Probabilistic optimization of a quarter car suspension with multi-objective framework and gradient based approximation

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Abstract. In this paper, a multi-objective robust optimization methodology is applied to the suspension optimization problem of a quarter-car numerical model. In order to increase the driver's comfort without compromising the drivability, the chosen objective function was the weighted RMS acceleration according to ISO 2631 with constrain regarding the suspension working space. The robust optimization is based in a probabilistic approach, more advanced compared to the interval based approach. Monte Carlo simulations are made to compare the statistics of the problem, as well as the failure probability. While the deterministic solution found 3.97% better mean acceleration values when compared to the robust optimization, the chosen solution generated by the multi-objective robust optimization results in a much lower failure probability: 10.55% for the Robust against 50% for the deterministic.

Keywords: multi-objective optimization, vehicle suspension, quarter car model, PSO, robust optimization.

1 Introduction

Vibration in vehicles is associated with many issues, like discomfort, excessive mental load, body ache, and even spine damage (Zamanian [1]). The vehicle suspension is designed to mitigate these problems, reducing the RMS (root mean square) acceleration on the passengers and driver. Active suspensions have shown to be more efficient, but are more complex and expensive than its counterparts. Thus, the passive suspension is still the most common type of suspension.

As stated in Chowdhury and Taguchi [2], the parameter optimization of the passive suspension is an important step in the vehicle design. The optimization step can be done using a variety of methodologies, where the most popular ones include the use of optimization algorithms, widely applied in engineering.

The numerical models and techniques for evaluating system responses in engineering are growing more and more precise year after year. Despite this evolution, the precision gained using advanced models and expensive approaches is often small compared to the uncertainty of such systems in real life. Besides that, the safety factor used to control uncertainties in engineering projects can lead to inefficient and heavy design if superestimated, and frail and risky design if underestimated. Hence, accounting for these uncertainties in the numerical study is of great importance.

The engineering area that accounts for both, optimization and uncertainty, is called robust optimization (RO) and is the focus of the present study.

Many authors have shown different frameworks to deal with RO in vehicle suspension. Cheng and Lin [3] used the software ADAMS to train a kriging surrogated model, which was then used in the optimization with a PSO (particle swarm optimization) algorithm. The use of a surrogated model is common in this research field since optimizing a vehicle model and account for uncertainties at the same time can be extremely expensive computational-wise. For this reason, the surrogated model technique can be seen in a variety of works, like: Cheng and Lin [3] (kriging), Gobbi *et al*. [4] (artificial neural network) and Park *et al*. [5] (artificial neural network). This approach can insert errors in the system, since the surrogated model admits errors depending on how it is trained.

Using Pearson or Spearman correlation coefficient to reduce the number of uncertain variables is also common (Gobbi *et al.* [4], Nohtomi *et al.* [6], Loyer and Jézéquel [7], Khalkhali *et al.* [8]). Again, reducing the number of uncertain variables can yield faster simulations, although this simplification can introduce errors in the final result.

To evaluate the robust part of the optimization (*i.e.* finding the mean and the variance of the objective functions and constrains), many authors use Monte Carlo approach (Jamali *et al.* [9], Nohtomi *et al.* [6], Khalkhali *et al.* [8]). Monte Carlo is still the most reliable method for analyzing the uncertainties, but it is also the most computationally expensive, and thus, many authors combine this approach with the surrogated model technique and the Pearson/Spearman correlation technique.

Another way to evaluate the robust part of the problem is using gradients and first-order Taylor Series expansion, like it is done by Loyer and Jézéquel [7], in frequency domain. According to Ang and Tang [10] this approach is fast, but the approximation is not very precise in the case where the objective functions or constrains are too non-linear and the uncertainty is sufficiently high.

Most of the time authors use single-objective optimization algorithms, like the PSO or QPSO for example, despite RO problems being multi-objective in nature (optimizing mean and variance accounts for 2 separate functions by themselves). Multi-objective algorithms demands much more function calls than its single-objective counterpart, and since the robust part of the problem is already very computational costly, this method is usually avoided.

2 Methodology

This work aims at using the Taylor Series expansion method, as well as the MOQPSO (Multi-Objective Quantum Particle Swarm Optimization, Grotti *et al.* [11]) to tackle the RO problem of a quarter car traveling on a irregular road type C (ISO 8608 [12]) at 20 m/s. The objective functions are the mean and variance of the weighted acceleration (ISO 2631-1 [13]), with constrain regarding the suspension working space to ensure that the drivability is not impaired. The first-order Taylor Series expansion approximation is compared to the traditional Monte Carlo method for the optimized design.

2.1 Irregular road profile

According to ISO 8608 [12], a good numerical representation of an irregular road profile can be achieved through the use of a simple stochastic process using PSD (Power Spectral Density) as follows in eq. 1:

$$G_\xi(\Omega) = \begin{cases} C\left(\dfrac{n}{n_0}\right)^{-w_1} & for\; n \leq n_0 \\ C\left(\dfrac{n}{n_0}\right)^{-w_2} & for\; n \geq n_0 \end{cases}, \tag{1}$$

where G_ξ is the spectral density (m²/cycle/m), n is the wave number (cycle/m), C is the general irregularity coefficient (m³/cycle), w_1 and w_2 are the wave length distribution. The PSD is divided in 2 parts in the discontinuity frequency n_0, (cycle/m), usually defined as $n_0 = \pi/2 \cong 0{,}16$ cycle/m. The PSD results in eq. 2:

$$x_a(t_i) = \sum_{i=1}^{ne} \sqrt{G_a(f_i)\Delta f}\, sin(2\pi f_i t_i + \varphi_i), \tag{2}$$

where φ_i are phase angles uniformly distributed between 0 and 2π, f_i are the spectral density frequencies, and ne are the number of spectral lines. The road chosen for this work is type C.

2.2 Human vibration exposure

The evaluation of the vibration on the human body has been studied extensively. Today, it is well known that the ride comfort is proportional to the RMS acceleration. Given a time instance t in the domain $t_1 \leq t \leq t_2$, the RMS acceleration, a_{rms}, for an acceleration signal, $a(t)$, is:

$$a_{rms} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2}[a(t)]^2 dt.} \tag{3}$$

ISO 2631-1 [13] goes beyond that, and states that the frequency of such vibration dictates how it is perceived by humans. In order to evaluate vibration in humans, a weighted sum approach is adopted:

$$a_w = [\sum_i (w_i a_i)^2]^{1/2}, \tag{4}$$

where a_w is the frequency weighted acceleration, and w_i is the weight of a given frequency a_i assigned to the i-th third of the octave band. Generally speaking, values between 2 and 10 Hz are the most perceived by the human body in terms of comfort. The acceleration signal must be measured on the seat (ISO 2631-1 [13]).

In order to evaluate vibration in humans, ISO 2631-1 [13] alerts that time exposure and the type of activity being performed (such as reading or resting, for example) should also be considered.

2.3 Quarter car suspension model

The suspension model of choice for this work is a quarter car with 3 degrees of freedom. A schematic depiction of the suspension model can be viewed in Fig. 1, where ξ is the excitation input coming from the road irregularities, m_1 is the unsprung mass, m_s is the sprung mass, m_c is the seat and driver mass, all in kg. z_1, z_s and z_c, in blue, are the degrees of freedom corresponding to m_1, m_s, and m_c. k_{ss}, k_{s1}, and k_{p1} are stiffness parameters of the driver's seat, suspension, and tire, and c_{ss}, c_{s1}, and c_{p1} are damping parameters of the driver's seat, suspension and tire, respectively.

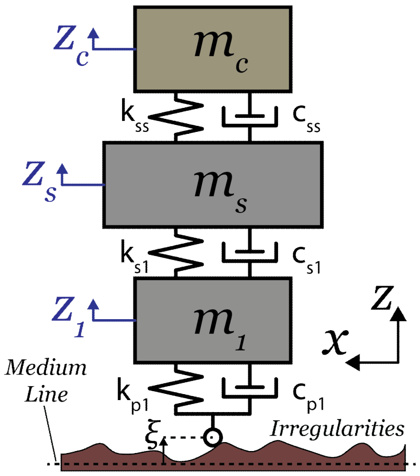

Figure 1. Schematic depiction of the quarter car suspension model used in the simulation.

The simulation time is set to T=5s, and the model velocity to v=20m/s. The simulation is solved in time domain using Newmark method. The numerical model can be written in matrix form in the following way:

$$\boldsymbol{K} = \begin{vmatrix} k_{p1} + k_{s1} & -k_{s1} & 0 \\ -k_{s1} & k_{s1} + k_{ss} & -k_{ss} \\ 0 & -k_{ss} & k_{ss} \end{vmatrix}, \tag{5}$$

$$\boldsymbol{C} = \begin{vmatrix} c_{p1} + c_{s1} & -c_{s1} & 0 \\ -c_{s1} & c_{s1} + c_{ss} & -c_{ss} \\ 0 & -c_{ss} & c_{ss} \end{vmatrix}, \tag{6}$$

$$\boldsymbol{M} = \begin{vmatrix} m_1 & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_c \end{vmatrix}, \tag{7}$$

and the equation of motion that governs the system:

$$\boldsymbol{M\ddot{X}} + \boldsymbol{C\dot{X}} + \boldsymbol{KX} = \boldsymbol{F_e}, \tag{8}$$

where $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{C}$ is the damping matrix, $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{X}$ is a vector containing the position of each degree of freedom, $\boldsymbol{\dot{X}}$ the first derivative of $\boldsymbol{X}$ with respect to time, $\boldsymbol{\ddot{X}}$ the second derivative of $\boldsymbol{X}$ with respect to in time, and $\boldsymbol{F_e}$ is the external forces vector given by:

$$\boldsymbol{F_e} = \begin{bmatrix} k_{p1} \\ 0 \\ 0 \end{bmatrix} \{\xi\} + \begin{bmatrix} c_{p1} \\ 0 \\ 0 \end{bmatrix} \{\dot{\xi}\}, \tag{9}$$

with $\dot{\xi}$ being the first derivative of ξ (external excitation input coming from the road irregularities) with respect to time. At last, the suspension working space can be calculated by $ws_1 = z_s - z_1$.

The nominal values for this model, as well as the search interval for project variables, and variance, can be seen in Tab. 1.

Table 1. Nominal values of the quarter car suspension model.

Parameter	Variable	Mean Value	Variance	Search interval	Unity
Sprung mass	m_s	730	37.5^2	-	[kg]
Seat and driver mass	m_c	75	6.25^2	-	[kg]
Unsprung mass	m_1	75.5	0	-	[kg]
Tire damping	c_{p1}	0	0	-	[N.s/m]
Tire stiffness	k_{p1}	175500	1250^2	-	[N/m]
Driver seat damping	c_{ss}	2500	100^2	$1000 < \mu(c_{ss}) < 4000$	[N.s/m]
Driver seat stiffness	k_{ss}	100000	4000^2	$50000 < \mu(k_{ss}) < 150000$	[N/m]
Suspension damping	c_{s1}	1250	50^2	$500 < \mu(c_{ss}) < 2000$	[N.s/m]
Suspension stiffness	k_{s1}	30000	600^2	$10000 < \mu(k_{s1}) < 20000$	[N/m]

2.4 Probabilistic gradient based uncertainty evaluation

There are two main strands for dealing with uncertainties: The probabilistic and the possibilistic approach. The possibilistic approach consists in finding the best and the worst combinations of the uncertain variables and parameters, without worrying about the statistics of the problem. This is useful when not much is known about the problem. However, when there is data about the problem (like the variance for example), a more advanced approach can be used: the probabilistic approach, which yields a more informative solution. In this work the gradient based uncertainty evaluation, which fits into the latter option, will be used. Let g be a general function of a vector $\boldsymbol{X}$ containing uncertain variables as follows:

$$Y = g(\boldsymbol{X}), \tag{10}$$

and defining expectancy, E, for continuous variables as:

$$E[g(\boldsymbol{X})] = \int_{-\infty}^{\infty} g(\boldsymbol{x}) \boldsymbol{f_X}(\boldsymbol{x}) dx, \tag{11}$$

where $\boldsymbol{f_x}$ is the PDF (probability density function).

According to Beyer and Sendhoff [14], Sundaresan [15], and Ang and Tang [10], it is possible to expand the function $g(\boldsymbol{X})$ using Taylor series on the mean value of $\boldsymbol{X}$. Truncating the series in the first order terms, the expectancy, E, and variance, Var, of g is found as such

$$E(Y) \cong g(\boldsymbol{\mu_x}), \tag{12}$$

$$Var(Y) \cong Var(\boldsymbol{X} - \boldsymbol{\mu_x})\left(\frac{dg}{d\boldsymbol{X}}\right)^2 = Var(\boldsymbol{X})\left(\frac{dg}{d\boldsymbol{X}}\right)^2, \tag{13}$$

with the derivatives being evaluated over mean of $\boldsymbol{X}$, $\boldsymbol{\mu_x}$.

Note that if $g(\boldsymbol{X})$ is approximately linear, this approximation should yield very good results for mean and variances of $g(\boldsymbol{X})$. Also, if $Var(\boldsymbol{X})$ is relatively small when compared to $g(\boldsymbol{\mu_x})$, this approximation should be adequate even when $g(\boldsymbol{X})$ is non-linear (Ang and Tang [10]).

This equations for mean and variance must be used to deal with the constrain problem in an analogous way.

2.5 Multi-objective optimization approach

As stated in the introduction, RO is a very expensive computational-wise problem. All methods used to account for uncertainties has shown to be very demanding, and most of the time the framework for solving RO problems is built around minimizing this heavy computational load. In this work, however, the gradient based method is used, which lessens the computational burden. This makes it possible to use more advanced approaches and algorithms on the optimization. The multi-objective approach using Pareto optimal set is most of the time avoided, despite RO being a multi-objective problem in nature. In this work the MOQPSO (Grotti *et al.* [11], Santana *et al.* [16], Grotti *et al.* [17], Gomes *et al.* [18]) will be used to tackle the multi-objective problem. The algorithm is based on the QPSO (quantum particle swarm optimization), and has been tested against high end algorithms like MUGA and NSGA-II, showing great results.

3 Numerical Solution

The RO problem is defined as

$$Minimize \begin{cases} f_1 = \mu\big(rms(a_w)\big) \\ f_2 = var\big(rms(a_w)\big) \end{cases}, \tag{14}$$

$$Constrained\ by\ G(x) = P(\max(ws_1) \leq ws_{crit}) > 90\%, \quad ws_{crit} = 0{,}3\ m, \tag{15}$$

where a_w is the weighted acceleration on the driver's seat, μ corresponds to the mean value, var corresponds to the variance, ws_{crit} means the critical value for the working space, and $G(x)$ is the constrain function. Equation 15 reads as: The probability for the maximum working space being smaller than ws_{crit} must be more than 90%.

The values of variance, uncertain variables, and parameters, are listed in Tab.1 for a total of 4 uncertain project variables and 3 uncertain parameters. The variance values corresponds to a coefficient of variation, CV=0.04, the same shown in Khalkhali *et al.* [8]. The resulting Pareto Front can be seen in Fig.2, where the point denoted by A is chosen using a utility function with equal weights ($UF = f_1w_1 + f_2w_2$). The point in red denoted by B is found using a deterministic approach and a single objective algorithm, the PSO (Particle Swarm Optimization).

The optimization parameters used in the PSO are: Number of particles, N=100; momentum, ω=0.9; individual and group cognitive components, c_1=c_2=2.01; turbulence generator, α=0.8; turbulence decay, α_t=0.99; and stop tolerance tol=0.001. For the MOQPSO: Number of particles, N=40; cluster toleration, $toll$=10^{-6}; contraction expansion coefficient lower bound, β_1=0.3; contraction expansion coefficient higher bound, β_2=1.3; number of front particles guide coefficient, $guide\_perc$=0.15; close particles guide coefficient, $guide\_prox$=0.30; extreme particles guide coefficient, $extreme\_guide$=0.30; and mutation factor, m=0.12.

A total of T=200460 function calls were used in both PSO and MOQPSO optimizations. Note that the solutions found in Fig. are result of the gradient based approximation and Taylor series expansion, and thus, are approximated values.

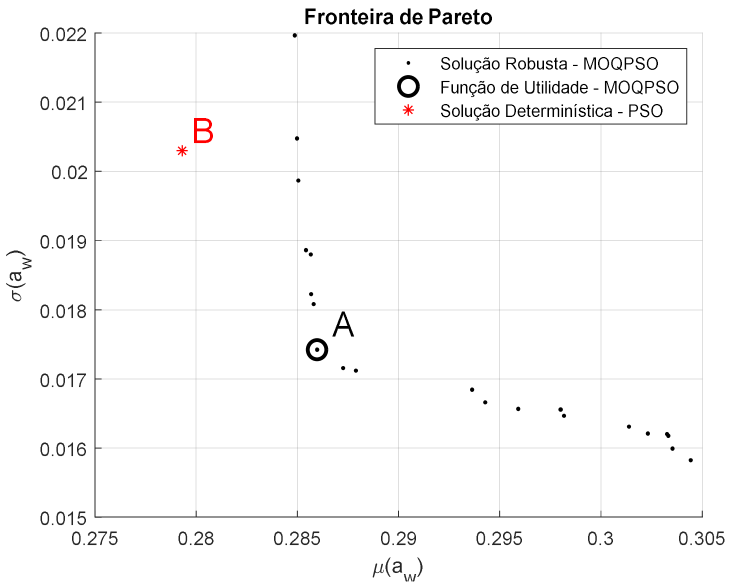

Figure 2. Solution of the RO problem in Pareto Front form (gradient based approximation).

Using LHS (Latin Hipercube Sampling) and Monte Carlo simulations (10000 function calls for each evaluation), it is possible to calculate the failure probability of each approach, the robust and the deterministic. The histograms for such Monte Carlo simulations can be seen in Fig.3, and the statistics of the solutions in Tab.2.

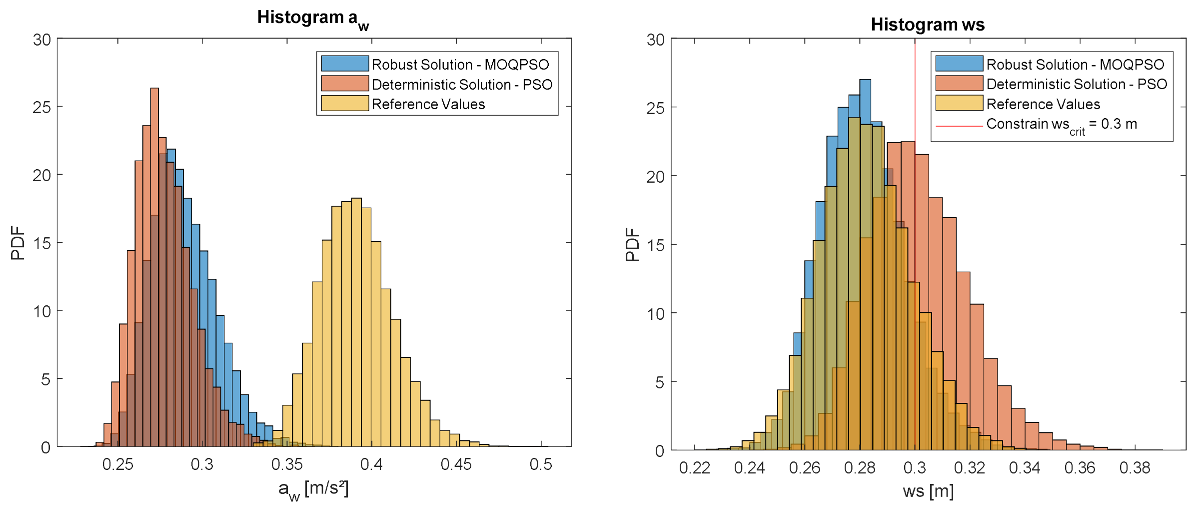

Figure 3. Monte Carlo histograms: at left, weighted acceleration histogram, and at right, working space histogram.

Table 2. Statistics for the solutions. Obtained using Monte Carlo (10000 function calls each).

	Failure Probability[%]	$\mu(a_w)$[m/s²]	$\sigma(a_w)$[m/s²]	$\mu(ws)$[m/s²]	$\sigma(ws)$[m/s²]
Reference Values	14.26	0.3908	0.0215	0.2823	0.0166
Deterministic (B)	50.00	0.2776	0.0169	0.3014	0.0179
Robust (A)	10.55	0.2891	0.0197	0.2810	0.0151

4 Conclusions

From Fig. 2, it is possible to notice that the deterministic solution, B, has achieved lower a_w when compared to the robust solution, A. Note that both solutions managed to decreased the acceleration from the reference values for 26.02% (A) and 28.96% (B), which can be checked in Tab. 2. Despite that, the failure probability (constrain violation probability) for solution A is found to be 50%, and for point B it is 10.55% (Tab. 2). This can be noted on Fig. 3 as well, where the deterministic histogram lays on the right side of the constrain line, indicating total violation. Hence, the 3.97% "better" solution resulting from the deterministic approach, is actually a risky design and should be avoided. This example shows how important it is to account for uncertainties in an optimization problem.

The Pareto Front approach also shows to be interesting, since it allows for the designer to choose between different sets of project variables depending on how much uncertainty there is on the application.

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