Comparison of different optimization methods applied to steel trusses structures

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Abstract. The objective of the present paper is the adaptation and development of algorithms to minimize the weight of trussed steel structures based on different optimization methods (mathematics and heuristics). Compare the efficiency between the Sequential Quadratic Programming (SQP) and the Genetic Algorithm, taking into account the quality of the solutions found, the complexity of implementation and the computational cost required. The results show that, when using the technical standards for compressed bars, it is necessary to adopt bars with more robust cross sections when compared to solutions whose restrictions derive from the classic formulation of Euler's critical load. A similarity between the results obtained by the SQP and the Genetic Algorithm is observed, but due to the combinatorial nature of the latter, the heuristic method requires longer processing time. In addition, the results obtained by the Genetic Algorithm are shown to be random during the iterations, and may converge in slightly heavier solutions than the SQP. Justified by the results presented, it is stated that the developed algorithms are good tools for structural optimization and can be used in real steel truss designs.

Keywords: Structural Optimization. Steel trusses. Optimal solution. Mathematical methods. Heuristic methods.

1 Introduction

In the most cases, the structural engineer defines an initial configuration based on yours experiences for the structural system to be designed and, through iterative processes, check if the technical standards conditions for the security and service are satisfy. Even though the proposed solution satisfies the conditions, it may not be the best in consumption of materials terms. For the optimal solution, is necessary to use structural optimization techniques, able to provide designs increasingly lighter and time and materials savings.

Optimization techniques are intended to minimize or maximize the value of an objective function (mathematical model), that can be subject or not to an equality and/or inequality constraints. A minimization problem can be expressed as:

$$\begin{array}{ll} \text{Minimize} & f(\mathbf{X}), \\ & h_{\mathrm{i}}(\mathbf{X}) = 0 \quad \mathrm{i} = 1,2,...\mathrm{m}, \\ \text{subjected to} & g_{\mathrm{j}}(\mathbf{X}) \leq 0 \quad \mathrm{j} = 1,2,...\mathrm{p}, \\ & \mathbf{X}_{\mathrm{L}} \leq \mathbf{X} \leq \mathbf{X}_{\mathrm{U}} \quad , \end{array} \tag{1}$$

being $f(\mathbf{X})$ the objective function, $\mathbf{X}$ the design variables vector with $\mathbf{X} \in \Omega$, where Ω is the viable solutions space. The others functions are the equality constraint ($h_{\mathrm{i}}(\mathbf{X})$), inequality constraint ($g_{\mathrm{j}}(\mathbf{X})$) and the side constraints with lower ($\mathbf{X}_{\mathrm{L}}$) and upper ($\mathbf{X}_{\mathrm{U}}$) limits. The nature of objective function and constraints may be linear or non-linear and the viable solutions space may be continuous or discrete.

Lee and Geem [1] states that the structural optimization has received considerable attention in the last two decades, so various optimization methods have been developed. However, these methods may be limited and no single method is completely efficient and robust for all types of structures problems. Therefore the motivation of this work is compare the efficiency between two different optimization methods (mathematical and heuristic) used to minimize the weight of trussed steel structures. Furthermore, the most of this structural optimization studies are applied to academic examples and consider just restrictions that derive from the classic formulation of Euler's critical. For this reason, another motivation is evaluate the optimization results applied in cases considering some recommendations of the brazilian technical standard ABNT NBR 8800:2008 [2], as the local instability effect in compressed bars.

In this work, used the optimization Toolbox of the software Matlab. To perform the structural analysis of the desired truss, the algorithm uses the Finite Element Method in an iterative process.

2 Optimization Methods

Given the relevance of optimization problems, several methods are developed by the scientific community. Even with the large number of methods developed, each one with its characteristic, it is possible to divide them in two groups: mathematical methods and heuristic methods.

Mathematical methods are based on finding the best solution through mathematical operators, such as the calculator of the first and second order derivatives of the objective function. The gradients direction allows the starting point arrives at an attractive solution, performing an iterative process. Adopting the same starting point, the algorithm will always results the same solution.

In the case of heuristic methods, the search to a good solution to the problem is done randomly. By employing probabilistic mechanisms, these methods find satisfactory results, but do not guarantee that the proposed solution, although excellent, is the best in the viable space. Unlike mathematical methods, heuristics can generate different satisfactory solutions with the same starting point, due to random search factors.

It is emphasized that the success of any optimization method, either mathematical or heuristic, is associated with its robustness. Although the difficulty linked to the implementation of an algorithm is a important point for its choice, attention must be paid to its capacity to solve the problem. The more robust the method selected, more reliable will be the solution obtained.

2.1 Sequential Quadratic Programming - SQP

According Sheta and Turabieh [3], the Sequential Quadratic Programming (SQP) algorithm is a powerful technique for solving nonlinear constrained optimization problems. The SQP solve the optimization problem iteratively, where the solution at each step is obtained by solving an approximation of the problem made of the second order derivative (Hessian) of the Lagrangian function using a Quasi-Newton updating method, guaranteeing so, the properties of global convergence. The SQP solves the following quadratic programming problem:

$$\begin{array}{ll} \text{Minimize} & \nabla^T f(\mathbf{X}^k)d + \frac{1}{2}d^T H(\mathbf{X}^k, \lambda^k, \mu^k)d, \\ \text{subjected to} & h_i(\mathbf{X}^k) + \nabla^T h_i(\mathbf{X}^k)d = 0 \quad \mathrm{i} = 1,2,...\mathrm{m}, \\ & g_j(\mathbf{X}^k) + \nabla^T g_j(\mathbf{X}^k)d \leq 0 \quad \mathrm{j} = 1,2,...\mathrm{p}, \end{array} \tag{2}$$

to determine the best search direction (d) from of the point $\mathbf{X}^k$ and update the solution in the iteration with $\mathbf{X}^{k+1} = \mathbf{X}^k + \alpha^k d^k$. The α is the step size and takes values in the interval $[0,1]$. The Han-Powell's algorithm is the most promising among the SQP methods and uses the Broyden-Fletcher-Goldfarb-Shanno (BFGS) to update the Hessian matrix $H(\mathbf{X}^k, \lambda^k, \mu^k)$ at each iteration.

2.2 Genetic Algorithm

Sheta and Turabieh [3] states the Genetic Algorithms is a heuristic optimization method that have the capability to search complex spaces with high probability of success in finding the minimum point. The idea of Genetic Algorithms was introduced by Holland [4] at the University of Michigan in 1975 that associated the natural solution as an optimization process. Basically, the Genetic Algorithms operate in a set of solution applying the survival principle where the fittest individuals produce an increasingly better solution generation after generation. The methods gained a great popularity due to their known attributes, like do not require derivative information about the fitness criterion and use probabilistic operators, like crossover and mutation, instead of deterministic ones.

The Sequential Quadratic Programming algorithm and Genetic Algorithm are present in the Toolbox of the software Matlab. It is possible to adjust the processing parameters of the algorithms, but in the present work default settings are used for comparison purposes, and just a few terms are adjusted. In SQP, the maximum number of iterations is 1500, the objective function error tolerance and the constraints error tolerance are 1.0x10^{-6}. In the Genetic Algorithm, the maximum number of iterations is 100x variables numbers, the objective function error tolerance and the constraints error tolerance are 1.0x10^{-6}, and the population size is 50. The crossover type adopted is the two-cut-point.

3 Steel Trusses Structures Design

To carry out a safe optimization structural design, it is necessary to define the constraints that will impose limitations on the structure's behavior. A safe structural is the one that has sufficient strength when subjected to stress, not collapsing or suffering excessive deformations. In general, the limitation and guidelines for the design of a structure are provided by technical standards based on models created for this purpose.

Trusses structures have great versatility of applications in the field of construction and an advantage is associated with its design, since the trusses are only subjected to axial efforts (tension and compression).

Several structural optimization researchers, like Lee and Geem [1] and Wu and Chow [5], adopted the Allowable Stress Design to impose the projects constraints. This traditional method consider the structural design satisfactory when the load stress (σ) in each section is lower than the resistance stress (f_k) reduced by a safety factor (γ). Thus, the equation that must be satisfied to ensure the safety of the bar n, with cross-sectional area A_n, subjected to axial effort (N) is:

$$\sigma = \frac{N}{A_n} < \frac{f_k}{\gamma}. \tag{3}$$

Equation 3 can be used to analyze a member for tension, however the collapse of a compressed member occur for various forms of instability before attain the materials yield strength. The most famous studies about compressed members stability is derived by Leonhardt Euler (1707-1783) that evaluated the balance of a compressed bar with a linear elastic behavior material and subjected by a perfectly centered load . According Pfeil and Pfeil [6], Euler's study shows that, under the imposed conditions, the member keeps straight and without lateral displacements until the applied load reaches a certain critical load, named Euler's critical load or critical buckling load, given by:

$$N_{cr} = \frac{\pi^2 EI}{L^2 K}, \tag{4}$$

where E is the Young's modulus, I is the second moment of area, L is the unsupported length of bar and K is the effective length factor, in bars of trusses, $K = 1$.

For many decades, the design of compressed bars was limited to using Euler's study, not considering resistance reducing factors. In addition, due the to the simplicity of working with just one equation sufficient to define the compressive strength of bars, most studies on the optimization of trusses use the Euler critical load, or even the following expression:

$$N_{cr} = \frac{kEA^2}{L^2}, \tag{5}$$

being k a constant that relates the second moment of area to the to the cross-sectional area.

Although the classic models presented are very applied in optimization problems, they have limitations, as the use of a single safety factor to define all the uncertainties related to the problem. The brazilian technical standards of structures are based on the Load and Resistance Factor Design, that use different safety factors for each loading combination and the resistance and loads are decreased and increased, respectively, with appropriate and independent coefficients, enabling the statistical analysis of each design variable.

As the focus of this present study is steel structures, the main current standard, ABNT NBR 8800:2008 - Design of steel and composite structures for buildings [2], is used. The standard impose that a tensioned bar may fail due yielding in the gross section and the design tensile strength ($N_{t,Rd}$) in this cases is:

$$N_{t,Rd} = \frac{A_g f_y}{\gamma_{a1}} > N_{sd}, \tag{6}$$

being A_g the gross area of member, f_y the steel minimum yield strength, γ_{a1} the steel yield reduction coefficient and N_{sd} the requested axial effort of design.

The ABNT NBR 8800:2008 [2] considers that the instability of any compressed member can occur in the part as a whole (global instability) and partially in a constituent element of its cross section (local instability). The design compressive strength is given by:

$$N_{c,Rd} = \frac{\chi Q A_g f_y}{\gamma_{a1}} > N_{sd}, \tag{7}$$

where χ and Q is the reduction coefficient that considers the global instability and local instability effect, respectively. The brazilian standard limits the use of compressed bars with a slenderness ratio greater than 200. For sake of brevity, the equations to calculate the reduction coefficients are not suppressed from this paper, but they can be found at ABNT NBR 8800:2008 [2].

4 Case Studies

With the intention to validate the results obtained by the algorithms developed, a classical test case that have been used by the trusses optimization studies in scientific community is analyzed. Posteriorly, in order to investigate cases that approach real designs of steel truss structures, two examples are presented. It is noteworthy that the optimizations is done both by SQP algorithm and by Genetic Algorithm to compare the efficiency. The iterative process of both algorithms is stopped when the relative error tolerance between solution in current and previous iteration is 1.0x10^{-6}.

4.1 Eighteen-bar planar truss - Classical problem

The 18-bar cantilever planar truss (Fig. 1) is a classical problem in the literature analyzed by Imai and Schmit Jr [7] (Multiplier Method) and Lee and Geem [1] (Harmony Search Algorithm). A set of vertical loads, $P = 88,96$ kN, is applied on the upper nodal points of the truss.

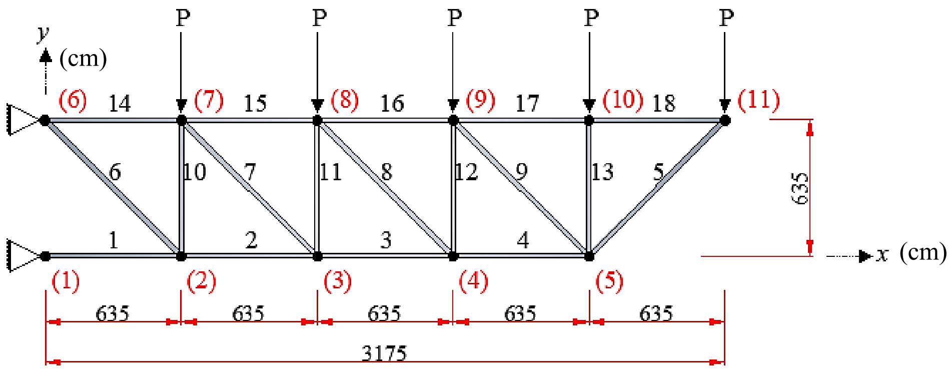

Figure 1. Eighteen-bar planar truss

The objective is minimize the structural weight from the cross-sectional areas of the bars. The Young's modulus and the material density of all members were 68.95 GPa and 27.15 kN/m³, respectively. The cross-sectional areas of the 18 members are linked into four distinct groups, in order to have the design variables: $x_1 = A_1 \sim A_5$; $x_2 = A_6 \sim A_9$; $x_3 = A_{10} \sim A_{13}$; $x_4 = A_{14} \sim A_{18}$. Design variables are analyzed in continuous domain and the side constraints is 0.6452 cm² $\leq x_i \leq$ 322.60 cm², for $i = 1, 2, 3, 4$.

The maximum yield stress of the bars is σ_{max} = 137.90 MPa and limits the behaviour of tensioned bars. For compressed bars, the Euler's critical load was imposed according Eq. 5, considering k = 4, value adopted by authors who studied this problem. The design variables (section areas) start the process with the value of 0.6452 cm². The optimal results obtained are compared to solutions presented by Imai and Schmit Jr [7] and Lee and Geem [1] in Table 1.

Table 1. Optimal design comparison for the 18-bar planar truss

Design variables (cm²)		Imai and Schmit Jr [7]	Lee and Geem [1]	SQP - This work	Genetic Algorithm - This work
1	$A_1 \sim A_5$	139.686	139.557	139.675	139.956
2	$A_6 \sim A_9$	45.614	45.532	45.616	45.606
3	$A_{10} \sim A_{13}$	80.650	80.585	80.642	80.714
4	$A_{14} \sim A_{18}$	64.507	64.391	64.511	64.500
Weight (kN)		28.614	28.577	28.608	28.638

Analysing the minimums weights obtained in this work, the results present an excellent agreement with those values found by the compared authors. SQP had a solution 0.0209% lighter than Imai and Schmit Jr [7] and 0.1084% heavier than Lee and Geem [1]. The Genetic Algorithm presented a solution 0.0838% and 0.2130% heavier than Imai and Schmit Jr [7] and Lee and Geem [1], respectively.

The differences of the results are due to the different optimization methods and tolerances for violating the constraints used in each job. The tolerance for violation of restrictions adopted by the compared authors is $1.0\text{x}10^{-1}$, while in this work, was adopted $1.0\text{x}10^{-3}$ of tolerance.

Despite presenting numerically close values, the SQP algorithm proved to be more efficient than the Genetic Algorithm in this example, not only for results a weight of the structure 0.105% lighter, but mainly due to the low computational cost, taking only 2.89 seconds in two iterations, compared to the 38.20 seconds needed for the Genetic Algorithm to perform seven iterations.

The cross-sectional areas grouped in x_2 and x_4 design variables are limited by the tension restriction, being A_6 and A_{14} the critical bars with axial efforts equal to 629.04 kN and 889.60 kN, respectively. Already the compressed bars, grouped in x_1 and x_3, are limited by the buckling restriction, the elements A_1 (-1334.40 kN) and A_{10} (-444.80 kN) are critical. Figure 2 illustrates the axial efforts acting on the truss.

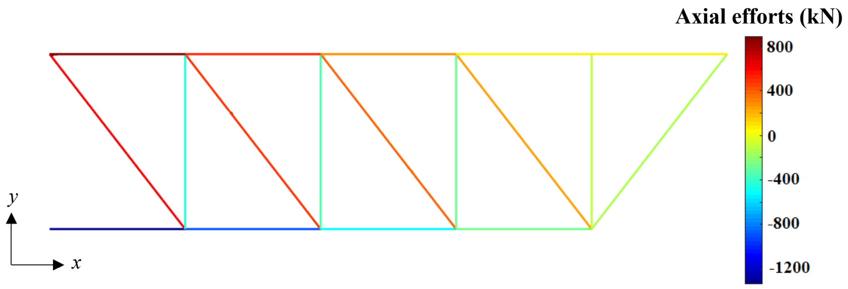

Figure 2. Axial efforts acting on the 18-bar planar truss

4.2 Eighteen-bar planar truss - Real steel structure design

In this case study, a 18-bar planar truss with a similar configuration to the first example is analyzed. The same set of vertical loads, $P = 88,96$ kN, is applied on the upper nodal points, as shown in Fig. 1. However, for this example, usual parameters in steel structure projects are used, while in the previous example, purely academic parameters are used, which are far from reality. The truss members are made by structural circular hollow tubes adopting the ASTM A36 steel as the material, where the Young's modulus (E), the density and the minimum yield strength (f_y) are 205 GPa, 78.5 kN/m³ and 250 MPa, respectively.

The optimization of steel structures faces discrete optimization problems, due to the commercial availability of precast profiles. In this case, the design variables are optimized in continuous domain and later converted to the commercial discrete value closest to the optimum, as long as this does not violate the design constraints. Again the areas are grouped into the same four sets represented by the design variables x_1, x_2, x_3 and x_4. The same initial estimate is adopted for all design variables, being 466.21 mm², corresponding to a cross-section with an outsider diameter (D) equal to 76.2 mm and a thickness (t) of 2 mm.

Since the objective is to optimize a real steel truss design, it is necessary to adopt the ABNT NBR 8800:2008 [2] recommendations. So, tensioned members should comply the Eq. 6 to not fail due yielding in the gross section, and compressed members should attend the Eq. 7 condition, to not fail due the global or local buckling. For comparison purposes, the classical formulation of Euler's critical load (Eq.4) and the simplified formulation (Eq. 5), adopting k=4, are applied in this example. The optimization is done by the SQP and Genetic Algorithm. Over again, the iterative process of both algorithms is stopped when the relative error tolerance between solution in current and previous iteration is $1.0\text{x}10^{-6}$. Table 2 presents the results of algorithms and constraint models applied to the problem in continuous and discrete domain.

Table 2. Optimal design comparison for the 18-bar planar truss - Parameters of real structures

	ABNT 8800:2008 [2]		Classical Euler's critial load		Simplified Euler's critial load	
Design variables	SQP	Genetic Algorithm	SQP	Genetic Algorithm	SQP	Genetic Algorithm
x_1 continuous (mm²)	15301.13	15416.44	5871.36	5913.62	8100.46	8123.13
x_1 discrete (mm²)	15301.13	15301.13	6283.97	6283.97	8105.43	8105.43
D (mm)	339.70	339.70	323.80	323.80	254.00	254.00
t (mm)	15.00	15.00	6.30	6.30	10.60	10.60
x_2 continuous (mm²)	2767.79	2766.79	2767.79	2766.79	2767.79	2766.79
x_2 discrete (mm²)	2806.07	2806.07	2806.07	2806.07	2806.07	2806.07
D (mm)	165.10	165.10	165.10	165.10	165.10	165.10
t (mm)	5.60	5.60	5.60	5.60	5.60	5.60
x_3 continuous (mm²)	4002.98	4002.98	2237.79	2237.79	4678.04	4681.22
x_3 discrete (mm²)	4002.98	4002.98	2237.79	2237.79	4704.35	4704.35
D (mm)	273.00	273.00	193.70	193.70	273.00	273.00
t (mm)	4.75	4.75	3.75	3.75	5.60	5.60
x_4 continuous (mm²)	3914.24	3913.24	3914.24	3927.13	3914.24	3923.86
x_4 discrete (mm²)	3933.59	3933.59	3933.59	3933.59	3933.59	3933.59
D (mm)	141.30	141.30	141.30	141.30	141.30	141.30
t (mm)	9.50	9.50	9.50	9.50	9.50	9.50
Weight (kN)	66.994	66.994	39.138	39.138	48.972	48.972
Number of iterations	3	12	3	4	3	3
Timer (seconds)	4.25	51.89	3.55	50.17	4.33	55.78

The results show that in continuous domain, the Sequential Quadratic Programming can converge to values different from the Genetic Algorithm, but both optimization methods fall back on the same values when converted to the closest discrete domain value.

As expected, the tensioned bars (grouped in x_2 and x_4 variables) result in the same optimal solution for the three cases compared, since the same restrictions were adopted to positive axial efforts. The situation that consider the ABNT 8800:2008 [2] standards presents the heaviest solution, being the most conservative to x_1, while the situation that imposes the simplified Euler's critical load results the largest value for x_3 areas. Despite this, a more conservative result does not necessarily attend all restriction situations, it being strictly necessary to analyze the behavior of the section adopted against the limiting conditions imposed in each restriction model.

5 Conclusions

The purpose of this study is the development of optimization algorithms applied to steel truss structures, evaluating the performance of the Sequential Quadratic Programming and Genetic Algorithm. The results obtained when using different formulations for the design of tensioned and compressed bars were compared.

For validation of algorithms, the first study case was a classical eighteen-bar planar truss applied in optimization problems. The SQP and the Genetic Algorithm provided good results when compared to other authors, with differences in the second decimal place. The main difference between the analyzed algorithms is the processing time. The computational cost of SQP algorithm is in the range of 12 to 14 times faster than GA.

Knowing that the truss studied in the first case adopts parameters that escape the reality of a real structural project, explored a similar eighteen-bar planar truss with usual parameters in steel structure projects. The results show that, using normative restrictions for compressed bars, it is necessary to adopt bars with a more robust cross sections when compared to solutions whose restrictions derive from the classical formulation of the Euler's critical load.

The more robust solutions presented by the brazilian standard are due to the strictness that ABNT NBR 8800:2008 [2] imposes by limiting compressed bars with a slenderness ratio lower than 200 and by using reducing resistance coefficients due to global and local instability effects, while the Euler's critical load considers only global instability effects.

It was observed that the simplified Euler's critical load, using k=4, generated a good solution compared to the classical formulation, however it is not possible to state that this solution also respects the stability conditions of the standard and could be used in a real project, since the integrity of the structure does not depend only on the cross-sectional area of the bar, but also properties such as the slenderness ratio and the local and global stability parameters.

Again, a similarity is observed between the results obtained by the SQP algorithm and the Genetic Algorithm, but due to the combinatorial nature of the latter, the heuristic method requires more processing time. In addition, the results obtained during the iterations are shown to be random until the end of the process, and may converge in solutions slightly heavier than the SQP.

Justified by the good results presented, it is stated that the developed algorithms are good structural optimization tools and can be used in real steel truss projects. Furthermore, the study corroborates the scientific community by presenting a critical analysis of optimization methods of different natures (mathematical and heuristic) applied to structural problems based on different design methods.

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