A COMPARISON OF OPTIMIZATION ALGORITHMS FOR THE PRE-SIZING OF REINFORCED CONCRETE STRUCTURES

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Abstract. Structural engineering is an opportune field for numerical optimization as it pursuits economical designs that comply with safety and usability requirements. The search for the dimensions of the cross-sections of beams and columns is a major step on the design of reinforced concrete frames, which will implicate greatly on the stiffness of the structure and its displacements. Then, with the objective of enhancing the structural design process, this work analyzes three algorithms to optimize the problem of obtaining a minimum concrete volume while complying with the stability criteria of small displacements, imposed by the Gamma-Z parameter. The algorithms for the interior point method, active set and sequential quadratic programming are briefly discussed and their implementations in the pre-sizing of concrete buildings are compared in terms of efficiency and quality of the solution by several numerical simulations.

Keywords: Optimization algorithms; Structural analysis and design; Gamma-Z stability parameter.

1 Introduction

Solving an optimization problem is trying to find the best viable solution for a given system through mathematical procedures, which use numerical algorithms. In general, the algorithm choice is a key feature in the optimization process, in order that an inadequate choice can seriously increase time and computational effort to get a final solution.

In recent years, a large quantity of black box optimization programs emerged, containing different subroutines of optimization. In view of this situation, it is necessary that researchers, during formulation of their problems, explore the performance of these subroutines as to avoid future problems for inadequate application.

On these grounds, this paper engages in a performance analysis of three deterministic algorithms (Active-Set, Interior Point and SQP) found in a black box optimizer, when used to solve a structural pre-sizing problem. The features analyzed are efficiency and reliability of solutions, characteristics used in other similar works like Beiranvand et al [1], Agnarsson et al [2] and Liu et al [3], to compare algorithms.

2 Optimization Algorithms

According to Arora [4], a constrained nonlinear optimization problem can be defined like:

$$minimize\ f(\boldsymbol{x})$$

subject to:

$$\begin{aligned} h_i(\boldsymbol{x}) &= 0, \qquad i = 1,2,3,\dots,z \\ g_n(\boldsymbol{x}) &\leq 0, \qquad n = 1,2,3,\dots,o \end{aligned} \tag{1}$$

where $\boldsymbol{x}$ is the vector of design variables, $f(\boldsymbol{x})$is the objective function, $h_i(\boldsymbol{x})$gathers the equality constraints, $g_n(\boldsymbol{x})$ the inequality constraints, which include the upper and lower bounds for the design variables.

In order to solve Eq. (1), the three methods utilized distinguish in relation to how they handle the objective function and constraints. The main differences between the three algorithms, Active-Set, Sequential Quadratic Programming (SQP) and Interior Point, are presented below.

2.1 ACTIVE-SET

According to Nocedal and Wright [5], Active-Set methods are a class of algorithms used since the decade of 1970, which are quite effective for medium scale optimization problems. The recent version of this algorithm is developed based on Gill et al [6] and Gill et al [7] works, where a quadratic subproblem (QP) is configured to determine the step descendent vector in each iteration. The QP subproblem is developed through an approximation by way of the Taylor series, where the Lagrangian function is truncated in quadratic term. This condition results in Eq. (2):

$$min\ \frac{1}{2}\ \boldsymbol{d^T} H_k \boldsymbol{d} + \left(\nabla f(\boldsymbol{x_k})\right)^T \cdot \boldsymbol{d}$$

subject to:

$$\left(\nabla h_i(\boldsymbol{x_k})\right)^T \cdot \boldsymbol{d} + \left(h_i(\boldsymbol{x_k})\right)^T = 0, \qquad i = 1,2,3,\dots,z$$
$$\left(\nabla g_n(\boldsymbol{x_k})\right)^T \cdot \boldsymbol{d} + \left(g_n(\boldsymbol{x_k})\right)^T \leq 0, \qquad n = 1,2,3,\dots,o \tag{2}$$

where H_k is the Hessian matrix of Lagrangian updated by BFGS method, $\nabla f(\boldsymbol{x_k})$ the gradient of the objective function, $\nabla g_n(\boldsymbol{x_k})$ the gradient of the inequality constraints and $\nabla h_i(\boldsymbol{x_k})$ the gradient of the equality constraints, evaluated at a point $\boldsymbol{x_k}$.

In each iteration, the inactive constraints of Eq. (2) are discarded, considering that, to determine the step descendent vector, only equality constraints and inequality constraints equals zero are used. This way, the Karush-Kuhn-Tucker conditions for an iteration are equal to those presented in Eq. (3), where λ_i^k and λ_n^k are the Lagrange multipliers of active and inactive constraints respectively.

$$\frac{1}{2}\ H_k \cdot \boldsymbol{d} + \left(\nabla f(\boldsymbol{x_k})\right)^T + \lambda_i^k \cdot \left(\nabla h_i(\boldsymbol{x_k})\right)^T = 0$$
$$\lambda_i^k \cdot [\left(\nabla h_i(\boldsymbol{x_k})\right)^T \cdot \boldsymbol{d} + \left(h_i(\boldsymbol{x_k})\right)^T\ ] = 0$$
$$\dot{\lambda}_i^k \cdot \left[\left(\nabla g_i(\boldsymbol{x_k})\right)^T \cdot \boldsymbol{d} + \left(g_i(\boldsymbol{x_k})\right)^T\right] = 0 \tag{3}$$
$$\lambda_n^k = 0\ if\ \left(\nabla g_n(\boldsymbol{x_k})\right)^T \cdot \boldsymbol{d} + \left(g_n(\boldsymbol{x_k})\right)^T < 0$$

These conditions carry on the occurrence of null Lagrange multipliers, due to non active inequality constraints. Moreover, it is possible to compute the values of λ_i^k and **d** for the current iteration. On the condition of all λ_i^k being positive, then $\boldsymbol{x_k}$ is a possible local minimum. Otherwise, if any constraint is violated, i.e. $\lambda_i^k < 0$, this constraint will turn into an active constraint (A_i^k) used to solve the QP subproblem in the next iteration. And the solution vector is updated by Eq. (4):

$$\boldsymbol{x_{k+1}} = \boldsymbol{x_k} + \alpha \cdot \boldsymbol{d} \tag{4}$$

where α is the step length, which is determined by violated constraints and assumes values on the interval 0 and 1. The Active-Set can be presented through next pseudocode:

Start $\boldsymbol{x_0}$
Find the active constraints A_i^0
For k=0,1,2,…. **do**
  Solve KKT condition in Eq. (3) for $\boldsymbol{d_k}$ and λ_i^k
  if $\boldsymbol{d_k}$ = 0 **then**
    if $\lambda_i^k \geq 0$ **then**
      Stop $\boldsymbol{x_k}$ is the solution
    else
      Remove of $A_i^k$the index i due $\lambda_i^k < 0$
    end if
  else
    if $\lambda_i^k \geq 0$ **then**
      $\boldsymbol{x_{k+1}} = \boldsymbol{x_k} + \boldsymbol{d_k}$
    else
      Remove of A_i^k the index i due $\lambda_i^k < 0$

$\alpha = \left\{-\frac{A_i^k x_k - g_i(x_k)}{A_i^k \boldsymbol{d}}\right\}$ for $i = 1,2,3, \dots, o$

$\boldsymbol{x_{k+1}} = \boldsymbol{x_k} + \alpha \cdot \boldsymbol{d_k}$

end if

end if

end for

2.2 SQP

The SQP algorithm is based in Nocedal and Wright [5] and has many similarities with Active-Set, as both use a local QP sub-problem for determining the descent vector. The difference between them being the way in which the step length is determined, which in this case is performed via analysis of a function of merit.

The merit function $\Psi(x)$ combines the objective function and constraints as presented in Eq. (5):

$$\Psi(\boldsymbol{x}) = f(\boldsymbol{x}) + \sum_{i=1}^{z} r_i^k \cdot h_i(\boldsymbol{x}) + \sum_{n=1}^{o} r_n^k \cdot max(0, g_n(\boldsymbol{x})) \tag{5}$$

where r_i and r_j are penalty parameters related to active set constraints and defined by:

$$r_i^{k+1} = \left\{\lambda_i, \frac{r_i^k + \lambda_i}{2}\right\} \; and \; r_n^{k+1} = \left\{\lambda_n, \frac{r_n^k + \lambda_n}{2}\right\} \tag{6}$$

The value α in current iteration is determined to impose a decrease in merit function, as in Eq. (7):

$$\Psi(\boldsymbol{x_k} + a \cdot \boldsymbol{x_k}) \le \Psi(\boldsymbol{x_k}) \tag{7}$$

Moreover, another difference between the SQP and Active-Set algorithms is that during the solution of the problem on SQP, all steps are taken in the feasible region, while in Active-Set this condition is relaxed. This decision can be beneficial, especially when the objective fucntion is undefined out of bounds.

2.3 INTERIOR POINT

Interior Point methods were developed in the 90's, and have a good applicability in large scale problems. The idea is that these methods solve the Karush-Kuhn-Tucker conditions iteratively by successive applications of Newton's method, where the inequalities constraints are handled by the use of a barrier function and slack variables *s*. In the view Eq. (1) can be rewritten with in Eq. (8):

$$min\, f(x) - \mu \cdot \left(\sum_{j=1}^{o} ln\left(\boldsymbol{s_j}\right)\right)$$

subject to: (8)

$$h_i(\boldsymbol{x}) = 0, \quad i = 1,2,3, \dots, z$$
$$g_n(\boldsymbol{x}) - s_n = 0, \quad n = 1,2,3, \dots, o$$
$$s_n \ge 0, \quad n = 1,2,3, \dots, o \; and \; \mu \ge 0$$

where μ is an imposed variable to delimit the Barrier function. As the slack variables are imposed to always be positive, the solution steps remain within the feasible region. Furthermore, as μ decreases to a minimum value, the original objective function also decreases.

3 Optimization Problem

The proposed optimization problem consists in minimizing the concrete volume of 45 beams and 12 columns of a structural building composed of 5 floors, presented in Fig. 1. In all floors, the values for the accidental (live) load were defined according to NBR 6120:2018 [7], being a live load of 2.50 kN/m² (standard for commercial use) and an additional 1.00 kN/m² for flooring and ceiling were considered. For the roof slab, those values are respectively 1.50 kN/m² and 0.24 kN/m². The thickness for the walls is 11 cm and its unit weight 13 kN/m³. The concrete weight is equal to 25 kN/m³.

The distance between floors was adopted equal to 300cm, strength of concrete (fck) of 30 MPa and Young modulus equal to 30672 MPa. The wind loads, considered in two orthogonals directions between them, were determined according to ABNT NBR 6123:1988 [8], considering the basic speed equivalent to 30m/s, the terrain rugosity of type B and category IV. The structural analysis was realized through the Direct Stiffness Method, and according to γ_z calculus suggested in NBR 6118:2014 [10]. A detailed explanation of the structural analysis is disponible in Santos et al [11].

The restrictions applied are divided in two types: those of design conceptions and those from the codes. The first group is composed of linear equality and inequality restrictions, related to parameters such as maximum acceptable dimensions and cross-section geometry for structural members, among others. The second type of restrictions are nonlinear inequality conditions that evaluate the strength and stability requirements in a way to provide a structure simultaneously slender and obedient to the regulations.

The design variables are the dimensions of structural elements, for a total of 8 variables, being: the dimensions of columns from number 1 to 12, except 5 and 8; the columns dimensions by 5 and 8; horizontal beams dimensions (parallel to x-axis); and vertical beams dimensions (parallel to y-axis). The flowchart of Fig. 2 illustrates the sequence of the optimization process and organization of the design constraints.

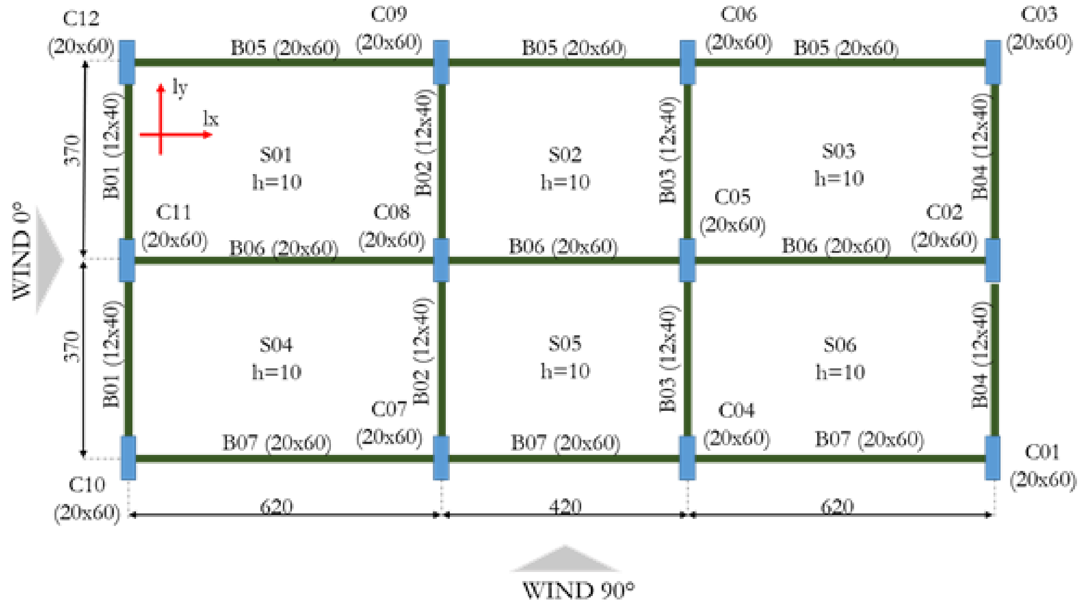

Figure 1. Floor plan for the structural building (dimensions in centimeters)

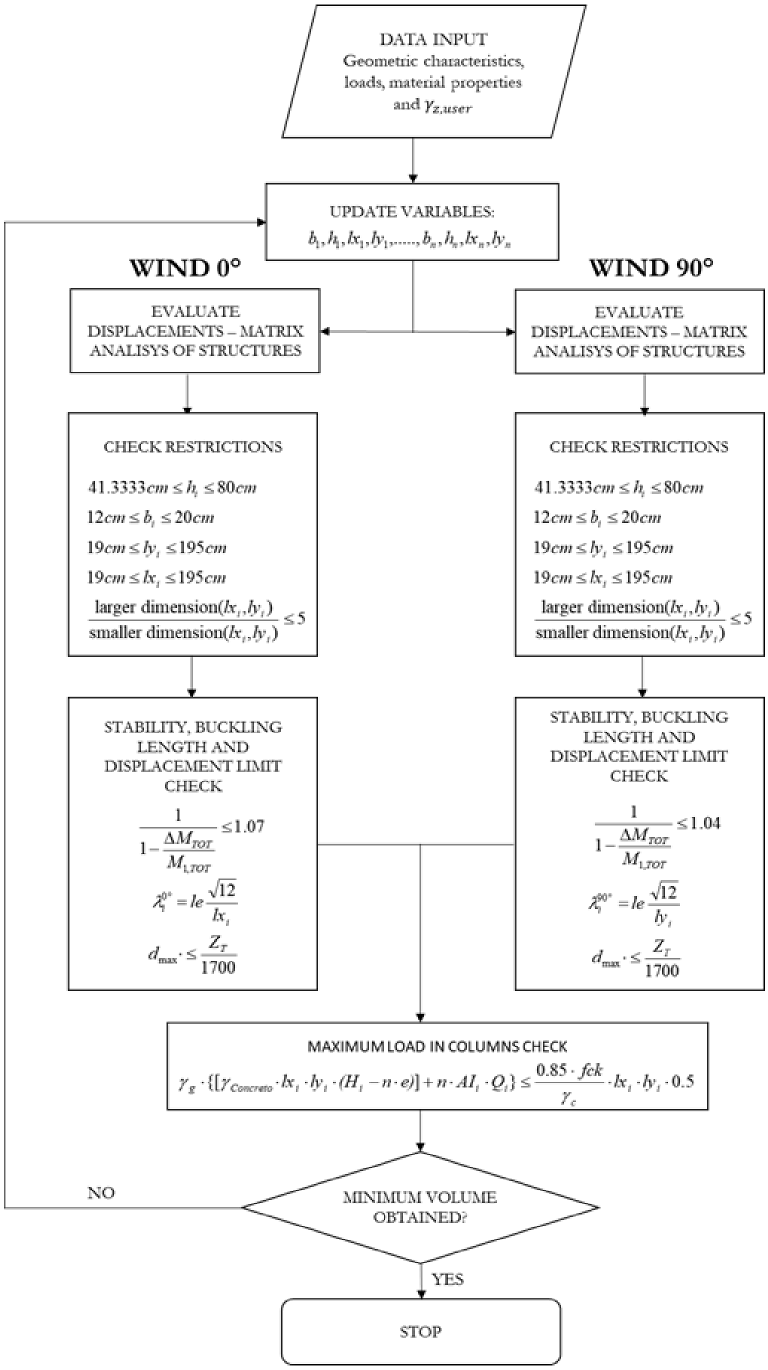

Figure 2. Flow-chart

In Fig. 2, i is the index used to identify each structural element, b_i beams width, l_i is the beam internal span, h_i the beam height, lx_i is the column dimension parallel to x-axis, ly_i is the column dimension parallel to y-axis, e is the thickness of the slab, H_i is the vertical distance between consecutive floors, γ_{user} initial global parameter of stability, $\lambda_i^{0°}$ a $\lambda_i^{90°}$ maximal slenderness of columns parallel in both axis, le equivalent buckling length of column, d_{max} horizontal maximum displacement of building, Z_T total height of building, AI_i influence area of column, Q_i load per square meter in each load, fck is the concrete's compressive strength, n number of floors, γ_g combination factor for each load, $\gamma_{concrete}$ specific weight of concrete and γ_c combination factor for concrete's strength.

4 Results

The performance of each algorithm was verified through three test cases, where the initial values of design variables were modified. In the first case, designated Min, the adopted dimension were 19cm for columns, 12cm for beam width and 41.333cm for beam's height , equivalent to 1/15 of the largest effective length of beams. The second case named Int, the values of the design variables were those of the structural elements presented in Fig. 1. The last case, entitled Max, used in the initial step columns with both dimensions of 195cm, which was purposefully chosen because as it is far from the expected optimal value, beam widths of 20cm and heights of 80cm. The Tables 1, 2 and 3 bring the results obtained for each situation, the analysis were realized through a CPU I5-8520U with RAM of 8GB.

Tab. 1 presents the results obtained by the Interior Point algorithm. Note that the Max situation presented the smallest value, 32.270m³, for the objective function, and the other cases, Min and Max, achieved the objective function equal to 33.084 and 33.084m³ respectively. In relation to analysis time, the Min case results were almost 2 and 3 times bigger than analysis Int and Max.

Table 1. Results for the Interior Point Method

	Interior Point		
	Min	Int	Max
Iter	168	107	73
F-Count	2038	1169	733
f(x)	33.084	33.084	32.270
Feasibility	0.00E+00	0.00E+00	0.00E+00
First Order Optimality	0.0004454	0.00002502	0.0002005
Status	Local minimum possible	Local minimum possible	Local minimum possible
Number of Analysis	10	10	10
Time (s)	14.771	8.847	5.832

The results of the Active-Set method are shown in Tab. 2, and just like the Interior Point Method, the Min case presented the largest value for the objective function. Furthermore, when comparing the Int and Max case to Active-Set, it is observed that the first one spends the lowest number of iterations, objective functions counts and time, while the Max situation presents the lowest value to the gradient norm.

Table 2. Results for the Active-Set Method

	Active Set		
	Min	Int	Max
Iter	28	46	67
F-Count	261	428	638
f(x)	34.264	33.084	33.084
Feasibility	-	-	-
First Order Optimality	0.000283	0.000407	0.000285
Status	Local minimum possible.	Local minimum possible.	Local minimum possible.
Number of Analysis	10	10	10
Time (s)	1.956	2.959	4.294

For the SQP algorithm (Tab. 3), a result similar to that observed in the Active-Set was obtained, where only the Min case did not return the smallest value of the objective function, and the Int situation was the most computationally efficient.. Again, the Int case was more effective in time, number of iterations and objective function counts, and the Max case obtained the lowest norm of gradient.

Table 3. Results for Sequential Quadratic Programming

	SQP		
	Min	Int	Max
Iter	36	49	60
F-Count	360	474	563
f(x)	34.264	33.083	33.083
Feasibility	4.84E-10	2.58E-10	3.66E-09
First Order Optimality	0.0002309	0.00003944	0.00007064
Status	Local minimum possible.	Local minimum possible.	Local minimum possible.
Number of Analysis	10	10	10
Time (s)	2.405	3.388	4.212

Regarding the design variables, it can be seen that in all analyses, the widths of the beams assumed the minimum value. It was also noted that, for the three algorithms, in the Int case results were the practice of the same values. Furthermore, the solution vector to SQP and Active-Set algorithms are similar for the three cases.

5 Conclusions

The results of the numerical experiment show that for this optimization problem the Interior Point algorithm performed worse than the Active-Set and SQP, both in terms of performance. The performance can be assessed by the processing time, and once again, the Interior Point Method rendered worse results.

As for the selection of the initial values of the design variables, in SQP and Active-Set, it can be noted that the Int case, when compared to the Max case, presented better results in relation to the number of iterations, time and number of evaluations of the objective function, presenting itself as a viable and computationally less costly solution. It is also highlighted that the Min case leads to a non-ideal solution for both algorithms. For the Interior Point the best solution is obtained by using the Max case.

Proposed as future research increase of number the starting points and so analyze other parameters like the succesfull rate.

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6 References

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