Slope Stability Analysis using Element-Free Galerkin Method and a visco-plastic approach with the shear strengh reduction technique

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Abstract. The majority of slope stability analyses are performed using either the computational approach of traditional Limit Equilibrium Methods (LEM) or the Finite Element Method (FEM). Several applications in geotechnical engineering involve large displacements, thus it becomes attractive to employ meshless methods. In this paper the meshless method Element-Free Galerkin (EFG) with a visco-plastic approach is applied to the slope stability analysis. An analysis is performed to demonstrate the capability of the EFG model to evaluate the slope stability, which is considered as a plane strain state for the stability analysis of homogeneous, isotropic and dry soil slope. A non-linear EFG approach with the shear strength reduction technique is applied to assess the factor of safety and potential slip surface. The results show good agreements with values found in the literature of classical methods (LEM and FEM), and with a meshless method. The failure criterion is assumed to be reached when the iterative process does not converge after a maximum number of iterations. The location and form of the potential slip surface are naturally obtained from the results of displacements and principal plastic strains.Therefore, the principal plastic stress or strain analysis becomes a strong alternative to verify the slope stability state.

Keywords: Slope stability analyses, Meshless, Element-Free Galerkin, Visco-plastic algorithm

1 Introduction

A slope failure can have catastrophic consequences, as has been observed in Brazil after the collapse of mining dams in Mariana and Brumadinho cities in 2015 and 2019, respectively (e.g. da Silva et al. [1]). The slope stability analysis verifies the stability state of a slope. Most slope stability analyses are performed using computational versions of the traditional *Limit Equilibrium Methods (LEM)* (Duncan and Wright [2]) or *Finite Element Method (FEM)* (Griffiths and Lane [3]). FEM presents advantages over LEM since the former considers the stress-strain relationship of the soil, and it is not necessary to assume hypotheses about the form and location of the failure mechanism. Despite its established use, FEM has limitations. Many applications of geotechnical engineering involve large displacements. In this aspect, FEM exhibits difficulties due to a mesh distortion problem inherent in the method. Considering this, it is attractive to use the so-called *Meshless/Meshfree (MFree) Methods*.

Recently, the MFree methods or particulate methods have been actively developed and applied in slope stability analysis. Several MFree methods have presented results that are in agreement with traditional methods regarding the factor of safety (FOS) and critical slip surface. Furthermore, MFree methods have the advantage of being able to capture the behaviors of large post-failure displacements (Bui et al. [4], Kwok et al. [5]). Bui et al. [4] and Li et al. [6] use *Smoothed Particle Hydrodynamics (SPH)* method (Lucy [7], Gingold and Monaghan [8]) in the analysis of slope stability. This type of analysis is also performed by Gago et al. [9] using the *Meshless Local Petrov-Galerkin (MLPG)* method (Atluri and Zhu [10]) through an elastoplastic constitutive model. Kwok et al. [5] use *Semi Lagrangian Reproducing Kernel Particle Methods (SL-RKPM)* (Guan et al. [11, 12]) in the analysis. The method *General Particle Dynamics (GPD3D)* method is proposed by Zhou et al. [13] to perform slope stability analysis.

In Qin and Wang [14]'s research, the *Element-Free Galerkin (EFG)* method is used in slope stability analysis, in which the main indices which influence slope stability are established and the analytical hierarchy process is used to determine the weight of the indices. By changing the data of the main indices, the changes of the nodal displacements are obtained. Then, it is possible to reach the final weight of the indices based on the interaction between the results obtained by the EFG method and the analytic hierarchy process. From this, a *fuzzy comprehensive evaluation* model is built. The purpose of this work is to use the EFG with the shear strength reduction technique to estimate the factor of safety. Material non-linearity is introduced by the visco-plastic model of Zienkiewicz and Cormeau [15].

2 Element-Free Galerkin Method

The *Element-Free Galerkin (EFG)* method, proposed by Belytschko et al. [16], is a numerical method used to solve differential equations. EFG is a global meshless method based on Galerkin's weak form and uses *Moving Least Square (MLS)*, which was developed by Lancaster and Salkauskas [17], to approximate the field variable. Several meshless methods have been developed because of the MLS approach. Various authors presented the EFG formulation in great detail (e.g. Belytschko et al. [16], Liu and Gu [18], Yorinori [19], Fries and Matthies [20], Nguyen et al. [21]). The procedure for solving *Boundary Value Problems* using the EFG method is briefly described below.

Once the geometry of the problem is defined, the problem domain is represented by a set of properly distributed field nodes that comprise the entire domain and boundary of the problem. In the MLS approach, a *local support domain* is assumed for a point of interest $\boldsymbol{x}$ (a sample point, field node or a quadrature point). The field nodes within the *local support domain* are used to compute the MLS shape functions at the point of interest $\boldsymbol{x}$. The approximation of the field variable at the point of interest, $\boldsymbol{u}(\boldsymbol{x})$, is done by a linear combination of the nodal values of the shape functions, $\boldsymbol{\phi}_i(\boldsymbol{x})$ and the nodal values of the field variable, $\boldsymbol{u}_i$, for the nodes that are within the *local support domain*.

$$\boldsymbol{u}(\boldsymbol{x}) = \sum_{i=1}^{n} \boldsymbol{\phi}_i(\boldsymbol{x}) \boldsymbol{u}_i \tag{1}$$

where n is the number of nodes within the local support domain of the point of interest $\boldsymbol{x}$. The construction of the shape functions is one of the most important issues in a MFree method because the function approximation process is defined based on an arbitrary set of nodes and therefore no *mesh* is required in the process.

The discrete system of equations is built in a process that involves the computation of nodal portions of the stiffness matrix, $\boldsymbol{K}$, and force vector, $\boldsymbol{F}$, which are added to the global system. In EFG, background cells are used to calculate the weak form integrals of the equilibrium equation. At the end of the process, the eq. (2) system is obtained.

$$\boldsymbol{KU} = \boldsymbol{F} \tag{2}$$

where $\boldsymbol{U}$ is the global vector of the field variable's nodal parameters.

Since the MLS shape functions do not have the Kronecker delta function property, special procedures are needed to impose the essential boundary conditions. The most used methods are the *Penalty method* and *Lagrange multipliers method*. For the same reason, after solving the system of equations from eq. (2) to $\boldsymbol{U}$, the approximation of the field variable at a point of interest is done by the interpolation showed in eq. (1).

3 Slope Stability Analysis using the Element-Free Galerkin Method and a visco-plastic approach

A computational model was developed to perform slope stability analysis using the visco-plastic algorithm (e.g. Griffiths and Lane [3], Zienkiewicz and Cormeau [15], Yorinori [19], Perzyna [22]). The factor of safety must be estimated, which is defined as the factor by which the soil strength parameters $\tan\phi_s$ and c must be reduced for the slope failure to occur, where c is the cohesion and ϕ_s is the friction angle of the soil. This process is currently known as the *shear strength reduction technique* (e.g. Griffiths and Lane [3], Matsui and San [23]). The factored shear strength parameters c_f and ϕ_f are given by:

$$c_f = \frac{c}{FOS} \tag{3}$$

$$\phi_f = \arctan\left(\frac{\tan\phi_s}{FOS}\right) \tag{4}$$

In the analysis, several FOS values are progressively tested. If the FOS value under test reduces the shear strength, resulting in failure, this will be adopted as the factor of safety for the slope under analysis.

The EFG method is used to approximate the field variables and to construct the discrete equation system. The global stiffness matrix is calculated only once, as is the vector of gravitational loads.

The elasto-plastic problem is solved by repeated elastic iterations over "time" using the *constant stiffness method*, in which material non-linearity is introduced by iteratively modifying the load vector. For each iteration, the stress state at each Gauss point is established. The state of stresses at the Gauss point is compared with the Mohr-Coulomb failure criterion by calculating the failure function, F. In terms of principal stresses (σ_1, σ_3), F is expressed by eq. (5) (assuming compression with a negative sign).

$$F = \frac{\sigma_1 + \sigma_3}{2}\sin\phi_f - \frac{\sigma_1 - \sigma_3}{2} - c_f\cos\phi_f \tag{5}$$

The function is defined so that it is negative within the failure envelope, zero over the envelope, and positive when stresses are outside the failure envelope. Positive values are not allowed and a yielding stress redistribution process must be used. Equation 6 schematically shows the relationships of the values assumed for F.

$$\begin{array}{ll} F < 0 & \text{Stresses inside failure envelope (elastic)} \\ F = 0 & \text{Stresses on failure envelope (yielding)} \\ F > 0 & \text{Stresses outside failure envelope} \\ & \text{(yielding and must be redistributed)} \end{array} \tag{6}$$

The redistribution of plastic stresses occurs by the visco-plastic algorithm (e.g. Griffiths and Lane [3], Zienkiewicz and Cormeau [15], Yorinori [19], Perzyna [22]). The plastic stresses are redistributed in the load vector that will be used in the next iteration.

At the beginning of each iteration, a convergence test is performed. The convergence test is used to verify the relative difference of the nodal displacement parameters between the current and the previous iteration. If the values of the relative differences for all degrees of freedom are smaller than the prescribed tolerance value (TOL= 10^{-4} is adopted in this paper), then the system is said to be convergent. A maximum value of iterations is prescribed and when reached it is assumed that there was no convergence (1000 iterations are adopted for this work). In this paper, the inability of the model to converge is admitted as the failure criterion, so the factor of safety that reduces the shear strength to the point where the model is unable to converge is the one adopted for the slope under analysis.

4 Results and discussions

A slope stability analysis is performed using the proposed model. The results obtained during the analysis are compared with the solutions of classical methods (LEM and FEM) and an MFree method (MLPG). Quadratic basis functions are used to compute the shape functions. The geometry of the slope to be analyzed is shown in Fig. 1. The slope height H = 1.0 m is adopted and the proportion factor for the foundation layer is D=1.5 (See Fig. 1). The slope is subject to the self-weight of the soil.

The boundary conditions are given as vertical rollers on the left and right boundaries, and full fixity at the base. The *Penalty method* is used to impose the essential boundary conditions. The slope is composed of dry, homogeneous, and isotropic soil. The soil parameters used in the analysis are friction angle, ϕ_s; cohesion, c; Young's modulus, E; Poisson's ratio, ν; dilation angle, ψ and unit weight of the soil, γ_s. The values adopted for these six parameters are presented in Table 1.

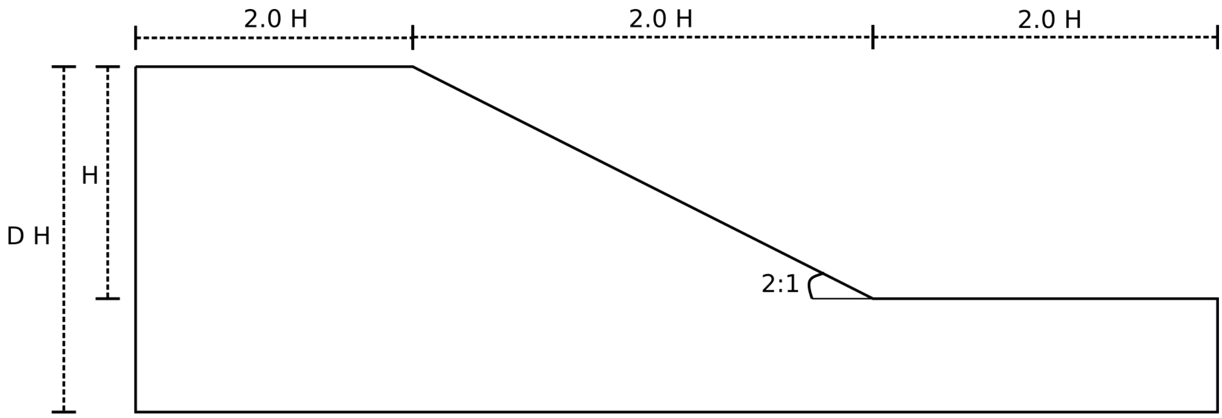

Figure 1. Slope geometry. Homogeneous slope with a slope angle of (2:1); H=1.0 m, D=1.5. $\phi_s = 20°$, $c/\gamma_s H = 0.075$.

Table 1. Soil parameters used in slope analysis with homogeneous, isotropic and dry soil.

Soil parameter	Nomenclature	Value	Unit
Friction angle	ϕ_s	20	$\circ$
Cohesion	c	1.5	kN/m^2
Young's modulus	E	5×10^4	kN/m^2
Poisson's ratio	ν	0.3	
Dilation angle	ψ	0	$\circ$
Unit weight	γ_s	20	kN/m^3

In the analysis, 820 field nodes are used for domain discretization. This value was chosen based on a convergence analysis of the displacement results. The shear strength reduction technique is applied for FOS values that are progressively increased, ranging from 1.40 to 1.60. From the value of 1.50, a more detailed approach is applied and therefore the FOS values are increased by an increment of 0.01 until the model is unable to converge. For the analysis, the dimensionless displacement is defined as $E\delta_{max}/\gamma_s H^2$, where δ_{max} is the maximum nodal displacement at convergence. The data obtained from dimensionless displacements, $E\delta_{max}/\gamma_s H^2$, and the number of iterations for convergence to be achieved for the different FOS values tested are presented in Fig. 2.

The results of dimensionless displacements reveal a sudden increase in its value when the model can't converge up to the established iteration limit, indicating the failure of the slope. This behavior occurs for FOS=1.60. Therefore, this is the value adopted as the factor of safety for the slope under analysis. Table 2 shows FOS values obtained by Gago et al. [9] for FEM, LEM and MLPG. These FOS values, when compared with the FOS calculated using the EFG model, reveal a good agreement with the FEM and LEM. The FOS value obtained by the MLPG is significantly lower than that achieved by the EFG model.

Table 2. Comparison between Factors of Safety obtained by Gago et al. [9] through different methods and by the proposed EFG model

Numerical Method	FOS	Reference
FEM	1.57	Gago et al. [9]
LEM - Jambu (Geostudio 2007)	1.62	Gago et al. [9]
MLPG com RBF	1.52	Gago et al. [9]
EFG	**1.60**	Proposed model

Figure 3 depicts the configuration of the slope at the rupture (FOS=1.60). The deformed configuration (Fig. 3 a) demonstrates that the failure mechanism occurs due to the slipping of the foot of the slope. The nodal displace-

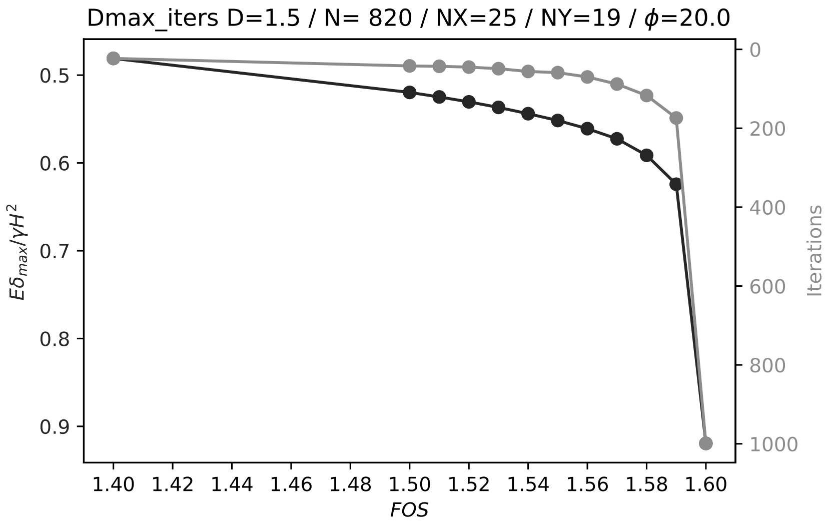

Figure 2. Dimensionless displacements (in black) and number of iterations do convergence (in gray) for different FOS values. At FOS=1.60, there is a sudden increase in the dimensionless displacement and the algorithm is unable to converge within the iteration limit

ment vectors (Fig. 3 b) indicate how the landslide occurs and the principal plastic strains (Fig. 3 c) reveal the shape and location of the potential slip surface.

5 Conclusions

Slope stability analysis is performed using a visco-plastic computational model of the Element-Free Galerkin method with the application of the shear strength reduction technique. The plane-strain state was used for the analysis of homogeneous and isotropic dry soil slope. The results obtained by the proposed EFG model showed that the FOS values determined are in agreement with the values found in the literature, giving reliability to the method. The EFG model was able to represent the slip failure mechanism at the foot of the slope. The location and shape of the potential slip surface were obtained naturally by the results of principal plastic strains or principal plastic stresses.

The EFG model proved to be adequate in solving the problem that involves large displacements. For these reasons, non-linear slope stability analysis using the EFG method offers real benefits over the classical LEM and FEM methods.

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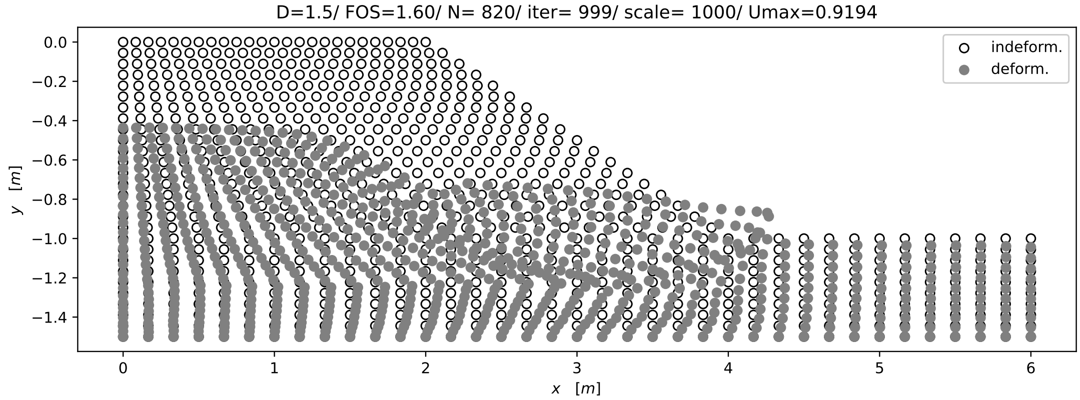
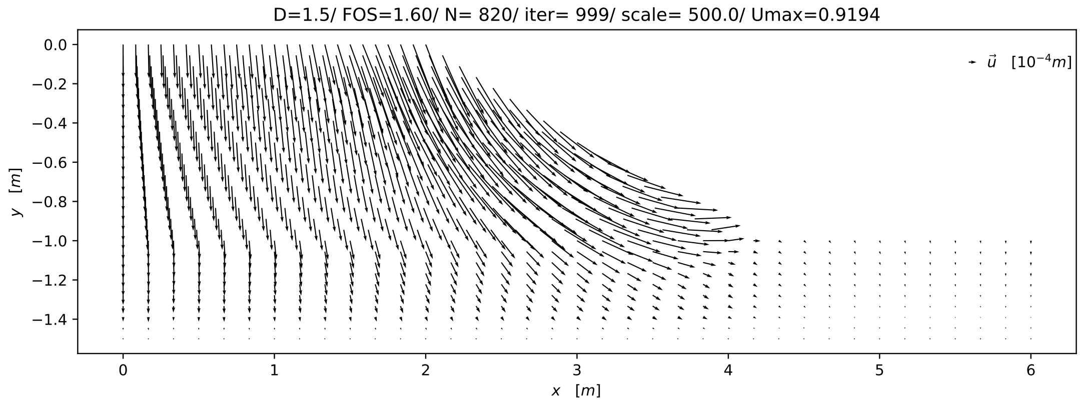
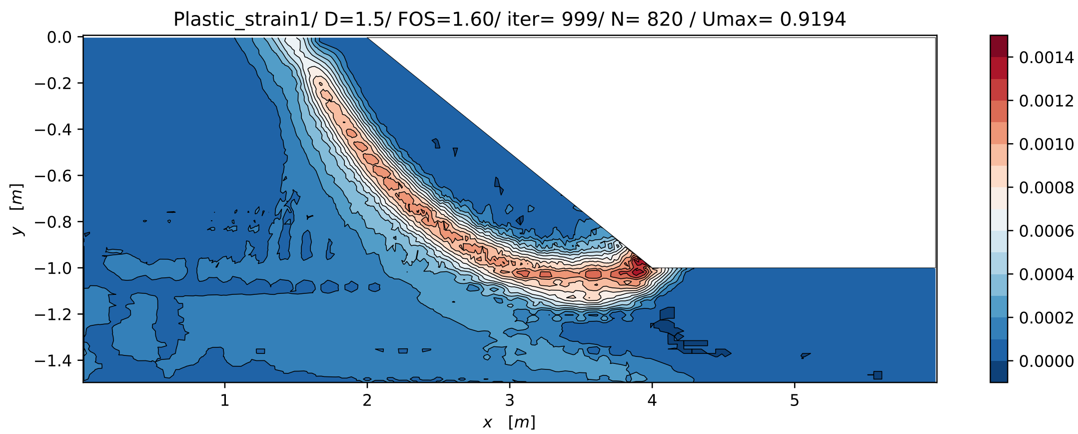


(a) Undeformed and deformed configurations



(b) Nodal displacement vectors



(c) Principal plastic strain ε_1^{vp}

Figure 3. Slope behavior during failure and identification of potential slip surface. Results of FOS=1.60 represent the behavior of the slope when failure occur.

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