

# Cost analysis on the optimum design of prestressed doubly-symmetric steel beams

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**Abstract.** In the last few years, the utilization of steel beams has shown great growth in civil construction. Although steel has a higher cost, its use is justified when wanting to overcome large spans. This aspect motivates the development of more economical alternatives as well as the implementation of prestressing tendons. The objective of this paper is to present the optimization problem aimed to reduce the total cost of prestressing doubly-symmetric I-sections of steel beams. MATLAB's native Genetic Algorithm technique was implemented in the mixed integer programming optimization problem, focusing to optimize the cross-section's geometrical properties (i.e., depth of cross-section, flange widths and thicknesses, and web thicknesses) and the number of tendons. The evaluation and validation steps used two examples from the literature. The design method, through constrained functions, verifies the Ultimate and Serviceability Limit States from the standard NBR 8800:2008. Therefore, results showed an efficient alternative for the structural engineering practice, giving a reduction in the total cost of 24.43 and 25.62% for the studied cases.

Keywords: prestressed, steel beams, optimization, genetic algorithm.

# 1 Introduction

Since the beginning of the 19<sup>th</sup> century, as a fast, precise, and excellent return construction process, steel constructions are used in the world. The advent of the Industrial Revolution started the use of steel, mainly for large-scale structural projects. In Brazil, the choice of steel structures has become expressive, allowing efficient and high-quality solutions.

Steel-built structures have a series of advantages over other types, such as the freedom of creation in the architectural project, the possibility to get large spans, and the facility for adaptations. Moreover, there is a reduction in the cost of foundations since steel profiles are lighter than those of the reinforced concrete.

Thus, investigations of optimal design of prestressed steel beams aiming for better materials use, implementing the intrinsic characteristics of steel structures to prestressing technique have been studied. The development of technologies for the application of prestressing in steel structures to achieve economic gain is considered a novelty. Sampaio Júnior [1] studied the economic design of prestressed steel beams, concluding a weight reduction from 15 to 30% comparing prestressed to conventional steel beams.

Troitsky [2] discusses the theory and design of prestressed steel bridges, presenting methods to apply, some types of anchorages, and prestress losses. Moreover, the supra-cited researcher infers the prestressing technique in steel bridges use as one of the best ways to achieve savings in steel, as well as reduced construction costs. Ponnada and Vipparthy [3] investigated the carrying capacity and differences between I-sections, such as doubly-symmetric as monosymmetric. Vipparthy and Ponnada [4] also presented a study showing the maximum increase in load capacity equal to 7.78% for I-sections and 57.08% for compression reinforced I-sections. Furthermore, Shah, Patel, and Jani [5] used the Limit State method, according to the Indian code [6], to prove the prestressing effectiveness.

Vipparthy and Ponnada [4], Ponnada and Vipparthy [7], Park et al. [8], and Alfouneh and Tong [9] pointed out the importance of understanding the behavior of steel beams when subjected to prestressing. Vipparthy, Venkateswarlu, and Ponnada [10] carried out the prestressed steel beams numerical and experimental analysis to arrive at an ideal optimization model for each situation.

The use of optimization techniques associated with metaheuristics applied to structural engineering problems is increasing. Luévanos-Rojas et al. [12], exemplify the computational resources uses to optimize the design of prestressed steel structures. Furthermore, Prendes-Gero et al. [13] used the Genetic Algorithm (GA) technique to find the minimum weight in a space frame supporting the external loads, using three different design guidelines, proving the possible versatility of the technique. As a result, the authors observed a gain of 10% compared to non-optimized models.

Kripakaran, Hall, and Gupta [14] also implemented the GA technique as a decision tool regarding the optimal model to be used for steel structures with different types of connection, aiming the cost-effective optimization. Agrawal, Chandwani, and Porwal [15] presented a study to optimize the weight of a welded steel beam through GA use, maximizing safety while minimizing the total cost. Mohammed, Abbas, and Abdul-Razzaq [16] investigated the prestressed steel beams optimization using the Finite Element Method via ANSYS software. The researchers perform an analysis with and without prestressing, pointed out advantages, and reveal the gains in the final solution. Yildirim and Ackay [17] emphasize maintaining high efficiency in the use of resources importance, using a fuzzy logic approach and GA technique aiming to find the minimum cost and duration. Skoglund, Leander, and Karoumi [18] used MATLAB's native GA technique to optimize costs, material quantity, and CO<sub>2</sub> emission. The researchers followed the European codes in hybrid steel beams and different types of steel with the same cross-sectional area to evaluate the feasibility of using high-strength steel. Such a study showed substantial cost savings and reduced the environmental impact.

Therefore, the objective of this paper is to present the optimization problem formulation involving prestressed steel beams according to standard NBR 8800:2008 [19]. Literature examples are analyzed to verify the efficiency of the proposed formulation. The program was developed in the MATLAB platform such as the interactive graphical interface generation and the optimization problem solution via the MATLAB's native GA technique.

### **2** Optimization problem formulation

The formulation has considered the doubly-symmetric I-section steel beams. The number of prestressed tendons, the depth of cross-section, as well as the superior and inferior flange widths were considered as discrete variables (i.e., integer variables). While the superior and inferior flange thicknesses and the web thicknesses were considered as continuous variables.

#### 2.1 Objective function

The objective function aimed to reduce the total cost of steel as well as its installation. The volume of steel, the number of prestressing tendons, and its installation were considered as follows in eq. (1):

$$f(x) = (Ct_s \cdot A_s + Ct_t \cdot n_t \cdot \mu_t) \cdot L_{span} + (n_t \cdot Ct_{ti})$$
(1)

Where:  $Ct_s$  is the steel cost per m<sup>3</sup> [R\$]; A<sub>s</sub> is the cross-sectional area [m<sup>2</sup>];  $Ct_t$  is the prestressing tendons cost per weight [R\$];  $n_t$  is the number tendons;  $\mu_t$  is the specific weight of the tendons [kN/m];  $L_{span}$  is the length of span [m] and,  $Ct_{ti}$  is the cost of the tendons installation per unit [R\$].

The steel costs for the beam and prestressing tendons were R\$ 12.88 and R\$ 12.69, respectively. The tendons installation of 9.5 mm and 15.2 mm is R\$ 125.66 and R\$ 158.65, respectively.

#### 2.2 Constraint functions

The Ultimate and Serviceability Limit States were followed by standard NBR 8800:2008. The coefficients of a permanent and variable combination of actions were equal to 1.3 and 1.25, 1.4, and 1.5, respectively. Equations (2)-(15) show the constraint functions used by GA.

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(**a**)

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$$C(1): M_{Sd} / M_{Rd} - 1 \le 0$$
<sup>(2)</sup>

$$C(2): M_{Sd,e}/M_{Rd} - 1 \le 0$$
<sup>(3)</sup>

$$C(3): V_{Sd} / V_{Rd} - 1 \le 0 \tag{4}$$

$$C(4): N_{Sd}/N_{Rd} - 1 \le 0 \tag{5}$$

$$C(5):\delta_{tot}/\delta_{lim} - 1 \le 0 \tag{6}$$

$$C(6): -\delta_{tot,e}/\delta_{lim} - 1 \le 0 \tag{7}$$

$$C(7): \left(N_{Sd}/N_{Rd} + 8/9 \cdot \left(M_{Sd,e}/M_{Rd}\right)\right) - 1 \le 0$$
<sup>(8)</sup>

$$C(7): \left( (N_{Sd}/2 \cdot N_{Rd}) + (M_{Sd,e}/M_{Rd}) \right) - 1 \le 0$$
<sup>(9)</sup>

$$C(8): \left(N_{Sd}/N_{Rd} + 8/9 \cdot (M_{Sd}/M_{Rd})\right) - 1 \le 0$$
<sup>(10)</sup>

$$C(8): \left( (N_{Sd}/2 \cdot N_{Rd}) + (M_{Sd}/M_{Rd}) \right) - 1 \le 0$$
<sup>(11)</sup>

$$C(9): (d/b_f)/4 - 1 \le 0 \tag{12}$$

$$C(10): 1 - (d/b_f)/1.5 \le 0$$
<sup>(13)</sup>

# $\mathcal{C}(11): \left(\sigma_t / f_v\right) - 1 \le 0 \tag{14}$

$$C(12): \left(-\sigma_c/f_y\right) - 1 \le 0 \tag{15}$$

Where:  $M_{Sd}$  is the design bending moment [kNm];  $M_{Rd}$  is the design resistance to bending moment [kNm];  $M_{Sd,e}$  is the prestressing bending moment [kNm];  $V_{Sd}$  is the design shear force [kN];  $V_{Rd}$  is the design resistance to shear [kN];  $N_{Sd}$  is the design axial force [kN];  $N_{Rd}$  is the design resistance to axial load [kN];  $\delta_{tot}$  is the total displacement on the y-axis due to the load [mm];  $\delta_{lim}$  is the maximum vertical displacement [mm];  $\delta_{tot,e}$  is the prestressing displacement on the y-axis [mm]; d is the depth of a cross-section [mm];  $b_f$  is the flange width [mm]; h is the depth of a web [mm];  $t_w$  is the web thickness [mm];  $\sigma_t$  and  $\sigma_c$  are the maximums tensile and compressive strength [kN/m<sup>2</sup>], respectively; and,  $f_y$  is the yield strength of the steel [kN/m<sup>2</sup>].

The first C(7) and C(8) conditions, eq. (8) and (10), are adopted by GA if the relation between  $N_{Sd}$  and  $N_{Rd}$  is greater or equal than 0.2 [19]. Therefore, if the respective relation is less than 0.2, the second C(7) and C(8) conditions are adopted, eq. (9) and (11). Also, the C(9) and C(10) conditions limit the cross-sectional dimensions.

### **3** Numerical analysis of results

The first example presented by Rezende[20] is an I-section doubly-symmetric steel beam, and the second example analyzed was presented by Ferreira [18], an I-section prestressed monosymmetric steel beam. The tensile and compressive strength for tendons were adopted equally 1900 MPa and  $C_b$  coefficient equal 1. Thus, the flange thicknesses ( $t_{INF}$  and  $t_{SUP}$ ) and widths ( $f_{INF}$  and  $f_{SUP}$ ), as well as the depth of a cross-section (d) and the number of tendons lower and upper limits were stated equal to [1.6, 4.44], [10, 55], [55, 200] centimeters, and [0, 20] units, respectively.

#### 3.1 Example 1 – I-section doubly-symmetric prestressed steel beam [20]

The first example was presented by Rezende [20]. Thus, were adopted (i) a permanent and overload of 9.6 kN/m and 18 kN/m, (ii) span equal to 20 m, (iii) tendons diameter of 15.2 mm, (iv) tendons position of 30 mm above the bottom section of the lower flange, (v)  $f_y$  equal to 250 MPa, (vi) elasticity modulus equal to 200000

MPa and (vii) prestressing losses of 20%. Table 1 shows the GA optimized results.

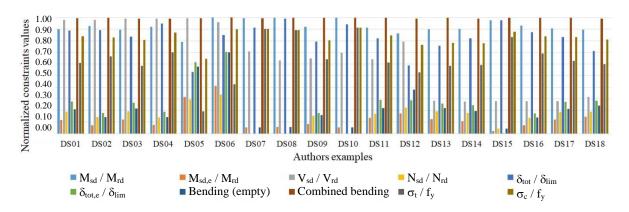
Example	d (mm)	$f_{\rm INF}(mm)$	$f_{\mathrm{SUP}}(\mathrm{mm})$	t <sub>INF</sub> (mm)	t <sub>SUP</sub> (mm)	$t_w(mm)$	tendons	I <sub>x</sub> (e+4mm <sup>4</sup> )	δ tot (mm)	σ <sub>c</sub> (MPa)	σ <sub>t</sub> (MPa)	Total Cost (R\$)
DS00	1000	400	400	22.40	22.40	8.0	2	486331.40	64.00			54024.98
DS01	1040	360	360	19.49	19.49	8.0	4	432185.33	50.69	-209.53	152.36	46456.68
DS02	1060	300	300	25.00	25.00	8.0	3	470474.23	51.07	-207.02	166.08	48091.54
DS03	1060	320	320	22.40	22.40	8.0	4	455671.42	47.81	-201.94	145.85	47306.31
DS04	1050	370	370	19.00	19.00	8.0	3	442768.67	54.23	-217.72	175.07	46223.06
DS05	1040	440	440	16.00	16.00	8.0	8	437408.29	30.42	-161.30	47.52	48565.61
DS06	1000	430	430	12.50	12.50	8.0	7	323877.86	48.54	-224.75	105.90	40824.60
DS07	1240	310	310	19.49	19.49	9.5	0	587093.23	52.18	-225.47	225.47	47501.52
DS08	1130	290	290	25.00	25.00	9.5	0	542424.28	56.66	-223.03	223.03	50068.68
DS09	1150	290	290	22.40	22.40	9.5	3	519903.86	45.25	-200.57	160.35	48923.46
DS10	1220	320	320	19.00	19.00	9.5	0	569262.36	53.80	-228.73	228.73	47296.30
DS11	1120	360	360	16.00	16.00	9.5	4	453003.40	46.94	-211.21	153.57	46089.78
DS12	1360	340	340	12.50	12.50	9.5	4	574218.18	33.64	-191.35	131.87	44727.85
DS13	1130	300	300	19.49	19.49	12.5	4	495764.89	43.29	-195.74	146.00	53112.49
DS14	1050	280	280	25.00	25.00	12.5	4	471958.33	47.01	-194.99	147.46	55480.69
DS15	1100	300	300	22.40	22.40	12.5	1	512613.10	55.82	-219.58	207.76	54323.48
DS16	1100	320	320	19.00	19.00	12.5	3	480046.87	49.95	-210.02	172.87	52853.73
DS17	1100	350	350	16.00	16.00	12.5	4	455933.99	47.46	-208.02	156.71	51537.48
DS18	1230	350	350	12.50	12.50	12.5	4	506525.27	40.58	-202.93	150.03	50046.13

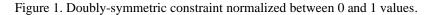
Table 1. Doubly-symmetric results based on Rezende [20] example (DS00).

The number of tendons remained higher most of the time. However, the algorithm compensated this increase by reducing the volume of steel in the beam, as represented by the moment of inertia  $I_x$ . The DS07, DS08, and DS10, resulted in no one tendons. This reason is due to the superior limit of the depth of a cross-section stated at 200 cm. In this sense, there is no reason to get prestressing in those cases.

Deflections showed lower values than the reference model, DS00. The greater reduction is 33.58 mm (DS05), almost 53% different from Rezende's model. The compression provided values greater or equal than tension values, meaning that the compression is more required than tension for this type of design. Furthermore, the limit of steel yield was not exceeded.

Therefore, the total cost has presented a reduction in most of the examples, excepted by DS14 and DS15. The best reduction is R\$ 13200.38, found in example DS06 (i.e., 24.43%). These two extremes are explained before by the  $I_x$  decrease. Thus, the cost of tendons such as their implementation is less than the cost of the volume of steel. Figure 1 presents the constraints analysis.





The conditions limits illustrated by Fig. 1 indicated the combined bending as a condition that governs the others due to its stability through the examples. Sometimes the shear, as well as the momentum conditions and displacements, are also governing conditions. This behavior is understanding due to the high momentum values in front of the other actions stated in the combination of actions process.

#### 3.2 Example 2 – I-section monosymmetric prestressed steel beam [21]

The second example studied by Ferreira [21] variated the web thickness between 16 and 12.5 mm because smaller values exceeded the GA constraints. Thus, point loads of 150 kN located at 11, 12.5, and 14 meters from the left support were employed, as well as a permanent, empty, and serviceability overload of 12.86 kN/m, 3 kN/m, and 15 kN, respectively. Furthermore, (i) the span was equal to 25 m, (ii) the tendons diameter equal to 15.2 mm, (iii) tendons position of 100 mm below the bottom section of the inferior flange, (iv)  $f_y$  equal to 345 MPa, (v) modulus of elasticity equal to 205000 MPa, and (vi) prestressing losses of 12.3%. Table 2 shows the GA optimized results.

Table 2. Doubly-symmetric results based on Ferreira [21] example (DS00).

Example	d (mm)	$f_{\rm INF}(mm)$	f <sub>SUP</sub> (mm)	t <sub>INF</sub> (mm)	t <sub>sUP</sub> (mm)	t <sub>w</sub> (mm)	tendons	I <sub>x</sub> (e+4mm <sup>4</sup> )	δ tot (mm)	σ <sub>c</sub> (MPa)	σ <sub>t</sub> (MPa)	Total Cost (R\$)
DS00	1000	380	500	32.00	44.40	16.0	18	853611.00	58.11	-169.84	328.18	134317.52
DS01	1210	310	310	39.82	39.82	16.0	18	1038055.18	69.61	-254.06	108.77	118056.96
DS02	1170	300	300	44.40	44.40	16.0	19	1012765.33	69.49	-248.03	98.74	121553.88
DS03	1260	320	320	39.82	39.82	12.5	16	1120211.86	69.95	-257.31	120.02	110545.52
DS04	1190	310	310	44.40	44.40	12.5	18	1042745.55	70.17	-251.09	100.59	114313.04
DS05	1240	310	310	37.50	37.50	16.0	17	1051585.88	71.34	-262.31	122.19	115270.03
DS06	1290	330	330	37.50	37.50	12.5	15	1157792.70	70.48	-260.81	131.13	109230.62
DS07	1320	330	330	31.50	31.50	16.0	15	1127893.83	71.20	-270.58	143.96	111668.59
DS08	1390	350	350	31.50	31.50	12.5	13	1260938.21	68.85	-267.51	151.34	104840.53
DS09	1400	360	360	25.00	25.00	16.0	14	1178925.00	68.99	-276.61	154.55	107825.48
DS10	1520	390	390	25.00	25.00	12.5	10	1420564.06	68.98	-278.78	187.63	101257.04
DS11	1470	370	370	22.40	22.40	16.0	12	1254444.70	70.95	-287.36	182.15	106163.28
DS12	1580	410	410	22.40	22.40	12.5	9	1491047.92	68.26	-282.45	199.72	99903.72

In general, the monosymmetric beams get lower costs than doubly-symmetric, still having the same conditions. Besides the monosymmetric model adopted by Ferreira [21], the GA was able to found better results than the aforementioned author in the objective function investigated by this research.

The number of tendons was greater or equal than DS00 in only three cases, DS01, DS02, and DS04. Unlike Example 1, the number of tendons herein was excessive. Thus, the GA decreases the tendons and increases the volume of steel to support the actions. The volume rising can be identified in the  $I_x$  column.

On the other hand, for each case, the deflexions were superior to DS00. The deflexion growth is explained by the compression and tension analysis. The doubly-symmetric results showed large compressions than DS00, getting the maximum difference up by 117.52 MPa. This behavior indicates the unnecessary number of tendons in DS00 as mentioned before. Particularly, the steel in the beam was not requested enough. Consequently, the tension presented superior values than necessary, getting at 229.44 MPa of differing.

Therefore, the best result is the DS12 model, where presented R\$ 34413.80 of total cost decrease (i.e., 25.62% of reduction). This same model presented the minimum number of tendons, which is satisfactory considering the points still discussed. Figure 2 presents the constraints analysis.

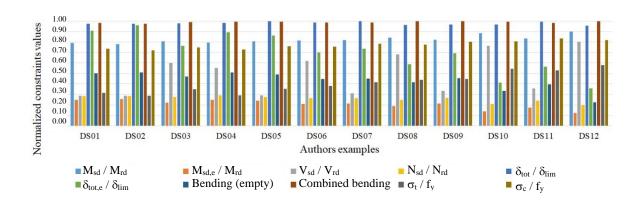


Figure 2. Doubly-symmetric constraint normalized between 0 and 1 values.

The same behavior is identified in Fig. 2 as well as Fig. 1 analysis. The governing conditions are the momentum values, deflexion, the combined bending, and shear values. However, the others conditions have been presented consistently through the examples. Some of them, such as DS07, DS08, DS10, and DS15, from Example 1 are near to 0 (i.e., lower normalized limit). Thus, in Example 2 there is no example presenting the same behavior. The difference is understood due to the conditions supported by the beams, requiring different types of actions when different types of loads are submitted.

## 4 Conclusions

This paper has presented an optimum design formulation using MATLAB's native GA technique applied to doubly symmetric I-sections of steel beams supported by standard NBR 8800:2008 [19].

The implemented models such as in Rezende [20] and Ferreira [21] reduced the total cost of beams by up to 24.43 and 25.62%, respectively. Despite the increase of 74.68% in the volume of steel, in the first case, the reduction in the number of tendons has balanced the objective function to minimize the cost of materials. On the other hand, the second case got a reduction of 33.40% in the volume of steel.

In this respect, the GA technique was successfully applied to get the minimum total cost of beams with a low computational cost. Moreover, this conclusion motivated future works focusing on minimizing materials as well as its cost in civil constructions.

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