Modeling and optimization of multimodal piezoelectric energy harvesters from broadband vibration

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Abstract. Piezoelectric energy harvesting technologies from ambient vibration sources have attracted considerable attention in recent years for powering low-power autonomous electronic devices. In real-world applications, environmental vibration excitation may feature in a broadband spectrum presenting random characteristics. In this work, circular-shaped and pizza-shaped configurations are investigated as candidate designs for multimodal piezoelectric energy harvesting systems to harness energy from wideband vibration sources. A comparison with a conventional beam-type energy harvesting system is performed. The system dynamics are modeled, designed, optimized, and investigated using the finite element method implemented in ANSYS Workbench. Numerical simulations are carried out to investigate the system performances aiming to establish suitable performance conditions, considering harmonic and random excitations. The modeling approach is interesting to design other energy harvesting systems, especially with complex geometries. The proposed multimodal harvester has potential to harvesting energy from broadband vibration excitations showing performance advantages compared with classical single-mode energy harvesters.

Keywords: Energy harvesting, multimodal harvester, piezoelectric, finite element analysis, random vibration

1 Introduction

Several techniques for harvesting energy from environmental excitations have been studied intensively as an alternative to power sources like batteries or wired connections in small and low-power electronic systems such as microelectromechanical systems (MEMS) and wireless sensor networks (WSNs). Piezoelectric vibration-based energy harvesting systems have gathered considerable focus due to their structural simplicity and high conversion capacity. One of the main challenges is to develop efficient devices capable of adapting to diverse sources of environmental excitation. Usually, devices presented in the literature are related to piezoelectric energy harvesting systems based on a cantilever beam, which cannot explore all energy potential available in the environment. Since ambient vibration excitations may feature in a broadband frequency spectrum.

Piezoelectric beams are the most common vibration-based configuration studied in the literature of energy harvesting systems due to their structural simplicity and power density. These devices can be modeled as linear electromechanical oscillators designed to operate under one of their resonant frequencies. The system operates efficiently in resonant conditions, losing performance when the excitation frequency energy spread over a broadband spectrum (Erturk and Inman) [1].

Several strategies have been investigated to improve the performance of linear energy harvesters, including multi-modal, frequency-tuning, and nonlinear systems. Multimodal systems constitute an alternative to broaden the operational frequency range of linear energy harvesters as discussed in (Kim, Jung, Lee and Jang) [2] and (Wu, Tang, Yang and Soh) [3]. The multimodal system can be designed aiming at close resonant peaks, increasing the operational frequency range (Zhu, Tudor and Beeby) [4]. One interesting approach is to ensemble an array of single-degree-of-freedom (SDOF) oscillators with different resonant frequencies (Rezaei and Talebitooti) [5] and

(Ferrari, Ferrari, Guizzetti, Marioli and Taroni) [6]. The system inefficiency when only one or a few oscillators contribute to the generated electrical power is one main drawback. Different oscillators can be coupled together in multimodal energy harvesters to provide broader frequency bandwidth, generating energy more efficiently (Tang and Yang) [7]. Jang, Rustighi, Brennan, Lee, Jung [8]; and Kim, Jung, Lee and Jang [2] developed a linear energy harvesting device designed with two very close natural frequencies. Upadrashta and Yang [9] proposed a novel trident-shaped device configuration with three close resonant frequencies to generate electrical energy from low frequency and wideband spectrum excitations. However, frequency response curves of multimodal harvesters are characterized by valleys between resonant peaks that cause performance drops in the overall spectrum. Therefore, this could be addressed by designing the system to have the resonant peaks spread along the desired frequency spectrum, but close enough in order to mitigate the antiresonant valleys.

Moreover, the concept evolution of multimodal energy harvesting system are becoming more complex in order to increase the system performance. In this regard, the Finite Element (FE) method is an interesting modeling approach for the design, analysis, and optimization of these energy harvesting systems according to the required application. Several works based on the FE analysis can be indicated on energy harvesting from ambient vibration. Numerical and experimental results in good agreement are reported by (Abdelkefi, Barsallo, Tang, Yang and Hajj) [10], (Upadrashta and Yang) [9,11], (Zhu, Worthington and Njuguna) [12] and (Zhu, Worthington and Tiwari) [13].

This work proposes a multimodal piezoelectric energy harvesting device to harness energy from a wideband ambient vibration source. The main contribution is to propose a finite-element-based analysis to design, optimize, and investigate multimodal energy harvesting systems to operate in a wideband spectrum. Circular-shaped and pizza-shaped configurations are employed as candidates for the device, comparing their performance with classical cantilever beam devices. Different types of ambient vibration sources are investigated, aiming to establish suitable performance conditions for the proposed harvester design.

2 Design concept evolution and optimization

This section describes the concept evolution of a multimodal piezoelectric energy harvesting device toward energy extraction from wideband vibration excitation. The analysis considers a conventional cantilever beam configuration, well known in the literature of vibration-based energy harvesting, as a reference performance case. Others energy harvester designs are exploited, starting from a circular-shaped geometry that evolves to different pizza-shaped configurations based on geometric and electric optimization. The finite element method is employed to design, develop, optimize and investigate the system to establish suitable performance conditions considering different types of vibration excitations found in real applications. Figure 1 shows a schematic picture of some design configurations: cantilever beam; circular-shaped plate; 4-slice pizza-shaped (4-SPS); 8-slice pizza-shaped (8-SPS); 4-irregular-slice pizza-shaped (4-ISPS); 8-irregular-slice pizza-shaped (8-ISPS).

The design concept evolution starts from a circular-shaped plate geometry (Fig. 1.b) that evolves to pizza-shaped configurations by dividing the plate into four and eight slices, originating the 4-SPS and 8-SPS configurations showed in Figures 1.c–d. The circular plate partition in slices provides versatility in adjusting the resonant frequencies of the system. This design strategy can be evolved by considering irregular slices with different radius for each slice of the pizza-shaped configuration. Based on this concept, two additional configurations are considered the 4-ISPS – Figure 1.e; and the 8-ISPS – Figure 1.f. Another interesting aspect employed in the conceptual evolution of energy harvester designs is the use of tip mass. Besides allowing a proper tuning of the resonant frequencies by carefully changing the weight of the tip mass. A tip mass causes higher amplitude oscillations, increasing the strain level of the piezoelectric element and consequently, the generated output power. Tip masses are included in the cantilever beam, 4-ISPS, and 8-ISPS energy harvesters.

These device configurations can be modeled as discrete mechanical systems represented by an equivalent mass-spring oscillator for each vibration mode with its own resonant frequency. The frequency spectrum of the system is a combination of the spectrum of each oscillator. Therefore, the goal strategy consists of spread the resonant peaks over the frequency spectrum, but keep the gap between consecutive peaks close enough in such a way that the antiresonant valleys can be attenuated in order to extract energy efficiently in a broadband spectrum.

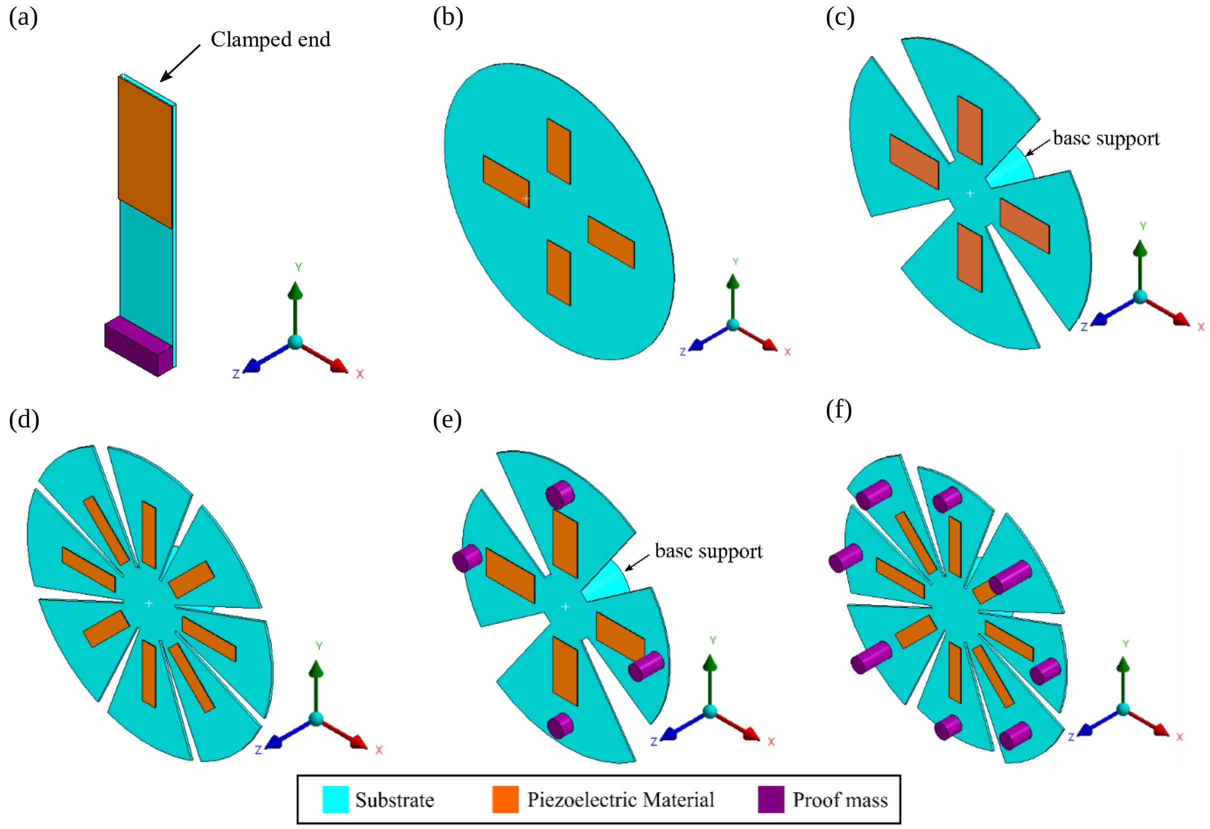

Figure 1. Piezoelectric energy harvesteing configurations: a) cantilever beam, b) circular-shaped plate, (c) 4-slice pizza-shaped, (d) 8-slice pizza-shaped, (e) 4-irregular-slice pizza-shaped and (f) 8-irregular-slice pizza-shaped energy harvesters.

2.1 Optimization procedure

The FE method is employed using ANSYS Workbench environment to either analyze and optimize the energy harvester devices. A piezoelectric analysis involves the interaction between structural and electrical fields that are represented, respectively, by a displacement vector, $\{u\}$, and electrical potential, φ. The general form of a single element electromechanical equation of motion, assuming linear approximation of the piezoelectricity theory, can be written as follows:

$$\begin{bmatrix} [m] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}\} \\ \{\ddot{\varphi}\} \end{Bmatrix} + \begin{bmatrix} [c] & [0] \\ [0] & -[c^d] \end{bmatrix} \begin{Bmatrix} \{\dot{u}\} \\ \{\dot{\varphi}\} \end{Bmatrix} + \begin{bmatrix} [k] & [k^c] \\ [k^c]^t & -[k^d] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{\varphi\} \end{Bmatrix} = \begin{Bmatrix} \{f\} \\ \{q\} \end{Bmatrix}, \tag{1}$$

where $[m]$ is the inertia matrix, $[c]$ is the structural damping matrix, $[c^d]$ is the dielectric damping matrix; $[k]$ is the stiffness matrix, $[k^d]$ is the permittivity matrix, $[k^c]$ is the piezoelectric coupling matrix where the subscript $[\ ]^t$ indicates that the matrix is transpose; $\{q\}$ is the electric charge and $\{f\}$ is the force vector.

Three types of analysis are used to achieve the main goals of this work: modal, harmonic, and transient analyzes. An optimization procedure is considered in this work based on the idea of harness energy from a wideband ambient vibration excitation from a piezoelectric energy harvester device. The main objective is the design of the energy harvester to optimize a multimodal device to operate in a broadband spectrum by setting as many resonant frequencies as possible in a required operational frequency range. The optimization is performed considering a frequency range defined by a lower limit, ω_L, and an upper limit, ω_U, containing p resonant frequencies in it. By considering that ω_i $(i = 1, \ldots, p)$ is a generic frequency within this interval, it is possible to define the optimization problem:

$$\min_{\omega} \; f_i(\omega)$$
$$\text{subject to } g_j^{\omega} \le 0 \text{ and } g_k^{M} \le 0. \tag{2}$$

The objective function is based on frequency determination as follows,

$$\begin{aligned} f_1 &= \omega_1 - \omega_L \\ f_i &= \omega_i - \omega_{i-1} \; (i = 2, \dots, p-1), \\ f_p &= \omega_p - \omega_U \end{aligned} \tag{3}$$

and the constraints are defined considering a minimal value of variation from consecutive frequencies, ω_g, and a maximum value of the weight of each tip mass, M_U. Therefore, it is written

$$\begin{aligned} g_j^{\omega} &= \omega_g - f_i \le 0 \quad (j = 1, \dots, p-1) \\ g_k^{M} &= M_k - M_U \le 0 \quad (k = 1, \dots, N_m), \end{aligned} \tag{4}$$

where N_m is the number of tip masses and M_k is the weight of the k-th tip mass.

The optimization procedure is built as a function of resonant frequencies of the system. Mechanical and geometric parameters like length, width, thickness, and the weight of tip masses play an important role to adjust the resonant frequencies in the desired range. Therefore, resonant frequencies are written as a function of the following parameters: length (or radius), thickness (h), tip mass (M). In addition, a range for each one of these parameters is defined by lower and upper bound values. Therefore, it is written,

$$\omega_i = \omega_i(r, h, M)$$
$$\text{subject to the constraints:} \tag{5}$$
$$r_L \le r \le r_U, \qquad h_L \le h \le h_U, \qquad M_L \le M \le M_U.$$

This optimization process is achieved using a Goal-Driven Optimization (GDO) method available in ANSYS Workbench, which is a deterministic method based on three main components: *Design of Experiments* (DOE), *Response Surface* (RS) and *Optimization*. Initially, the DOE is used to determine a set of sampling points (design points) that allows one to generate and solve a table of N_p design points. Response surfaces are built from the DOE data and provide approximated values of output parameters in the design domain by fitting surface curves. The optimization component of the GDO is a constrained, multi-objective optimization technique to obtain the best possible designs from a sample set given the objectives or constraints set for parameters.

The optimization procedure is performed in two stages. Initially, geometric parameters (length, thickness, and weight of tip mass) are determined by setting resonant frequencies to align in a desired frequency range in order to extract energy from a wideband spectrum. Once the optimized geometry of the system is acquired, the second stage consists of searching for the optimal value of resistance load, which represents the electric storage circuit, that is coupled to the harvester model to measure output power. Harmonic analysis is developed to obtain the steady-state response of the system. The total output power is established as a function of the resistance load. Then, the following single-objective optimization problem is solved to determine the optimal value of the resistance load, which generates maximal power output.

$$\max_{R} \; P(R)$$
$$\text{subject to the constraint:} \tag{6}$$
$$R_L \le R \le R_U,$$

where R is the value of resistance load; R_L and R_U are lower and upper bound values for the resistance load; P is the peak output power taken from the frequency spectrum response curve.

3 Performance investigation of energy harvesting configurations

This section is destined to investigate the energy harvesting systems considering excitation uncertainties representing diverse sources of ambient vibrations. Simulations are carried out under harmonic and random excitations to obtain the system responses in terms of output power and power density. The system responses are tested considering harmonic and random excitations. The proposed configuration is compared with the classical piezoelectric energy harvesting device to establish the advantages of operating in a broadband frequency spectrum.

The power density (PD) is one of the most commonly used performance metric for energy harvesters, which is defined as the output power over unit of volume. The PD metric is written as a function of the RMS output power and the volume of piezoelectric material. Figure 2 presents a comparative analysis of the PD generated for all energy harvester configurations in the frequency range of 100 – 150 Hz. Results show that the cantilever beam configuration is more efficient in terms of peak PD. It is noticeable that the pizza-shaped designs achieve wider frequency bandwidth compared to conventional energy harvesters. The frequency bandwidth is expanded by designing the resonant peaks to be spread enough in the concerned frequency range but sufficiently close to mitigate the valleys among them. This is advantageous when the ambient vibration spectrum is distributed away from the resonant frequency of the system and the beam harvester loses performance. Therefore, the proposed multimodal piezoelectric pizza-shaped device shows potential to extract energy from ambient vibration sources with a wideband spectrum.

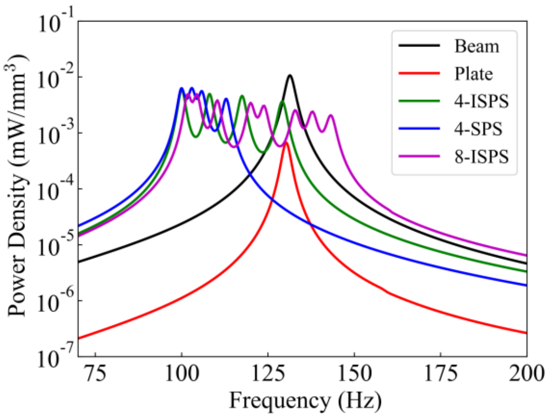

Figure 21. Comparison analysis of power density response curves of all five EH configurations.

Ambient vibration sources may present fluctuations with varying amplitudes and with energy spread over a broad frequency spectrum. Therefore, the performance of the harvesters is investigated considering deterministic and nondeterministic excitations: pure harmonic, harmonic with random-frequency; and random-amplitude excitation. The excitation is defined as $u_b(t) = u_0 \sin(\Omega t)$ where u_0 is the base excitation amplitude. Uncertainties are considered by representing its frequency as a Gaussian white noise, defined by $\Omega = N(\bar{\Omega}, \sigma)$, with a mean value $\bar{\Omega}$ and standard-deviation σ. A pure harmonic excitation is obtained for $\sigma = 0$ and values of $\sigma \neq 0$ define a harmonic random-frequency excitation.

Figure 3.a shows a histogram for the frequency distribution considering two values of standard-deviations: $\sigma = 1$ and $\sigma = 3$. The harmonic random-frequency excitation for an amplitude of $u_0 = 14.7$ μm and $\sigma = 3$ is showed in Figure 3.b. Subsequently, the energy harvesters are tested considering a random-amplitude excitation condition by assuming $u_b(t)$ as a Gaussian white noise $u_b(t) = N(\bar{u}, \sigma)$ with a bandlimited frequency spectrum. Figure 5.b presents the time-domain of a bandlimited random-amplitude excitation obtained by applying a windowing of 100 – 150 Hz in the frequency spectrum of a Gaussian white noise signal with u_0= 0 and $\sigma = 49$ μm. The power spectrum density (PSD) of Gaussian white noise and the PSD after applied the windowing are shown in Figure 5.a.

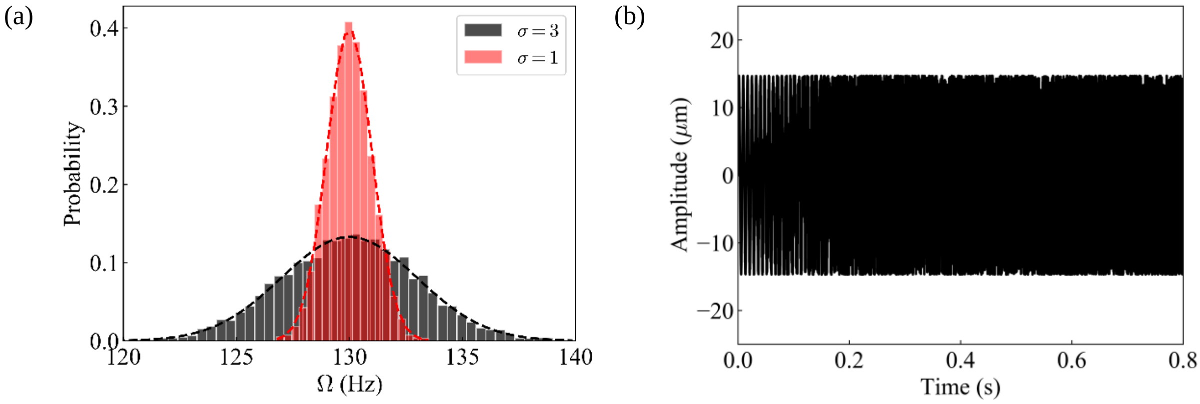

Figure 3. (a) Histogram of random-frequency excitation showing a Gaussian distribution with a mean value of 130 Hz and standard-deviations of $\sigma = 1$ and $\sigma = 3$; (b) random-frequency excitation for $\sigma = 3$.

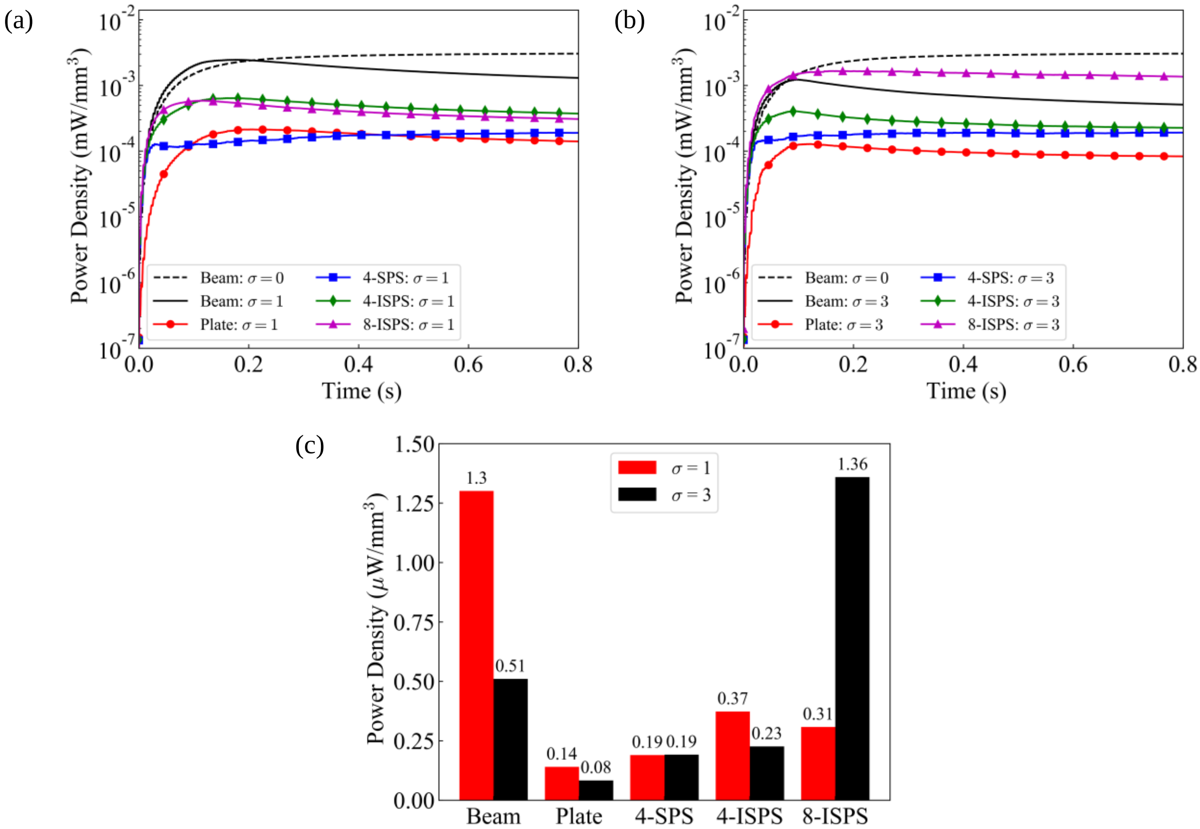

Figure 4. RMS power density curves for random-frequency excitation with $\overline{\Omega} = 130$ Hz, and standard-deviations of (a) $\sigma = 1$ and (b) $\sigma = 3$; (c) accumulated RMS power density comparison for all EH configurations.

Transient simulations are carried out to determine time-history response curves of all configurations in terms of RMS PD as shown in Figure 4.a–b. The beam device presents better performance for a random-frequency excitation with $\sigma = 1$, where the energy is concentrated near the first resonant frequency of the beam. The circular-shaped plate and pizza-shaped configurations present lower levels of PD. For this excitation condition, only one or few piezoelectric patches of the pizza-shaped configurations are essentially contributing to the overall output power, and therefore, an inefficiency is observed in terms of generated energy by unit of volume. Moreover, when the frequency spectrum of ambient vibration is broadened by increasing the standard-deviation to $\sigma = 3$, the proposed 8-ISPS configuration presents higher levels of PD since more piezoelectric patches are contributing to the overall output power, while the remaining configurations are less efficient. Figure 4.c shows the accumulated RMS power density for a time of 0.8 s for all concerned EH configurations. It is noticeable from Figure 4.c that the proposed 8-ISPS can be designed to provide good performance metrics based on the available excitation frequency spectrum.

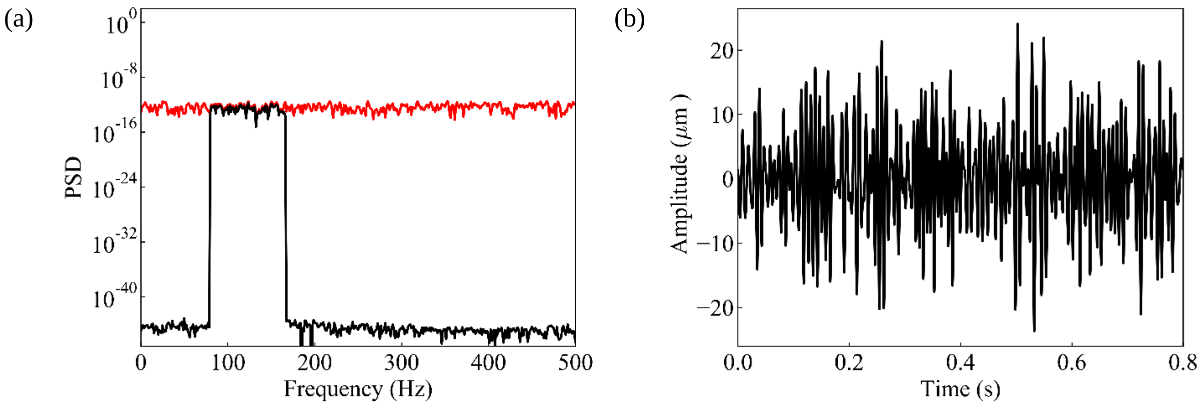

Figure 5. (a) Curves of PSD spectrum for a Gaussian white noise without windowing (in red) and with windowing in the range of 80 – 165 Hz (in black); (b) time-history signal for a band-limited random-frequency base excitation.

Finally, uncertainties in the amplitude of vibration are considered to test the energy harvesting configurations under ambient excitation with fluctuations in the amplitude. The time-history RMS PD responses and accumulated RMS PD after 0.8 s are presented, respectively, in Figure 6.a and Figure 6.b. Similar to the previous case, the proposed 8-ISPS device shows better performance compared with the circular-shaped plate and the remaining pizza-shaped configurations, presenting a minor difference of PD when compared with the conventional beam device. The main interesting observation regarding the proposed 8-ISPS configuration is related to the different performance obtained for both excitation conditions investigated. For the random-frequency excitation, the resonant phenomenon is more pronounced, since more modes are contributing to the overall output power. For the random-amplitude excitation, it seems that only one vibration mode is really contributing to the overall output power at a time, and therefore, close values of PD are observed for both cantilever beam and the proposed 8-ISPS.

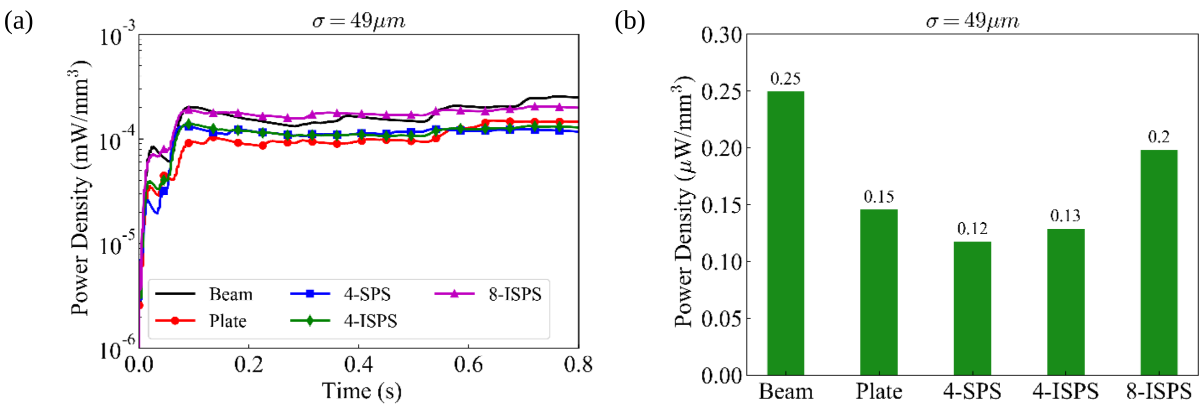

Figure 6. (a) power density time-history curves for random-amplitude excitation with $\sigma = 49\ \mu m$; (b) accumulated power density comparison for all EH configurations.

4 Conclusions

This work proposes a multimodal eight-slice pizza-shaped (8-ISPS) piezoelectric energy harvesting device to generate electricity from excitations with uncertainties representing diverse sources of ambient vibrations. The classical cantilever beam, the circular-shaped and pizza-shaped configurations are analyzed as reference cases to exploit the design evolution and the performance of the proposed device. The proposed 8-ISPS energy harvester device shows potential to extract energy from excitations with wideband frequency spectrum. The ambient excitation characteristics play an important role in the system's performance and need to be considered in the design concept for enhancement purposes.

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References

[1] A. Erturk and D.J. Inman. "An experimentally validated bimorph cantilever model for piezoelectric energy harvesting from base excitations". *Smart Materials and Structures* 18(2): 025009, 2009.
[2] I. H. Kim, H. J. Jung, B. M. Lee and S. J. Jang. "Broadband energy harvesting using a two degree-of-freedom vibrating body". *Applied Physics Letters* 98(21): 214102, 2011.
[3] H. Wu, L. Tang, Y. Yang, and C. K. Soh 2013. "A novel two-degrees-of-freedom piezoelectric energy harvester". *Journal of Intelligent Material Systems and Structures*. 24(3): 357–368, 2013.
[4] D. Zhu, M.J. Tudor and S.P. Beeby. "Strategies for increasing the operating frequency range of vibration energy harvesters: A review". *Measurement Science and Technology* 21(2): 022001, 2009.
[5] M. Rezaei and R. Talebitooti. "Wideband PZT energy harvesting from the wake of a bluff body in varying flow Speeds". *International Journal of Mechanical Sciences*, 163:105135, 2019.
[6] M. Ferrari, V. Ferrari, M. Guizzetti, D. Marioli and A. Taroni. "Piezoelectric multifrequency energy converter for power harvesting in autonomous microsystems". *Sensors and Actuators, A: Physical* 142(1), 329-335, 2008.
[7] L. Tang and Y. Yang. "A multiple-degree-of-freedom piezoelectric energy harvesting model". *Journal of Intelligent Material Systems and Structures*, 23(14):1631–1647, 2012.
[8] S. J. Jang, E. Rustighi, M. J. Brennan, Y. P. Lee and H. J. Jung. "Design of a 2DOF Vibrational Energy Harvesting Device". *Intelligent Material Systems and Structures*. 22(5), 443-448, 2011.
[9] D. Upadrashta and Y. Yang. "Trident-shaped multimodal piezoelectric energy harvester". *Journal of Aerospace Engineering*. 31(5): 04018070, 2018.
[10] A. Abdelkefi, N. Barsallo, L. Tang, Y. Yang and M. R. Hajj. "Modeling, validation, and performance of low-frequency piezoelectric energy harvesters". *Journal of Intelligent Material Systems and Structures*. 25(12): 1429-1444, 2014.
[11] D. Upadrashta and Y. Yang. "Finite element modeling of nonlinear piezoelectric energy harvesters with magnetic interaction". *Smart Materials and Structures* 24(4): 045042, 2015.
[12] M. Zhu, E. Worthington, J. Njuguna. "Analyses of power output of piezoelectric energy-harvesting devices directly connected to a load resistor using a coupled piezoelectric-circuit finite element method". *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*. 56(7): 1309–1317, 2009.
[13] M. Zhu, E. Worthington and A. Tiwari. "Design study of piezoelectric energy-harvesting devices for generation of higher electrical power using a coupled piezoelectric-circuit finite element method". *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*. 57(2): 427–437, 2010.