Use of a recurrence based fractional derivative model in the analysis of the influence of geometrical parameters in the transient response of viscoelastic beams

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Abstract. This paper aims to analyze the influence of geometrical parameters on the dynamical behavior of viscoelastic beams. Viscoelastic materials are widely used in vibration control of dynamic systems. Usually, a viscoelastic layer is applied on the surface of the structure and a second layer with elastic properties is used to restrict viscoelastic displacements. This restrictive layer intensifies energy dissipation once it introduces shear deformation in the viscoelastic material. Naturally, geometrical parameters, mainly the width of the viscoelastic layer may influence the shear deformation and the efficiency of the vibration control treatment. For this analysis, a recurrence based fractional derivative model is used, the system is discretized in finite elements and solved using Newmark's method. This procedure makes it possible to compare the mechanical behavior of several viscoelastic beams with different geometries, and determine how they affect vibration control treatment.

Keywords: viscoelastic material, vibration control, fractional calculus, finite element method.

1 Introduction

Viscoelastic materials have been widely used in vibration control, however, modeling their time domain behavior is not an easy task. The dynamical behavior of these materials is characterized by a memory effect, i. e., previous stress and strain states influence current mechanical response. This feature is not well represented by a complex modulus and even the use of the inverse Fourier transform from the frequency domain fails to describe the time domain behavior [1].

With the evolution of Fractional Calculus in last century, a new time domain approach was developed: the so-called Fractional Derivative Models (FDM). Bagley and Torvik [2, 3] were pioneers to model the behavior of viscoelastic materials using FDM. Then, other authors have studied this subject [4, 5]. The Fractional Calculus based models have shown to be the most powerful tool in modeling viscoelastic systems [6]. However, classical FDM present some limitations with regard to applicability and computational time consumption, as discussed by Nunes [7].

Willing to overcome these difficulties, a more efficient method for modeling viscoelastic material behavior was developed at the Structural Mechanics Laboratory at the Federal University of Uberlândia – LMEst/UFU [7]. This new model is based on a recurrence approach and it is capable of making the computational analyses faster. Therefore, this paper aims to analyze the influence of geometrical parameters in the dynamical response of damped structures using this improved FDM, which represents a straightforward way to incorporate the damping effects of viscoelastic materials in structural dynamic models.

2 Recurrence FDM

According to Nunes [7, 8], based on a four parameters fractional derivative model [3, 9], shear and elongation constitutive law for a viscoelastic material is described in Eqs. (1)-(2), where τ and γ are shear stress and deformation, σ and ε are normal stress and deformation, β^{G}_{j+1} and β^{E}_{j+1} are the recurrence terms described in Eqs. (3)-(4). Seven parameters are needed for this model: E_0 and E_∞ represent low and high frequencies elongation modulus; G_0 and G_∞ are low and high frequencies shear modulus; a_E and a_G are elongation and shear parameters; and α is the fractional derivative order.

$$\tau_t = \sum_{j=0}^{N_t} \beta_{j+1}^{G} \gamma_{t-j\Delta t} \tag{1}$$

$$\sigma_t = \sum_{j=0}^{N_t} \beta_{j+1}^{E} \varepsilon_{t-j\Delta t} \tag{2}$$

$$\beta_{j+1}^{G} = \frac{2G_\infty \Delta t^{-\alpha}}{1+a_G \Delta t^{-\alpha}} A_{j+1} - \sum_{i=0}^{j} \frac{a_G \Delta t^{-\alpha}}{1+a_G \Delta t^{-\alpha}} A_{i+1} \beta_{j+1-i}^{G} \tag{3}$$

$$\beta_{j+1}^{E} = \frac{E_\infty \Delta t^{-\alpha}}{1+a_E \Delta t^{-\alpha}} A_{j+1} - \sum_{i=0}^{j} \frac{a_E \Delta t^{-\alpha}}{1+a_E \Delta t^{-\alpha}} A_{i+1} \beta_{j+1-i}^{E} \tag{4}$$

$$\beta_1^{G} = \frac{2G_0 + 2G_\infty \Delta t^{-\alpha}}{1+a_G \Delta t^{-\alpha}} \tag{5}$$

$$\beta_1^{E} = \frac{E_0 + E_\infty \Delta t^{-\alpha}}{1+a_E \Delta t^{-\alpha}} \tag{6}$$

3 FEM modeling

Consider a three-layered sandwich beam with two elastic faces and a viscoelastic core. Based on the Classical Laminate Theory (CLT), one describes the displacement field of this structure in term of four variables: u, axial displacement; w, transversal displacement; θ, rotation and β, shear deformation of the viscoelastic layer. The system is discretized in finite elements and linear shape functions are used for u and β, cubic functions for w, and θ is related to w as its first derivative. Eqs, (7)-(10) describe the displacement fields, where {q(e)} is the DOF vector and N(x) are interpolating function matrices.

$$w(x,t) = \left[N_w(x)\right]\left\{q_{(e)}\right\} \tag{7}$$

$$u^{(1)}(x,z,t) = \left[N_u(x)\right]\left\{q_{(e)}\right\} - z\left[N'_w(x)\right]\left\{q_{(e)}\right\} \tag{8}$$

$$u^{(2)}(x,z,t) = \left[N_u(x)\right]\left\{q_{(e)}\right\} - z\left[N'_w(x)\right]\left\{q_{(e)}\right\} + \left(z - \frac{h}{2}\right)\left[N_\beta(x)\right]\left\{q_{(e)}\right\} \tag{9}$$

$$u^{(3)}(x,z,t) = \left[N_u(x)\right]\left\{q_{(e)}\right\} - z\left[N'_w(x)\right]\left\{q_{(e)}\right\} + h_2\beta(x,t)\left[N_\beta(x)\right]\left\{q_{(e)}\right\} \tag{10}$$

Kinetic energy is described in Eq. (11) and mass matrix is defined in Eq. (12), where k is referred to each of the three layers; ρ, b, h, li and A represent the density, the width of the beam, the thickness, the finite element length and the transversal area of each layer.

$$T_e = \frac{1}{2}\left\{\dot{q}_e\right\}^T \left[M_e\right]\left\{\dot{q}_e\right\} \tag{11}$$

$$\left[M_e\right] = \sum_{k=1}^{3} \rho_k \left(b\int_{z_k}^{z_k+h_k} \int_0^{li} \left[N_u^{(k)}\right]^T \left[N_u^{(k)}\right] dx\, dz + A_k \int_0^{li} \left[N_w^{(k)}\right]^T \left[N_w^{(k)}\right] dx \right) \tag{12}$$

Based on Continuum Mechanics, strain fields are defined as the first derivative of displacement fields. Deformation energy is defined as the sum of its components for each layer. For the elastic layers, Eq. (13) shows the deformation energy and Eq. (14) describes the stiffness matrix, where E represents the Young's modulus.

$$V_e^{(e)} = \frac{1}{2}\{q_e\}^T\left[K_e^{(e)}\right]\{q_e\} \tag{13}$$

$$\left[K_e^{(e)}\right] = b\sum_{k=1,\,3} E_k \int_{z_k}^{z_k+h_k}\int_0^{l_e}\left[N'^{(k)}_u\right]^T\left[N'^{(k)}_u\right]dxdz \tag{14}$$

Using recurrence FDM described in Eqs. (1)-(2), viscoelastic layer deformation energy is described in Eq. (15). For this layer, it is not possible to define a single stiffness matrix due to viscoelastic memory effect, therefore one stiffness matrix (Eq. (16)) is used for each time step and it is updated by the recurrence terms.

$$V_e^{(v)} = \frac{1}{2}\{q_e(t)\}^T\sum_{j=0}^{N_l}\left[K_e^{**(v)}\right]_j\{q_e(t-j\Delta t)\} \tag{15}$$

$$\left[K_e^{**(v)}\right]_j = b\left(\beta_{j+1}^E\int_{z_k}^{z_k+h_k}\int_0^{l_e}\left[N'^{(2)}_u\right]^T\left[N'^{(2)}_u\right]dxdz + \beta_{j+1}^G\int_{z_k}^{z_k+h_k}\int_0^{l_e}\left[N_\beta^{(2)}\right]^T\left[N_\beta^{(2)}\right]dxdz\right) \tag{16}$$

Global mass and stiffness matrices are assembled and the system's equation of motion is found using Lagrange's equation, which results in Eq. (17).

$$[M]\{\ddot{q}(t)\} + \left[K^{(e)}\right]\{q(t)\} + \sum_{j=0}^{N_l}\left[K^{**(v)}\right]_j\{q(t-j\Delta t)\} = \{f(t)\} \tag{17}$$

This equation is solved using Newmark's constant average acceleration method and the displacements at each time step are found.

4 Simulations

Consider a cantilever sandwich beam with two elastic faces and a viscoelastic core whose mechanical properties are shown in Table 1. This structure is and subjected at its free end to a unit impulse force with the peak at 2 ms. The FE model is discretized in 25 elements. The time interval is divided into 0.1 ms steps and memory length uses 500 points of time. The viscoelastic material is considered to be at 27 ºC. For this temperature, Nunes (2020) determines the FDM parameters by a curve-fitting procedure. The author finds the following values: G_0 = 423,632.8 Pa, G_∞ = 30,177.8 Pa s^α, a_G = 0.00022 s^α, α = 0.6766. The thickness of each layer is made to vary.

Table 1. Mechanical properties of the structure

	1st layer	2nd layer	3rd layer
Material	Steel	ISD-112	Steel
Length	0.5 m	0.5 m	0.5 m
Thickness	0.015 m	0.02 m	0.01 m
Width	0.04 m	0.04 m	0.04 m
Young's modulus	210 GPa	-	210 GPa
Poisson's ratio	80 GPa	-	80 GPa
Density	7,850 kg/m³	1600 kg/m³	7,850 kg/m³

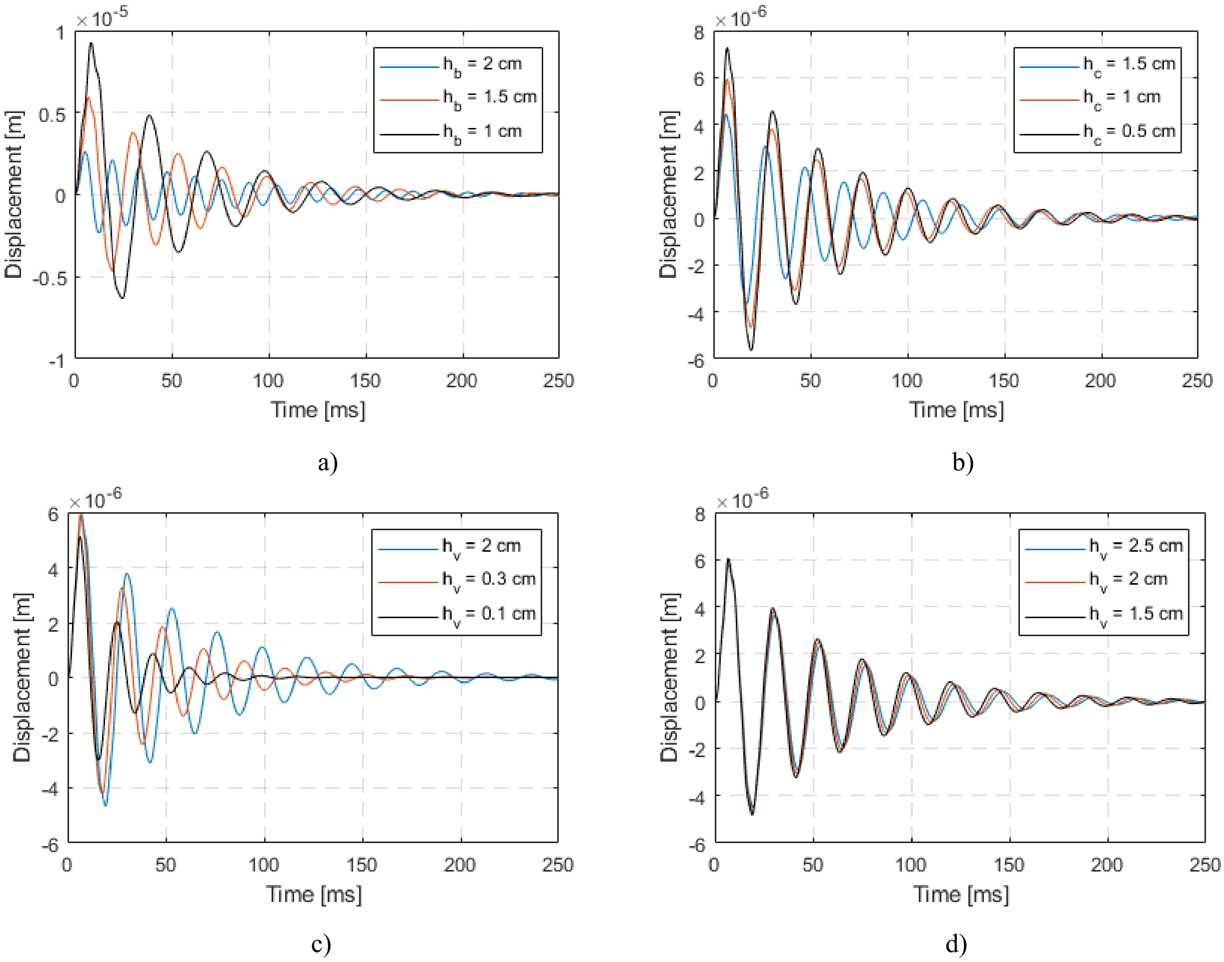

Figure 1. Influence the thickness of viscoelastic a) b), base c) and constraining d) layers on the time domain response

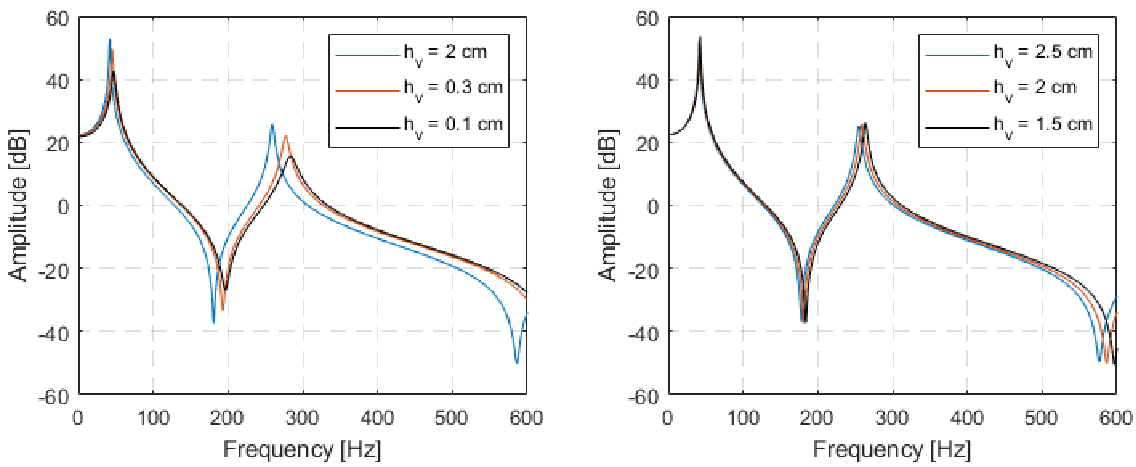

Figure 2. Influence of the viscoelastic layer width on the system response.

Fig. 1 shows the time domain responses of the system. The reduction in amplitude is expected when the thickness of the elastic faces are increased [Fig. 1 a) and b)], since these parameters affect the stiffness of the system, increasing it. When the thickness of the viscoelastic core varies, however, the effect is reversed. This occurs because the damping of the sandwich structure is due to shear deformations in the viscoelastic material, which are greater when the velocity gradient in the viscoelastic layer increases, and this occurs when its thickness decreases [Fig. 1 c)]. However, from a certain value, the increase in thickness does not have major effects on the behavior of the system [Fig. 1 d)]. Fig. 2 shows the frequency domain response of the system, indicating that this behavior occurs not only for the impulse time domain response, but also for a wide frequency spectrum, which includes the resonance peaks.

5 Conclusions

This paper proposed the analysis of the influence of geometrical parameters in the time domain response of a viscoelastically treated beam using an improved and more efficient fractional derivative model. The simulations performed in this work show a great influence of the thickness of the viscoelastic layer in the dynamical behavior of the dampened structure. It can be seen that for a thin viscoelastic layer, the amplitude of the system increases as its thickness increases. These results also reaffirm the capability of the recurrence FDM developed in the Federal University of Uberlândia in describing viscoelastic material damping effects.

As future works the authors suggest a stochastic analysis to consider the presence of parametric uncertainties in the model in order to obtain more realistic results and the analysis of dampened structures as laminate composites treated with viscoelastics.

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