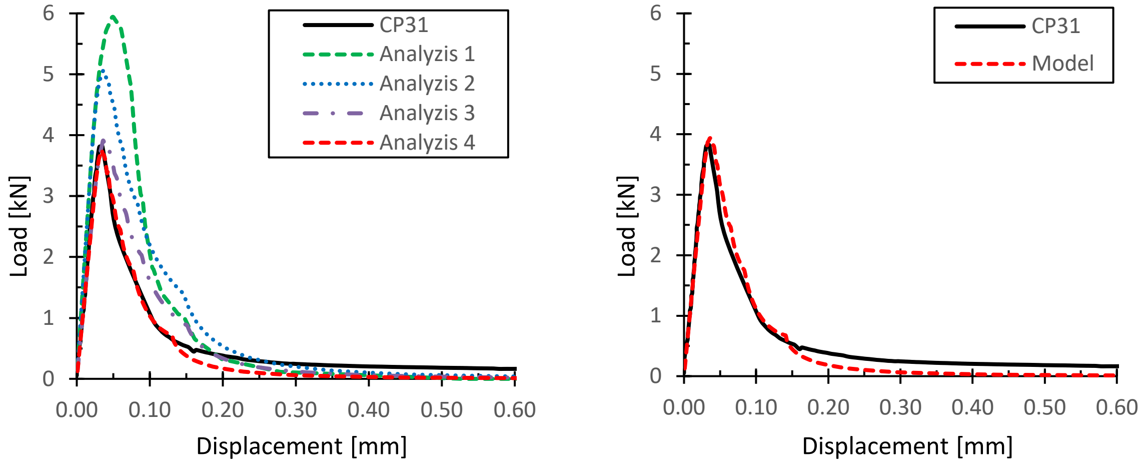
Figure 4 - Modeled beam. (a) model with a central interface element. (b) model at the end of the load, with measured displacement

The beam was modeled using two different materials, one for the sample body and another for the interface element. The body of the beam was configured as a concrete with a linear elastic behavior, because in this way the deformation will be null in the concrete body outside the interface element in the last stage of loading, when the crack is completely open. Thus, it is guaranteed that the cracking mode will be ideally discrete. On the other hand, the interface element material must be configured with the crack opening parameters as normal stiffness modulus, tensile strength, and fracture energy, which were extracted from the experimental tests.

A predefined tension softening function was chosen for the interface element, which represents the physical behavior described on Figure 3. The selected Tensile Stress-Strain Curve was Hordijk [6]. The nonlinear behavior of the interface element is important to ensure the better fit of the load vs. displacement curve in comparison to the experimental result.

By comparing the load vs. displacement curves between the model and the experimental test, it was possible to analyze for each sample its fracture energy that produced a similar result. For each hydration degree, one or more samples were analysed.

The Figure 5 shows the results of this procedures. One analyzis was done for each set of values of the concrete parameters, especially the fracture energy. According to the shape of the resultant curve, the fracture energy value was changed in the software until the optimized shape was obtained. As it can be saw, this procedure was individually done for each sample from the flexion test.



(a) (b)

Figure 5 – Example of procedure for obtaining the best load vs. displacement curve fit comparing model and experimental result. (a) several analyses. (b) best fit of the model

4. Relationship between degree of hydration and fracture energy

After modeling each sample from experimental data and determining its fracture energy by the described procedures, the results of each sample are grouped according to its hydration degree and shown through the Table 1. The table respectively shows the hydration degree, the sample tag, the experimental data (tensile strength(ft), compression strength(fc), elastic modulus(E)), the FEM results from the optimized curves (tension strength and fracture energy), the fracture energy according to the fib model code equation and finally the variation between FEM and model code Gf values.

Table 1 - Experimental and modeling data.

Hydration	Sample	Experimental Data			FEM		fib Model Code	Variation
[%]		Ft [N/mm²]	Fc [MPa]	E [GPa]	Ft [N/mm²]	Gf [N/mm]	Gf [N/mm]	[%]
100.00%	CP35	3.65	32.20	22.22	3.7	**0.120**	0.14	14.3%
100.00%	CP34	3.56	32.20	22.20	3.5	**0.070**	0.14	50.0%
100.00%	CP33	3.62	32.20	22.22	3.7	**0.115**	0.14	17.9%
100.00%	CP32	3.79	32.20	22.20	3.7	**0.120**	0.14	14.3%
100.00%	CP31	3.80	32.20	22.20	3.8	**0.080**	0.14	42.9%
100.00%	CP30	4.24	32.20	22.20	4.0	**0.080**	0.14	42.9%
80.27%	CP29	3.05	19.60	17.57	2.9	**0.110**	0.12	11.8%
79.72%	CP28	2.81	19.29	17.43	2.8	**0.080**	0.12	35.7%
79.44%	CP27	2.86	19.14	17.36	2.5	**0.070**	0.12	43.6%
79.16%	CP16	2.85	18.99	17.29	2.8	**0.080**	0.10	20.0%
68.70%	CP24	2.32	13.75	14.54	2.2	**0.050**	0.11	54.5%
66.10%	CP15	2.20	12.59	13.84	2.2	**0.070**	0.11	36.4%
63.30%	CP13	2.10	11.41	13.07	2.2	**0.045**	0.11	59.1%
45.83%	CP19	1.17	5.41	7.97	1.0	**0.040**	0.11	63.6%
44.16%	CP18	1.02	4.96	7.46	0.9	**0.013**	0.10	87.0%
43.33%	CP23	1.12	4.75	7.21	1.2	**0.015**	0.11	86.4%
42.22%	CP22	0.97	4.48	6.86	1.0	**0.020**	0.11	81.8%
41.39%	CP21	0.97	4.48	6.86	1.0	**0.011**	0.11	90.0%
31.19%	CP12	0.46	2.30	3.35	0.6	**0.008**	0.10	92.0%
27.97%	CP10	0.40	1.85	2.30	0.4	**0.007**	0.08	91.3%

As a reference value, the analytical value for the fracture energy was calculated according to the equation 1 ,obtained from the *fib* Model Code for Concrete 2010 [7], whose f_{cm} is the mean compressive strength of the concrete.

$$G_F = 73 * f_{cm}^{0,18} \tag{1}$$

Analyzing the Table 1 we can see that the values of the crack energy increase with the hydration degree both for FEM and model code results. Although, the order of magnitude seems to be discrepant principally at the first ages of the concrete, where the variation values are higher.

The discrepancy of these two cases is even more evident if the evolution curve Gf vs. Hydration is analyzed for both cases (Figure 6). According to the tendency lines on the graph, while the pattern of growing for Gf by the model code equation is approximately linear, the pattern presented by the inverse analysis is nonlinear. Although, the order of magnitude in both cases are similar for high hydration degrees.

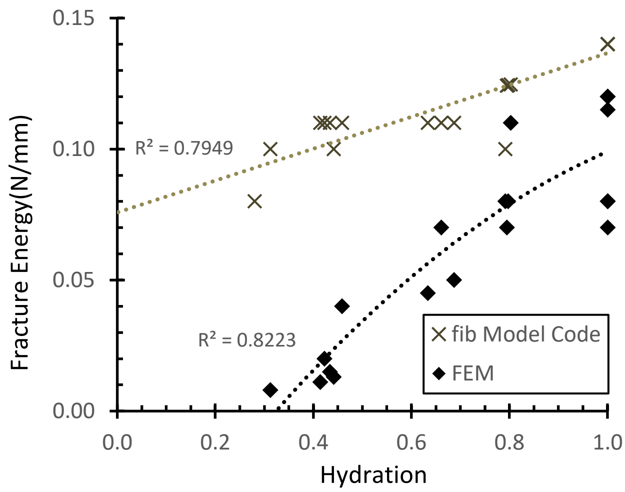


Figure 6 - Evolution curve for the fracture energy of the concrete according to the hydration degree. Comparison between FEM and Model Code results

5. Conclusions

The inverse analysis seemed to be a good method for obtaining the fracture energy of the concrete with a reasonable precision. This method proved to be reasonable as the coefficient of determination of the curve (R^2), shown by Figure 6, remained in the order of 0.82, even greater than the approximation foreseen in the model code, which resulted in the order of 0.79.

The growing pattern of the concrete fracture energy seems to be nonlinear, initially equal to zero from the hydration degree about 0.3. This result is more consistent than the model code equation prevision because the fracture energy should not be different from zero at early ages, when it has not yet acquired any resistance.

Besides that, the fact that these two trends become closer as the degree of hydration approaches 1 can be an evidence that, after hardening, the finite element model will coincide with the predicted analytical results.

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