A MULTI-SCALE MIXED METHOD FOR A TWO-PHASE FLOW IN FRACTURED RESERVOIRS CONSIDERING PASSIVE TRACER

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Abstract In this research, the mathematical model represents a two-phase flow in a fractured porous reservoir media, where the Darcy law represents the flow in both fractures and matrix. The flux/pressure of the fluid flow is approximated using a hybridized mixed formulation coupling the fluid in the volume with the fluid flow through the fractures. The spatial dimension of the rock matrix is three and and is coupled with two-dimensional discrete fractures. The transport equation is approximated using a lower order finite volume system solved through an upwind scheme. The C++ computational implementation is made using the NeoPZ framework, an object oriented finite element library. The generation of the geometric meshes is done with the software Gmsh. Numerical simulations in 3D are presented demonstrating the advantages of the adopted numerical scheme and these approximations are compared with results of other methods.

Keywords: Reservoir Simulation, Mixed Finite Elements, Multi-scale Finite Elements, Discrete Fracture

1 Introduction

Strongly motivated by the force of industries such as hydrocarbon reservoirs exploration, the modeling of fractured porous media has been a consistently growing object of research within the computational mechanics community. In general, such research efforts are stimulated by the characteristic of fractures representing lower-dimensional discontinuities in the porous medium, which introduce secondary permeability that significantly affects its flow patterns.

Discrete fracture models (DFMs) idealize fractured media by emphasizing realistic fracture geometries [1], which contrasts with the other popular alternative of dual-porosity models. DFMs propose to model fractures explicitly as (d-1)-dimensional elements in a d-dimensional permeable domain [2–5]. They attempt to overcome the shortcomings of conventional dual-continuum models, which are evident, for example, in the presence of a small number of large-scale fractures [6].

Considering the distribution uncertainty and huge variability of natural fractures in formations, combined approaches have been developed to reduce the prohibitive computational cost caused by direct simulation of small-size fractures [7]. In these frameworks, continuum-based methods cooperate with discrete fracture methods so each can tackle the part of the problem for which they are designed, which motivates the exploration of both methodologies separately to further improve their joint application.

In the context of Mixed Finite Element (MFE) formulations for Darcy flow in porous media, the purpose of the introduction of a multi-scale hybrid strategy is to cope with the complex geometry and largely heterogeneous nature observed for mega-scale reservoir modeling problem. The simulation is divided into macro domains that facilitate the use of the hierarchy of meshes and approximation spaces. Methods like the Multi-scale Hybrid Mixed method (MHM) make use of coarse-scale normal fluxes between subregions and resolve fine-scale features inside each subregion through the solution of completely independent local Neumann problems, whose boundary conditions are set by the trace variable. The so-called MHM-H(div) method [8–10] adopts the MFE formulation of the local Neumann problems, using flux and pressure representations by divergence-compatible FE pairs based on refined meshes inside the sub-regions. Thus, it has been shown by [9] that relevant aspects of MFE methods for subsurface applications are inherited by the MHM-H(div) method

The success of this approach is mainly due to some relevant properties valid for these methods. For instance, it is well recognized that local mass conservation, continuous fluxes, and strong divergence-free enforcement for incompressible flows, which are crucial aspects for simulations based on realistic reservoir geology, are naturally verified by MFE methods. [11]

In this direction, the goal of this work is to construct multi-scale mixed finite element approximations for fractured reservoir simulations. This work builds upon the numerical framework given by [10] by taking simpler and more limited hypotheses for the transport model, and by introducing the coupling of flow through discrete fractures. The MHM finite element construction for the pressure-flux system is discussed in detail by [3] for 2D problems, from where we derive our tridimensional implementation.

2 Mixed Finite Element Formulation

The physical process of subsurface flow considered for our choice of reservoir simulations is the isothermal, incompressible flow, with neglected capillarity effects and rigid rock formation, characterized by Darcy's law. A steady-state flow with the injection of a linear passive tracer for the transported saturation. The relevant variables are the total fluid flux $\mathbf{u}$, the pressure field p, and the oil saturation c. The equidimensional mixed problem, following that chosen by [12], is to find the triple $(\mathbf{u}, p, c)$ such that:

$$\begin{cases} \mathbf{K^{-1}u} + \nabla p = \mathbf{0} & \text{in } \Omega \\ \nabla \cdot \mathbf{u} = 0 & \text{in } \Omega \end{cases} \tag{1}$$

$$\phi \frac{\partial c}{\partial t} + \nabla \cdot (c\mathbf{u}) = 0 \qquad \text{in } \Omega \times [0, T] \tag{2}$$

Where Ω represents the spatial domain, $[0, T]$ is the time domain, $\mathbf{K}$ is the rock permeability tensor assumed symmetric and positive definite, and ϕ is the porosity of the medium. Such a system is completed by the boundary conditions for pressure and the normal component of the flux:

$$\begin{aligned} \mathbf{u} \cdot \mathbf{n} &= 0 \quad && \text{on } \partial\Omega_N \\ p &= p_D && \text{on } \partial\Omega_D \end{aligned} \tag{3}$$

together with the boundary injection of the transported saturation and its initial state distribution:

$$\begin{aligned} c &= c_{in} \quad && \text{on } \partial\Omega_{in} \\ c(\mathbf{x}, t{=}0) &= c^0 && \text{in } \Omega \end{aligned} \tag{4}$$

With the goal of reducing the size of the global algebraic system, our method of choice (MHM-H(div)) takes a 2-scale discretization of the computational space. It groups the fine elements, and its interface normal fluxes are the global degrees of freedom. The fine mesh is a partition of each coarse element, and its degrees of freedom are viewed as internal to the coarse elements and, as such, condensed into the global algebraic system without affecting its size. Discrete fractures are two-dimensional elements that exist as the interface between fine elements, and thus are part of the fine mesh.

Following the methodology introduced by [9, 13] we approximate the interior mesh problem over an H(div) conforming space, which allows us to carry the multi-scale computations by restricting the fine-scale shape functions to the space of macro fluxes at the interface of macro-regions. Thus, the variational formulation for the MHM-H(div) method matches that of a mixed formulation over an H(div) space. The added feature is that in the multi-scale setup the flux approximation space is partitioned between macro fluxes associated with the boundary of the macro domains and internal fluxes and pressures are associated with the interior [3].

As such, the starting point for our model is the variational mixed problem for the tridimensional porous matrix. For that, we take the following approximation spaces:

$$\begin{aligned} \mathscr{V}(\mathcal{T}_d) &\subset H(\text{div}, \Omega_d); \\ \mathscr{P}(\mathcal{T}_d) &\subset L^2(\Omega_d) \end{aligned} \tag{5}$$

piece-wise defined over a hybrid, conformal at each discretization level, finite element mesh $\mathcal{T}_d$ (d=2 for fractures and d=3 for porous medium); Here, the the inner-product spaces $H(\text{div}, \Omega)$ and $L^2(\Omega)$ hold their usual meaning, and the divergence-compatibility property is verified $\nabla \cdot \mathscr{V} \subset \mathscr{P}$. Detailed constructions of such spaces can be found in [14].

Representing by $(\cdot, \cdot)_{\mathcal{R}}$ the $L^2(\mathcal{R})$ inner-product, and omitting the integration domain whenever it covers the complete domain of the test functions (e.g. $\mathcal{R} = \mathcal{T}_3$) with no ambiguity risked, and using $\langle \cdot, \cdot \rangle_{\partial \mathcal{R}}$ for the inner-product at the trace space over element boundaries or interfaces. The mixed problem that thus governs the pressure-flux system for the porous rock matrix follows:

Find the pair $(\mathbf{u}_3, p_3) \in \mathscr{V}(\mathcal{T}_3) \times \mathscr{P}(\mathcal{T}_3)$ such that:

$$\begin{aligned} (\mathbf{K}_3^{-1}\mathbf{u}_3, \boldsymbol{\psi}_3) - (p_3, \nabla \cdot \boldsymbol{\psi}_3) + \langle \boldsymbol{\psi}_3 \cdot \mathbf{n}, p_2 \rangle_{\Omega_2} &= - \langle \boldsymbol{\psi}_3 \cdot \mathbf{n}, p_{D2} \rangle_{\partial \Omega_D} \quad && \forall \boldsymbol{\psi}_3 \in \mathscr{V}(\mathcal{T}_3) \\ -(\varphi_3, \nabla \cdot \mathbf{u}_3) &= (\varphi_3, f_3) && \forall \varphi_3 \in \mathscr{P}(\mathcal{T}_3), \end{aligned} \tag{6}$$

where we use the subscripts 3 and 2 to discriminate the tridimensional porous rock domain from the bi-dimensional surface of the fractures respectively.

The system is completed by the variational constitutive and conservation laws for the pressure-flux pair within the fracture surface:

Find the pair $(\mathbf{u}_2, p_2) \in \mathscr{V}(\mathcal{T}_2) \times \mathscr{P}(\mathcal{T}_2)$ such that:

$$\begin{aligned} (\mathbf{K}_2^{-1}\mathbf{u}_2, \boldsymbol{\psi}_2) - (p_2, \nabla \cdot \boldsymbol{\psi}_2) &= - \langle \boldsymbol{\psi}_2 \cdot \mathbf{n}, p_{D_1} \rangle_{\partial \Omega_{2,D}} \quad && \forall \boldsymbol{\psi}_2 \in \mathscr{V}(\mathcal{T}_2) \\ -(\varphi_2, \nabla \cdot \mathbf{u}_2) + \langle \varphi_2, \mathbf{u}_3 \cdot \mathbf{n} \rangle_{\Omega_2} &= 0 && \forall \varphi_2 \in \mathscr{P}(\mathcal{T}_2) \end{aligned} \tag{7}$$

For the saturation transport we select a element-wise constant functions space $\mathscr{P}_0(\mathcal{T}_d)$. The resulting discrete finite volume formulation considers a linear passive tracer, which is computed through a first-order upwind scheme:

Find the pair $(c_3^{n+1}, c_2^{n+1}) \in \mathscr{P}(\mathcal{T}_3) \times \mathscr{P}(\mathcal{T}_2)$, such that:

$$\left(\phi_3 c_3^{n+1}, v_3\right) + \Delta t \langle c_3^{n+1} \mathbf{u}_3 \cdot \mathbf{n}, v_3 \rangle_{\Gamma_3} = (\phi_3 c_3^n, v_3) - \Delta t \langle c_{in} \mathbf{u}_3 \cdot \mathbf{n}, v_3 \rangle_{\partial \Omega_{3,in}} \qquad \forall v_3 \in \mathscr{P}_0(\mathcal{T}_3) \tag{8}$$

$$\left(\varepsilon_2 \phi_2 c_2^{n+1}, v_2\right) + \Delta t \langle c_2^{n+1} \mathbf{u}_2 \cdot \mathbf{n}, v_2 \rangle_{\Gamma_2} - \Delta t \left(\sum \tilde{c}^{n+1} (\mathbf{u}_3 \cdot \mathbf{n}), v_2 \right) = (\varepsilon_2 \phi_2 c_2^n, v_2) \qquad \forall v_2 \in \mathscr{P}_0(\mathcal{T}_2), \tag{9}$$

where ε_2 is the fracture aperture, the superscript n enumerates the time-step, and we have represented the (d-1)-dimensional-interface between d-dimensional-volumes by Γ_d. Again, subscripts 2 and 3 were used to distinguish between parameters like porosity (ϕ_d) for the fracture and the porous rock. The upwind saturation $\tilde{c}$ is evaluated based on the orientation of the flux:

$$\tilde{c} = \begin{cases} c_3 & \text{if} \quad \mathbf{u}_3 \cdot \mathbf{n}|_{\Gamma_3} > 0 \\ c_2 & \text{if} \quad \mathbf{u}_3 \cdot \mathbf{n}|_{\Gamma_3} < 0 \end{cases} \tag{10}$$

These equations, in their essence, directly follow from the usual equi-dimensional presentation of the fluid flow in porous media given in Equations (1) and (2). The coupling of the multidimensional flows in between rock and fracture is realized by observing the reciprocal conservation of mass: Where there is fluid exchange from the rock to the fracture, flux that leaves the tridimensional domain enters as a source term for the bidimensional conservation law; Reciprocally, the pressure of the fluid in the fracture surface acts as a boundary pressure for the porous matrix fluid flow.

3 MHM-H(div) Computations

The MHM method is a technique developed towards the numerical approximation of partial differential equations whose solutions exhibit multi-scale features. Within the MHM framework, the normal component of the flux over the macro elements is approximated by piecewise continuous functions. The extension of these flux functions in the interior of the macro domains constitutes the MHM basis functions [3].

Mixed finite element approximations in H(div) space have been widely studied in previous articles such as [15–17]. The Multi-scale Hybrid Mixed (MHM) method has also been studied in [3, 9, 10].

In general terms, the MHM-H(div) technique consists of the following steps:

1. Define a skeleton (red lines in Figure 1) that bound each of the macro domains.
2. In the skeleton of the coarse-scale, a macroscopic linear flux is computed (red lines in Figure 1).

3. The associated degrees of freedom with the flow of the fine-elements that are in the limit of two macro elements (dashed lines) are duplicated in such a way that they exist in both macro elements, and, in turn, those micro fluxes are restricted to the macroscopic flow.
4. The global problem is solved (computation of macro-flows and a constant pressure per macro-element).
5. The internal pressures and flows are computed considering the solution of the macroscopic flows as boundary conditions.

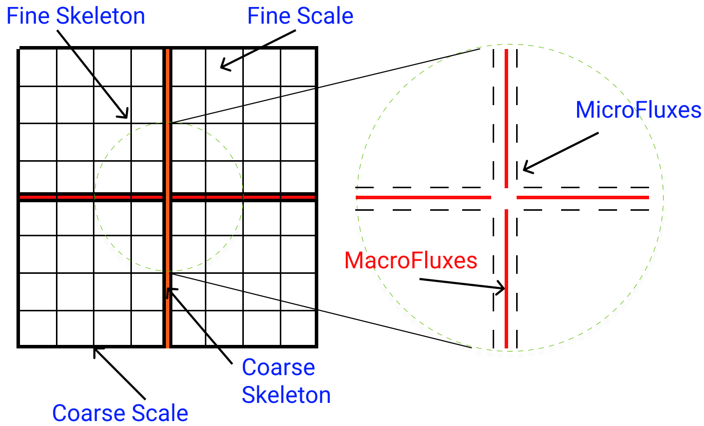


Figure 1. Fine and Coarse mesh

For the modeling of flow through fractures using the MHM technique, the fractures must be contained in macro elements. In other words, each fracture element must be an interface between 2 fine elements that belong to the sub-mesh of the same macro element (Figure 2). This implies that fractures cannot occupy the plane of the skeleton in the interface between two macro elements. We note, however, that this is a critical case that should be dealt with on follow-up papers.

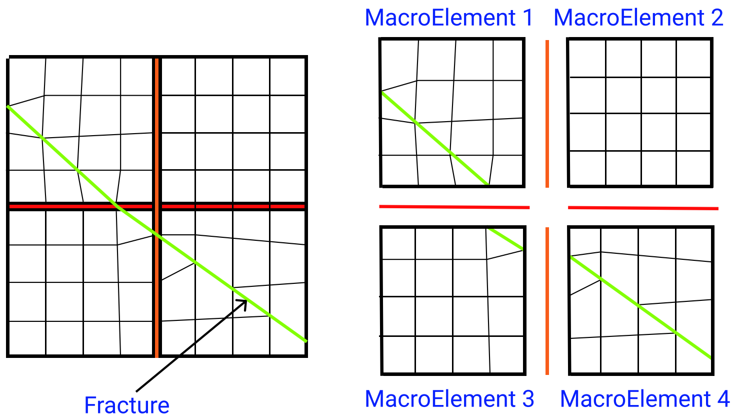


Figure 2. Fine and Coarse mesh with fractures

4 Results

In complex cases, the flow through fractures does not have an exact solution. Mainly for this reason, to validate the implementation of this method, we will use the results of the Benchmark Case1 defined by [12] in which the authors participate with results of the HDiv method.

4.1 Benchmark case 1 definition

For our chosen Benchmark academic reservoir, characteristics are attached in Table 1. Boundary conditions are set as $p_{D,in} = 4$m and $p_{D,out} = 1m$ in the regions defined in the problem illustration in Figure 3. The rest of the boundary observes a zero-flux condition. The 2-scale mesh used for computations is illustrated in Figure 4. In Figure 4b, a clip of the mesh is given so the interior and conformity with fracture elements are visible.

Part of the reservoir	Permeability$[m^2]$	Porosity
ResSup1	10^{-6}	0.2
Frac	Normal 10^{-3} ,Transversal 20.0	0.4
ResSup2	10^{-6}	0.2
ResInf	10^{-5}	0.25

Table 1. Reservoir and fracture properties

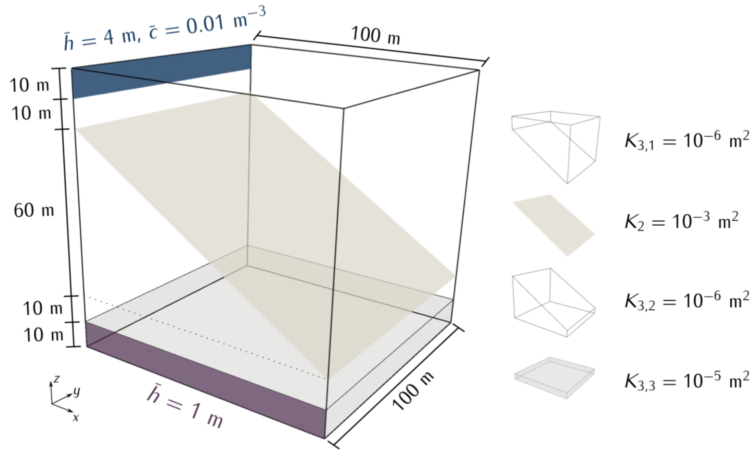
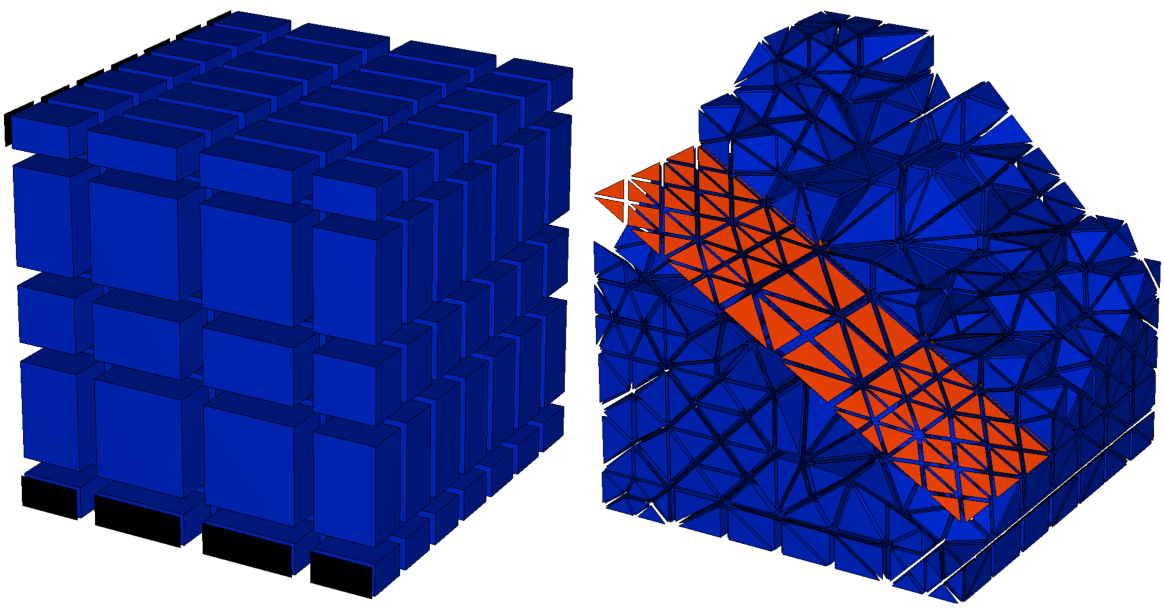


Figure 3. Benchmark as defined by [12]



(a) Coarse mesh (b) Cut-through of unstructured fine mesh

Figure 4. Computational domain setup for MHM-H(div) simulation

4.2 MHM-H(div) Results

Figure 5 shows the pressure along the line joining the points (0,100,100) and (100,0,0). Figure 6 shows the saturation along the same line and Figure 7 shows a graph of the saturation in the fracture over time (in years). To carry out these simulations, a coarse mesh with 140 subdomains (Figure 4a) and approximately 5400 unstructured elements on the fine-scale (4b) has been generated. Asymptotically, if we were to increase the number of subdomains, the solution should tend towards the H(div) solution; This expected result is due to elimination of flow restrictions, imposed as one of our multi-scale features.

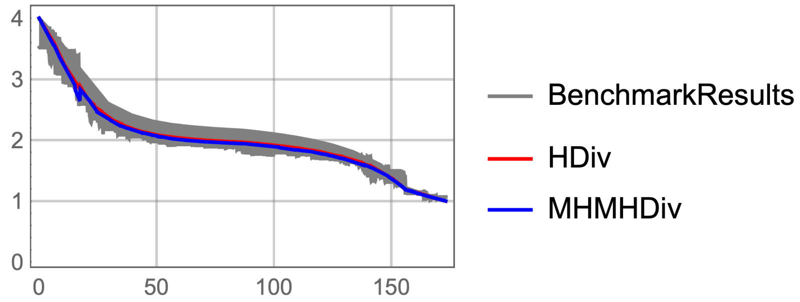


Figure 5. Pressure comparison

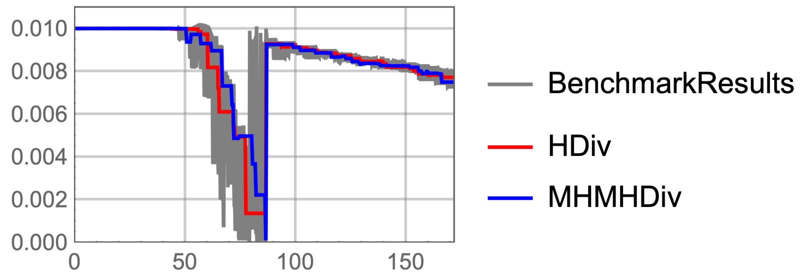


Figure 6. Comparison of saturation

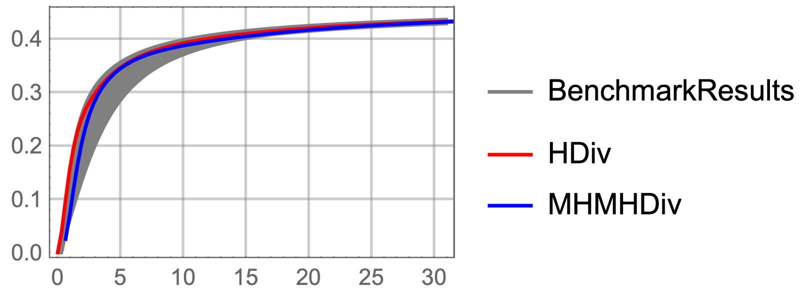


Figure 7. Comparison of saturation in the fracture over time

5 Conclusions and Outlook

For the example case described in Section 4, employing the MHM-H(div) method, in comparison to the classical H(div) method, reduces the number of equations in the global system from 226470 to 1678. This expressive reduction is further commendable when we account for the highly parallelizable content introduced by the restriction to sub-regions. It amounts to an observed decrease in resolution time from 8 minutes down to 3 seconds (using the same machine).

The solution obtained with MHM-H(div) is within the range of results obtained in the Benchmark, which indicates that the quality of the approximation is acceptable for the proposed problem.

On the other hand, the pre-processing necessary to create the MHM data structure is considerably more complex. This is especially true in accounting for the restrictive nature of discretely representing fractures within a predetermined coarse mesh. Indeed, a related paper from the authors has been published discussing solutions on the topic [18]. Nonetheless, the pre-processing effort is clearly justified by the return in computation time.

Follow-up publications should address resolution time gained for non-linear problems. Early (unpublished) results indicate even more potential for improvements due to optimization of the several systems of equations, which must be solved sequentially, inherent to these types of problems.

The terms which introduce the consideration of fracture intersections were omitted from the numerical formulation for simplicity since the benchmark example taken contains only a single fracture. To account for intersections, however, not many modifications are required to the numerical framework as it is here presented. This is to be achieved, mainly, through the introduction of a Lagrange multiplier with the physical interpretation of the pressure along fracture intersections, with a conservation law that takes the sum of normal fluxes to zero at these domains. For more details, the reader is referred to [3] and [12].

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