Simulation of a Free Surface Laminar Flow Using The SPH Method

Willian Teles Pinto[1], Joel R. G. Vasco[1]

[1] *Graduate Program in Environmental and Sanitary Engineering, Federal University of Goiás*
Universitária Ave., 1488, 74605-220, Goiânia. Goiás, Brazil
williantelesamb@gmail.com, joelvasco@ufg.br

Abstract. The Smoothed Particle Hydrodynamics Method (SPH) is a Lagrangian numerical method used to solve engineering problems, notably on free surface flow. This article deals with numerical simulation using the SPH method to one-dimensional laminar flow on a free surface in an infinite channel. The numerical code developed discretizes the fluid domain in particles with constant mass, which do not move during the simulation and only present variation in their velocity. The boundary conditions at the bottom of the channel were implemented using ghost particles to reflect the no-slip condition. Ghost particles with corrections are implemented on the free surface to ensure that the shear stress on the free surface is zero. This correction ensures that the numerical results obtained in the SPH method are the closest to the analytical ones. The transient velocity profile was obtained by adopting the Eulerian framework in the SPH method and compared with the analytical ones, showing agreement between the results.

Keywords: free surface, laminar flow, SPH, boundary conditions.

1 Introduction

Smoothed Particle Hydrodynamics (SPH) is a particle, Lagrangian, and meshless numerical method that was originally created to simulate nonaxisymmetric phenomena in astrophysics [1][2]. 17 years later, the SPH method was successfully applied to free surface flows [3], solving the propagation of the bore wave through a dam break simulation. Since then, many free surface flows have been solved with the SPH method, such as hydraulic jumps [4], wave propagation and breaking [5][6], sloshing [7] and slamming [8].

Free surface flows in hydrodynamics are hard to simulate due to difficulties in determining boundary conditions. According to Colagrossi [9], despite the increasing diffusion of the SPH method, there is still a lack of detailed description and theoretical analysis of the terms associated with the free surface.

Therefore, this paper aims to apply the SPH method to a free surface laminar flow in an infinity channel. The SPH model is a one-dimensional (1D) code based on the heat equation with imposed free surface boundary condition. For the numerical model, the transient velocity profile is obtained and compared with the analytical solution. It is shown that only with the correct free surface boundary condition the numerical velocity profile agrees with the analytical results.

2 Laminar flow on free surface

The 1D free surface laminar flow occurs in a channel with slope θ and infinite bottom in which a viscous fluid flows with a low Reynolds number (Figure 1).

The application of the one dimensional code based on the Eulerian framework and on the Navier-Stokes equations with free surface treatment for the flow in the x direction, disregarding the pressure gradient $\partial p/\partial x$, results in

$$\frac{\partial V_x}{\partial t} = \nu \frac{\partial^2 V_x}{\partial y^2} \tag{1}$$

where v is the kinematic viscosity of the fluid [m^2/s] and V_x is the linear velocity in the x direction [m/s]. The boundary conditions of eq. (1) are V_x (0, t) = 0 to reflect the no-slip condition, and the partial derivative $\partial V_x/\partial y$ for (h, t) is equal to zero. The initial condition is V_x (y, 0) = 0.

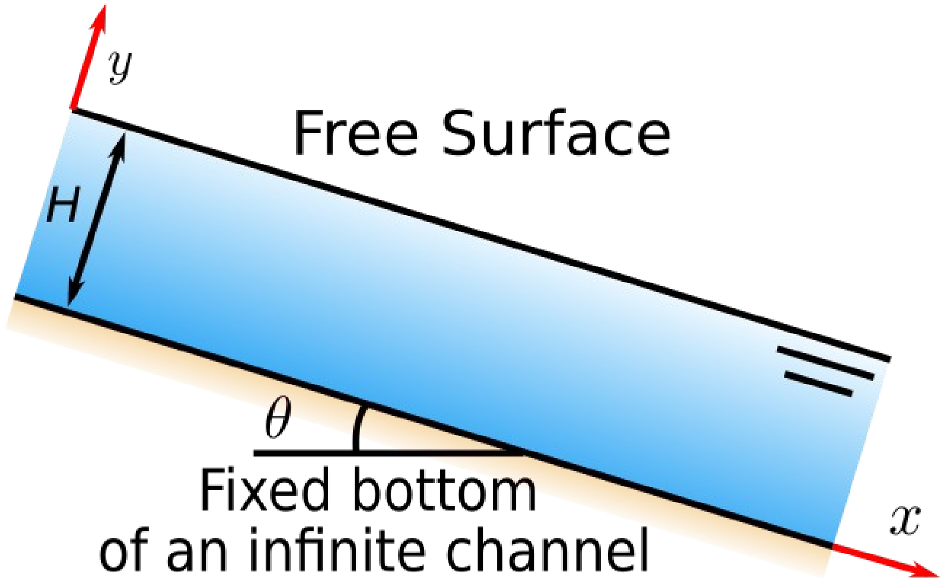

Figure 1. Description of the Free Surface Laminar Flow

2.1 Analytical Solution Equation

The analytical solution for the free surface laminar flow problem is

$$V_x(y,t)=\frac{-\gamma\, sin\theta\, y}{2\mu}(y-2H)+\sum_{n=1}^{\infty}-\frac{16\gamma\, sin\theta\, H^2}{(2n-1)^3\pi^3\mu}sen\left(\frac{(2n-1)\pi y}{2H}\right)\exp\left[-v\left(\frac{(2n-1)\pi}{2H}\right)^2 t\right]$$
(2)

where γ is the fluid specific weight [kg/m³] and μ is the dynamic viscosity [kg/m.s].

3 Smoothed Particle Hydrodynamics Method

In this section, the SPH equations applied to the free surface laminar flow in an infinite channel are presented.

3.1 Classical SPH equations

The classical SPH function, the integral interpolant, can be defined by

$$A_1(r)=\int A(r')W(r-r',h)\,dr' \tag{3}$$

where h is the smoothing length and the integration is over the entire space, and W is the kernel function which have to satisfy these two conditions

$$\int W(r-r',h)\,dr'=1 \tag{4}$$

and

$$\lim_{h\to 0} W(r-r',h)\,dr'=\delta(r-r') \tag{5}$$

where δ is the Dirac delta function, created by Paul Dirac in 1930 and often interpreted as unit impulse.

The Dirac Delta function was replaced by a 1D cubic spline kernel, which is a scalar function, specified by

$$W(r,h)=\frac{2}{3h}\begin{cases}1-\frac{3}{2}q^2+\frac{3}{4}q^3 & if\, q<1\\ \frac{1}{4}(2-q)^3 & if\, 1\leq q<2\\ 0 & if\, q\geq 2\end{cases} \tag{6}$$

with h is the smoothing length and $q = r_{ij}/h$ is the relative distance between particles, following the notation $r_{ij} = |y_i - y_j|$, where y_i is the vertical position of the particle i and y_j is the position of the neighboring particle j.

As suggested by Brookshaw [10] and Fatehi et al, 2011 [11], a SPH summation for the second derivative present in eq.1 can be approximated as

$$\left(\frac{\partial^2 V_x}{\partial y^2}\right)_i=\sum_{j\neq i}^{N_v} 2\frac{m_j}{\rho_j}\frac{(V_i-V_j)}{r_{ij}}\frac{(y_i-y_j)}{r_{ij}}\frac{dW_{ij}}{dr} \tag{7}$$

where Nv is the number of neighbors of the particle that directly depends on the smoothing length (h) and that in the analysis of particle i, only the neighbors particles are considered, being the particle itself disregarded.

3.2 Boundary Conditions

To simulate the boundary conditions ghost particles are created outside of the fluid domain ($0 \leq y \leq H$), above the free surface and below the channel. The velocities of the ghost particles created on the bottom of the channel are called V_{p1}, and V_{p2} for velocities of the ghost particles above to the free surface. To ensuring the boundary conditions, the velocities of ghost particle at the bottom of the channel are calculated in terms of the SPH method as

$$0=\frac{\sum m_j V_{xj} W(y_j,h)/\rho_j}{\sum m_j W(y_i,h)/\rho_j} \tag{8}$$

where zero order correction is applied to the W kernel, or Shepard correction.

Attributing that the velocity of the ghost particles on the bottom of the channel are equal, V_{p1} is

$$V_{p1}(t)=\frac{V_{(x,1)}W(0.5\,\Delta y,h)+V_{(x,2)}W(1.5\,\Delta y,h)}{W(0.5\,\Delta y,h)+W(1.5\,\Delta y,h)} \tag{9}$$

where $V_{(x,1)}$ and $V_{(x,2)}$ are the velocities of the first and second real particles, respectively, and Δy are the distance between the fluid particles.

Also assuming that the velocities of all ghost particles above the free surface are equal and the derivative on the free surface is zero, V_{p2} is

$$V_{p2}(t)=\frac{V_{(x,N)}\frac{dW}{dr}(0.5\,\Delta y,h)+V_{(x,N-1)}\frac{dW}{dr}(1.5\,\Delta y,h)}{\frac{dW}{dr}(0.5\,\Delta y,h)+\frac{dW}{dr}(1.5\,\Delta y,h)} \tag{10}$$

where $V_{(x,N)}$ and $V_{(x,N-1)}$ are the velocities of the real particle closest to the free surface.

4 Computational Simulations

The input data used on the computational simulations of this article are described in this topic.

4.1 Fixed input data

For the computational simulations, was admitted an channel with infinite bottom and slope θ = 0,1745 rad

(10º of bottom inclination) where a fluid flows with Reynolds equal to 1.

4.2 Particle distribution

The particles have been distributed in such a way that there are no particles exactly at the bottom of the channel ($y = 0$) or on the free surface ($y = H$). So the first particle is at a distance $\Delta y/2$ from the bottom of the channel, just as the last particle (N) is at this same distance $\Delta y/2$ below the free surface.

For the computational simulations, was admitted an infinite channel with a depth (H) equal to 0.6 m, in which 25 particles were equidistantly distributed on the y axis, resulting in a spacing between the particles (Δy) of 0.024 m (Figure 2). The smoothing length (h) used was 0.0288 m, resulting in a total of 4 ghost particles created, with two ghost particles located below the channel bottom and the same amount above the free surface of the channel.

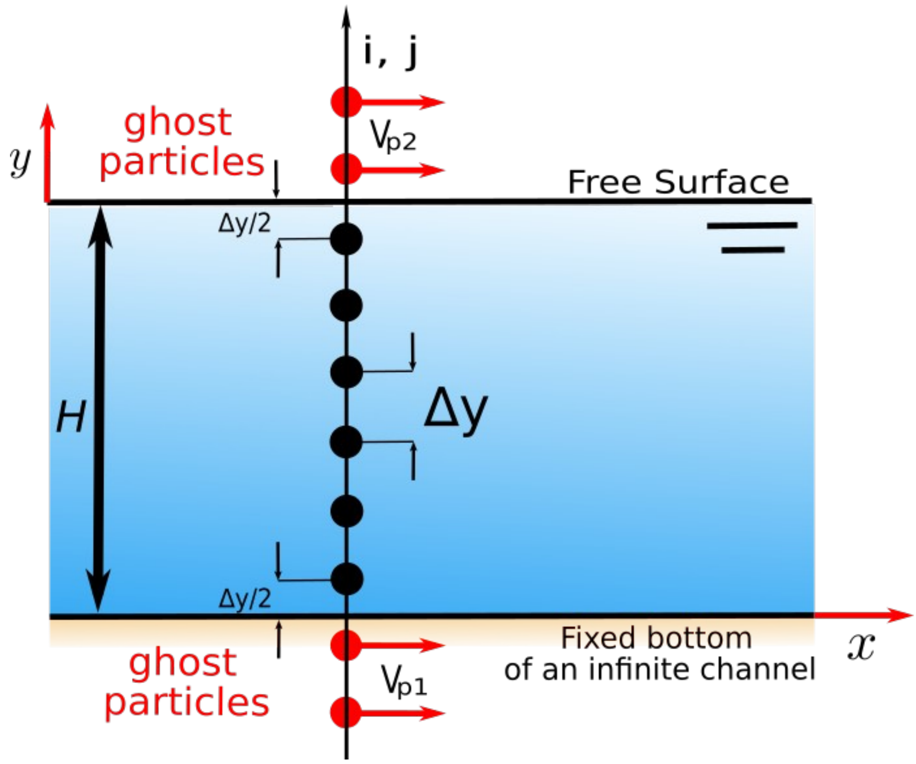

Figure 2. Description of particle distribution

5 Results and Discussions

After establishing the input data of the model, the analysis were performed with the SPH method and compared with the analytical solution for pre-established time points.

In order to make the axes of the graph dimensionless, the height of the particles in relation to the bottom of the channel (y) were divided by the maximum channel depth (H) and the particle flow velocity (Vx) divided by the average velocity (Vm). To also make the time dimensionless (t*), was applied the formula $t^* = t\ (g/H)^{0.5}$ (Figure 3).

From observing Fig. 3, the results obtained with the SPH Method have an agreement with the analytical solution for the initial times. However, as the flow approaches the steady solution, it is noticed that the difference between SPH and analytical solutions increases, especially the particles that are close to the free surface.

To alleviate this error in the SPH method, a second simulation was performed in which a correction was applied to ensure that the integral in the compact domain of the particle, i.e. the corrected gradient W, is null, resulting in a normalized kernel gradient (Figure 4).

Analyzing Fig. 4, it is noticed a better agreement between the SPH and analytic results due to the correctness of the kernel derivative. Even for particles that are close to the free surface, the method continues to converge.

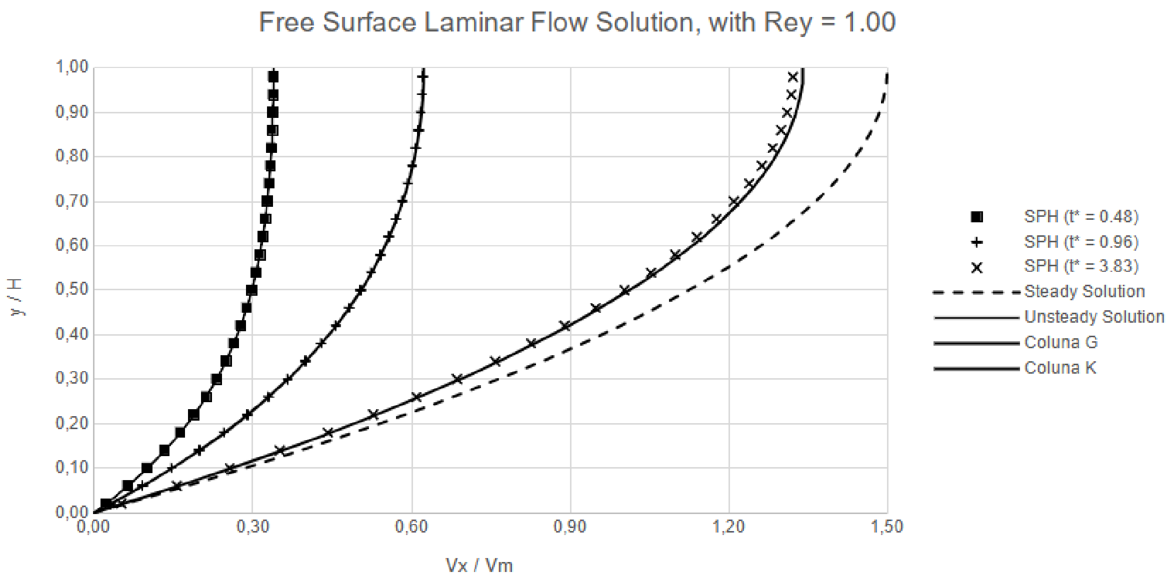

Figure 3. The results obtained in the SPH method application in a free surface laminar flow for different time steps.

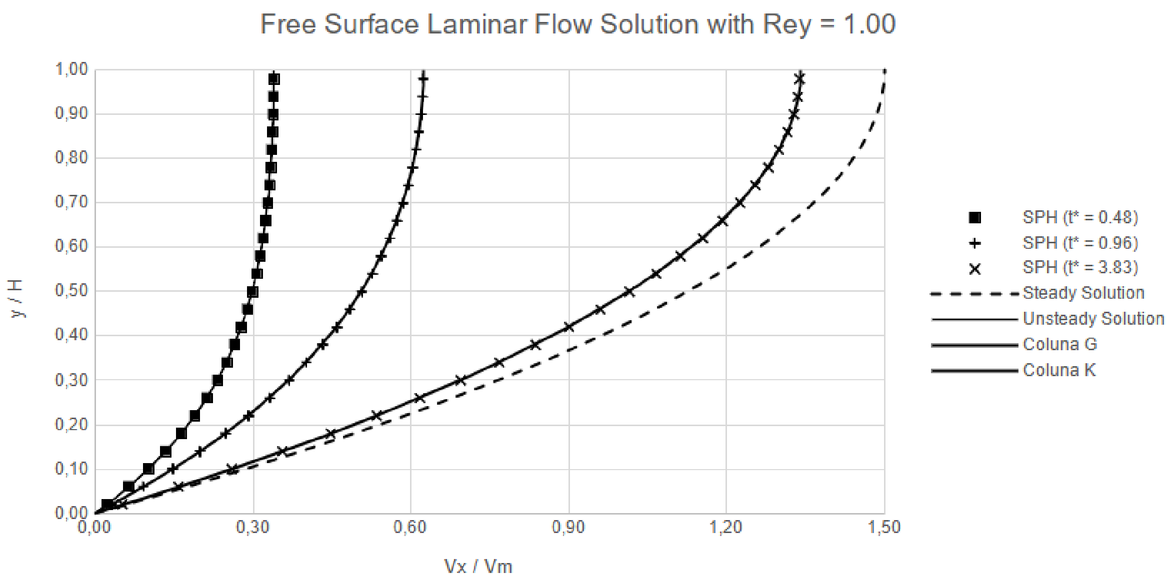

Figure 4. The results obtained in the SPH method application in a free surface laminar flow with the kernel derivative correction.

6 Conclusion

The transient velocity profile obtained with the SPH method and compared with the analytical ones showed great agreement between the results, mainly after the correction applied to the free surface. This demonstrates the need and importance of the correction on the free surface to obtain a result closer to the real situation.

The non-treatment of the free surface causes an error during the unsteady regime and that may not be noticed when analyzing only the steady state, as was done in the article written by Federico et al [4].

Acknowledgements. The authors would like to thank FURNAS Centrais Elétricas and the "Programa de Pesquisa e Desenvolvimento Tecnológico" (P&D) of the ANEEL for the financial support and the graduate program in Environment and Sanitary Engineering (PPGEAS) from the Federal University of Goiás (UFG).

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