Influence of Geometrical Dimensions of Reservoir on the Fluid-Structure Coupled Dominant Modes in Concrete Gravity Dams

Davidson de O. França Júnior[1], Selênio F. da Silva[2], Lineu J. Pedroso[1]

[1]*Postgraduate Program in Structures and Civil Construction (PECC), University of Brasilia (UnB)*
SG–12, CEP 70910-900, Campus Darcy Ribeiro, Brasília - DF, Brazil
davidson.francajunior@gmail.com, lineujp@gmail.com
[2]*Department of Civil Engineering, Federal University of Pará (UFPA)*
Rua Augusto Correa, 01, CEP 66075-110, Belém - PA, Brazil
selenio@ufpa.br

Abstract. Properly determining the dynamic structural response of a dam is complex as it involves aspects related to fluid-structure interaction (FSI). The coupled response depends on numerous variables, and among them is the fluid domain, which has a significant influence on the coupled dominant modes. In the design phase, such a reservoir is idealized as an acoustic cavity that must have dimensions and boundary conditions appropriate to the problem. However, depending on these geometric parameters considered for the reservoir, the natural frequency ranges and the order of the fluid vibration modes are different, directly influencing the coupled fluid-structure vibrations, this is because the coupled response depends on the decoupled frequency range of the structure and fluid domain in isolation. Thus, in this paper the uncoupled and coupled fluid-structure free vibrations of the Koyna gravity dam located in India are studied. The natural frequencies, vibration modes of the structure and hydrodynamic pressures are investigated and compared under different numerical modeling (reservoir dimensions) using the finite element method (FEM) by means of ANSYS®. For validation of the numerical approaches presented here the results obtained are compared with values available in the literature.

Keywords: Dams, Fluid-Structure Interaction, Reservoir Dimensions, Finite Element Method.

1 Introduction

Concrete gravity dams are generally large structures, which may have different uses and purposes, according to the needs of the region of their implementation. In the area of dam engineering, it is essential to ensure meticulous safety conditions, because the failure of such structures can generate considerable damage, not only structural, but also economic, social and environmental. In dynamic analysis, two relevant means can be highlighted in the investigation: the structure and the reservoir fluid. When analyzed together, these means form the fluid-structure coupled system, which is essential in analyses because it presents a high degree of complexity and criticality, directly impacting the design of the project.

The dam-reservoir interaction (DRI) is quite significant for the development of the dynamic response in free vibrations, because the presence of the pressure distribution of the liquid (hydrodynamic pressures) directly influences the vibration modes of the structure and, consequently, its natural frequencies. During the vibration motion, the structure initially moves by applying pressure to the fluid. In turn, the fluid responds with a hydrodynamic pressure at the interface with the structure. This response pressure is referred to as the impulsive pressure. Therefore, the dam-reservoir coupled frequencies depend, among other variables, mainly on the frequency range of the structure as well as on the frequency range of the reservoir domain (fluid). However, depending on these geometric parameters considered for the reservoir, the natural frequency ranges and the order of the fluid vibration modes are different, directly influencing the dam-reservoir coupled vibrations, because the coupled response depends on the uncoupled frequency range of the structure and fluid domain in isolation. Thus, the sequence of the coupled modes dominant structure additional mass (AM), cavity dominant (CD) or mixed modes (MM) undergo changes, arising from the diversity of frequency ranges.

Westergaard [1] considered only the inertial coupling aspects of the fluid (additional mass) assuming the displacement of the dam as rigid-mobile, with the fluid represented by Laplace's equation, where the effects of fluid compressibility are neglected. Later, in the works by Chopra [2], Chopra and Chakrabarti [3] and in Chopra [4] the effect of the frequency range and compressibility of the fluid on the dam-reservoir interaction was investigated, emphasizing that considering the fluid as incompressible can produce significant errors and the effects of compressibility cannot be ignored. Thus, for dynamic analyses of dam-reservoir systems involving more complex situations, several formulations have been improved and, among them, the finite element method (FEM) is a robust technique. In the works by Chopra et al. [5], Hall and Chopra [6], Sharan [7], Ftima and Léger [8] and many others, analytical and FEM analyses involving the dynamic effects on DRI are presented. In Brazil, many studies on dams, dynamic structure-fluid interaction and related studies have been developed, as for example, the works of Pedroso [9], Silva [10], Silveira et al. [11], Mendes et al. [12], Silveira et al. [13], among others.

Therefore, knowing that the fluid impounded in a dam is a practically infinite system due to the extensive length of the reservoir and that in design formulations and simulations the length of the reservoir needs to be limited to a certain distance away from the dam, i.e., it must be finite, in this paper the uncoupled and coupled fluid-structure free vibrations of the Koyna gravity dam, located in India, are studied. The natural frequencies, structural vibration modes and hydrodynamic pressures are investigated for different L/H (length/height) ratios of the reservoir using finite element method (FEM) through U-P formulation (structure displacement and pressure to fluid) by means of ANSYS® software. Helmoltz's formulation (wave equation) is used to validate the acoustic cavity. The results obtained by FEM are compared with analytical values for acoustic cavity and with values presented by Chopra [14] and by Huang [15] for uncoupled dam and dam-reservoir interaction. This study provides a discussion of the uncoupled and coupled dam-reservoir frequency ranges and their involved effects.

2 Theoretical Foundation

2.1 Structure Domain (Dam)

For continuous systems, the dynamic equation of motion by the finite element method is written in its matrix form, in which the global matrices are made based on the elementary matrices (of each finite element), which in turn have an approximation field due to the interpolation functions of the finite element and the resolution through numerical integration in the structural domain. In general, the equation of motion for uncoupled structure is given by:

$$[M_s]\{\ddot{U}\} + [C_s]\{\dot{U}\} + [K_s]\{U\} = \{F_E\} \tag{1}$$

where $\{F_E\}$ is an external force vector, $\{U\}$ is the structure displacement vector, $\{\dot{U}\}$ is the structure velocity vector, $\{\ddot{U}\}$ is the structure acceleration vector, $[M_S]$ is the structure mass matrix, $[K_S]$ is the structure stiffness matrix and $[C_S]$ is the structure damping matrix. In the system above undamped free vibrations are established if $\{F_E\} = 0$ and $[C_S] = 0$.

2.2 Fluid Domain (Reservoir)

The analysis of the acoustic behavior of cavities is related to the length of the acoustic waves along with the dimensions of the medium, in which they present natural frequencies and acoustic modes. Based on Pedroso [9] the wave equation in acoustic media for two-dimensional cavities (2D model) is given by:

$$\frac{\nabla^2 P}{\partial x^2} + \frac{\nabla^2 P}{\partial y^2} + \frac{1}{c^2}\,\frac{\partial^2 P}{\partial t^2} = 0\,. \tag{2}$$

The eq. 2 is the Helmoltz's formulation (wave equation), where c is the velocity of propagation of the wave in the acoustic cavity, P is the pressure and $\nabla^2 P$ is the Laplacian pressure operator. The complete solution for the natural frequencies and pressure vibration modes for two-dimensional cavity with rigid ($\partial P/\partial n = 0$) and open (P=0) boundary condition in the two opposite directions (x and y) are found in Pedroso [9], França Júnior [16].

2.3 Dam-Reservoir Interaction

The analysis of the dam-reservoir coupled problem is based on the (U-P) formulation, with pressure as the variable in the fluid domain and displacement as the variable for the structure. This formulation evaluates the fluid force by performing the integration of the pressures at the fluid-structure interface and, consequently, performs the coupling by applying this fluid force to the structure's equation of motion. With this, we have the fluid-structure (FS) coupling matrix, which couples the effects. The complete U-P formulation can be found in França Júnior [16], Mendes [12], and many others. In compact form the U-P formulation is given by:

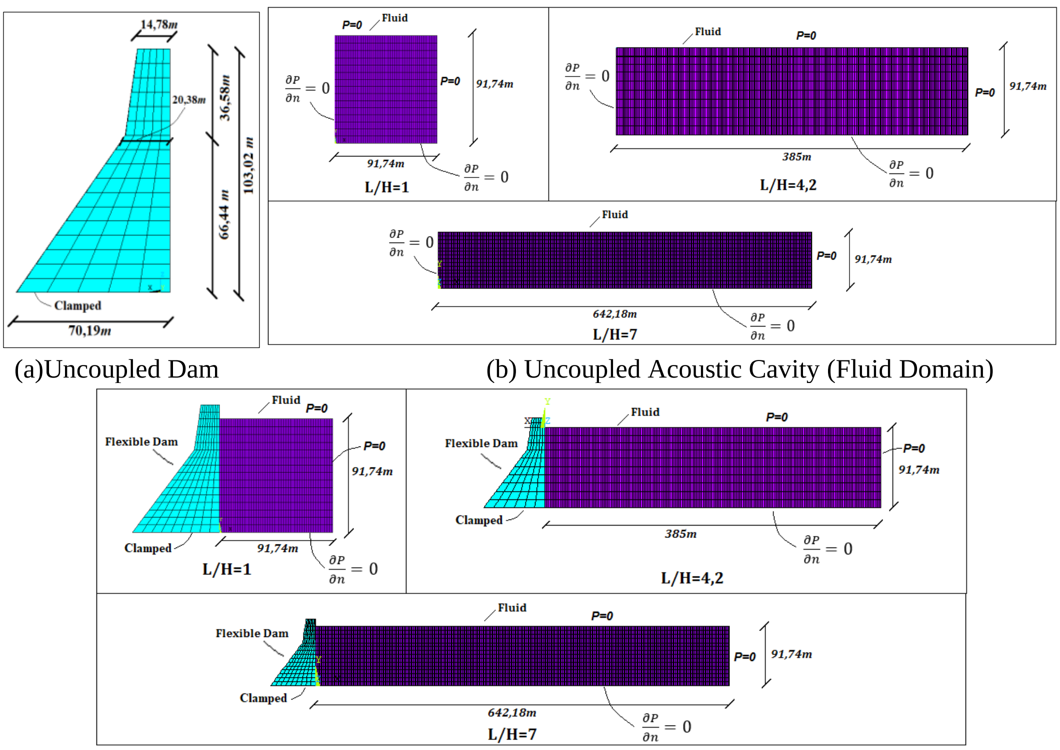
$$\begin{bmatrix} [M_s] & [0] \\ \rho_f [FS] & [M_f] \end{bmatrix} \begin{Bmatrix} [\ddot{U}] \\ [\ddot{P}] \end{Bmatrix} + \begin{bmatrix} [C_s] & [0] \\ [0] & [C_f] \end{bmatrix} \begin{Bmatrix} [\dot{U}] \\ [\dot{P}] \end{Bmatrix} + \begin{bmatrix} [K_s] & [-FS] \\ [0] & [K_f] \end{bmatrix} \begin{Bmatrix} [U] \\ [P] \end{Bmatrix} = \begin{Bmatrix} [F_E] \\ [0] \end{Bmatrix}. \tag{3}$$

Where $[P]$ is the fluid pressure matrix, ρ_f is the fluid density, $[M_f]$ is the fluid mass matrix, $[K_f]$ is the fluid stiffness matrix, $[C_f]$ is the fluid damping matrix, $[FS]$ is the fluid-structure coupling matrix. In the system above undamped free vibrations are established if $[F_E]$= 0, $[C_S]$= 0 and $[C_f]$= 0.

3 Description of the Models

The Koyna dam located in the state of Maharashtra, India. According to Chopra [14], the concrete of Koyna dam has physical properties such as: specific mass (ρ_e) equivalent to 2643 kg/m³, modulus of elasticity (E_c) of 31027 MPa, Poisson's ratio (ν_c) of 0,20. In this work, the modeling of the dam structure was done according to a plane state of deformation (Fig. 1), in which used the PLANE183 finite element to discretize the structure. The PLANE183 element used was in the quadrilateral geometry with eight nodes of two degrees of freedom at each node, capturing translations and/or rotations in the x and y nodal directions. The boundary condition of the dam was clamped at the base, with the foundation assumed rigid.

The rectangular reservoir attached to the dam was assumed to be 91,74 *m* high and with three distinct lengths of 91,74 *m*, 385 *m* e 642,18*m* (lengths adopted in this work). The distinct lengths reflect in three different length/height (*L/H*) ratios of the reservoir. The fluid specific mass corresponds to 1000 kg/m³ and the wave propagation velocity in the fluid (c) is equal to 1440 m/s, in agreement with Huang [15] and Silveira et al. [11].



(a)Uncoupled Dam (b) Uncoupled Acoustic Cavity (Fluid Domain)

(c) Fluid Structure Interaction

Figure 1. Geometric dimensions and boundary conditions of the models analyzed in this work.

The FLUID29 element used was in the quadrilateral geometry with four nodes of two degrees of freedom at each node, capturing translations in the x and y nodal directions and pressures. In Figure 5 it is evidenced that the fluid boundary conditions are zero pressure (P=0) at the free surface and at the finite end of the reservoir, as well as the rigid boundary condition ($\partial P/\partial n = 0$) is applied at the uncoupled cavity interface (dam positioning location) and at the reservoir base (rigid foundation at the reservoir base). In the coupled problem the fluid-structure condition is applied at the dam-reservoir interface.

4 Results and Discussions

4.1 Structure Domain (Dam)

The results obtained through FEM referring to the four initial natural frequencies and respective vibration modes of the dam structure agree with results presented by Chopra [14]. The small percentage difference can be observed, demonstrating that the modeling of the structure was validated. The frequencies are presented in Tab. 1.

Table 1. Natural frequencies (ω) of the uncoupled dam (Hz).

Vibration Mode	ω (Hz) Chopra [14]	ω (Hz) Numerical (FEM)	Difference (%)
1º (Flexure)	3,07	3,07	0,01
2º (Flexure)	8,20	8,16	0,49
3º (Axial)	10,75	10,81	0,53
4º (Flexure)	15,87	15,93	0,38

It is evidenced that the numerical model adequately represents the vibration phenomenon, as modal deformations of the dam were captured and compared with Chopra [14], presenting good agreement. It is evident from the graph (Fig. 2) that the four vibration modes of the decoupled dam are predominantly bending modes (U_X), except for the third vibration mode which is an axial mode, where the normalized displacement presented in the graph is in the vertical direction of the dam (U_Y).

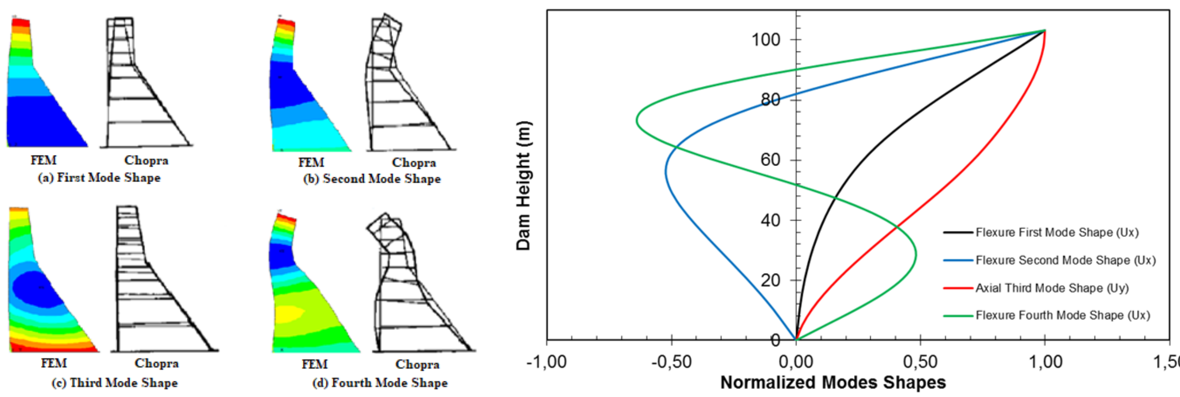


Figure 2. Vibration modes of the decoupled dam for the four natural frequencies.

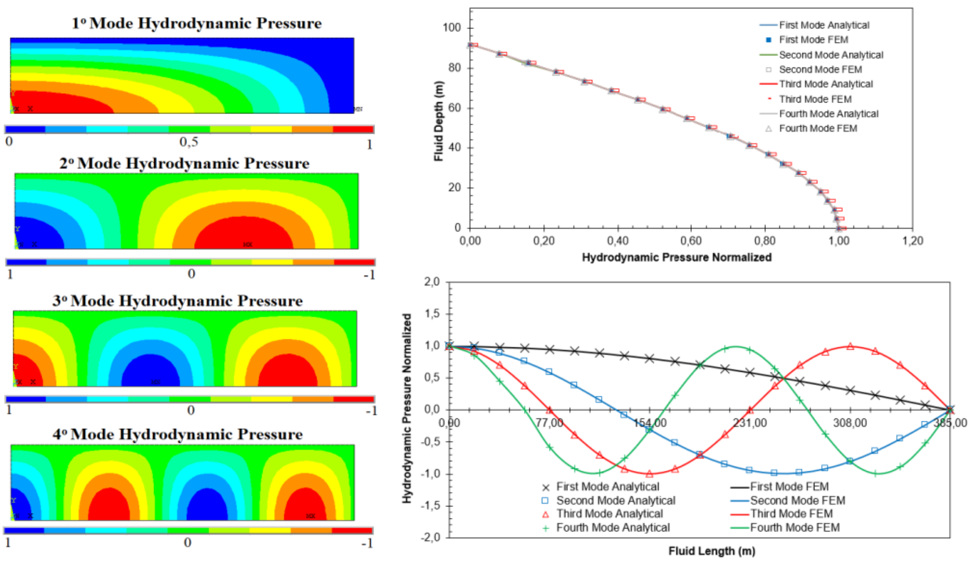
4.2 Fluid Domain (Reservoir)

The results concerning the four natural frequencies and vibration modes of the fluid domain (Tab. 2) were studied through the analytical solutions and the finite element method. In the results the small percentage differences of the numerical model compared to the analytical solutions demonstrates that the modeling was validated for the acoustic cavity.

Table 2. Natural frequencies (ω) of the uncoupled fluid (Hz).

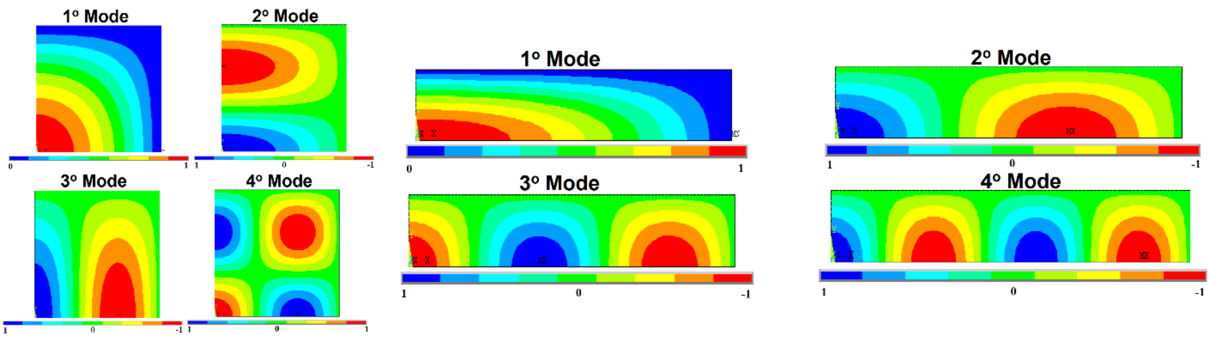
L/H=1				*L/H*=4,2				*L/H*=7			
Mode (i, j)	ω (Hz) Analytical	ω (Hz) Numerical	Diff. (%)	Mode (i, j)	ω (Hz) Analytical	ω (Hz) Numerical	Diff. (%)	Mode (i, j)	ω (Hz) Analytical	ω (Hz) Numerical	Diff. (%)
1º (1,1)	5,57	5,55	0,36	1º (1,1)	4,03	4,04	0,25	1º (1,1)	3,96	3,97	0,25
2º (1,3)	12,40	12,41	0,08	2º (3,1)	4,82	4,83	0,21	2º (3,1)	4,27	4,27	0,00
3º (3,1)	12,41	12,42	0,08	3º (5,1)	6,10	6,11	0,16	3º (5,1)	4,82	4,82	0,00
4º (3,3)	16,65	16,66	0,06	4º (7,1)	7,63	7,65	0,26	4º (7,1)	5,55	5,55	0,00

The modal pressure deformations for each vibration mode were compared and are presented in Fig. 3. In the graphs presented it is possible to notice that the hydrodynamic pressures obtained through the analytical technique coincide with the results obtained by the finite element method. At the rigid boundary interface, the hydrodynamic pressures are equally distributed along the height for the four pressure modes, i.e., the pressure at the cavity interface is independent of the mode, and this fact is explained by the rigid boundary condition assumed at this location. On the other hand, pressure waves are visualized in the longitudinal direction of the reservoir, where pressure waves propagate in the acoustic cavity with acoustic modes consistent with the rigid boundary condition and zero pressure in opposite directions. Such modes are critical for further evaluation of the fluid-structure coupled problem. In Fig. 4 the hydrodynamic pressure modes for the ratio's *L/H*=1 and *L/H*=7 is presented. For the cavity with ratio *L/H*=1 transverse modes arise and, in agreement with Tab. 2, the frequency range increases.



(a)Numerical Results (b) Comparison of Pressures (Transverse and Longitudinal Section)

Figure 3. Hydrodynamic pressures of the uncoupled fluid for the *L/H*=4,2 ratio.



(a) *L/H=1* (b) *L/H=7*

Figure 4. Hydrodynamic pressures of the uncoupled fluid for the *L/H*=1 and *L/H*=7 ratio.

4.3 Dam-Reservoir Interaction

In the coupled fluid-structure analysis, results were obtained for the four natural frequencies and vibration modes for different L/H ratios. To validate the models, the results studied by Huang [15] for an L/H=4.2 ratio were compared. The natural frequencies are shown in Tab. 3.

Table 3. Natural frequencies (ω) coupled dam-reservoir (Hz).

L/H=1			*L/H*=4.2					*L/H*=7		
Mode	ω (Hz) Numerical	Mode Type	Mode	ω (Hz) Huang [15]	ω (Hz) Numerical	Diff. (%)	Mode Type	Mode	ω (Hz) Numerical	Mode Type
1º	2,80	AM	1º	2,79	2,78	0,36	AM	1º	2,78	AM
2º	5,27	CD	2º	4,05	4,05	0,00	CD	2º	3,98	CD
3º	8,01	CD	3º	4,79	4,79	0,00	CD	3º	4,27	CD
4º	10,71	CD	4º	5,98	5,99	0,17	CD	4º	4,80	CD

Based on Tab. 3, it can be stated that the modeling was validated. When analyzing the table, it is clear that the additional mass mode (AM) is the fundamental mode of the structure, that is, for different *L/H* ratios, the first frequency remains of the same value and behavior during vibration, in which the structure predominates, and the fluid follows the structural deformation of the dam. The effect of the additional fluid mass occurs near the dam interface. On the other hand, from the second to fourth coupled mode the frequencies change for all different reservoir discretization's (*L/H*), that is, the cavity predominates over the system and changes the coupled modes to dominant cavity (CD) modes and, consequently, the frequencies.

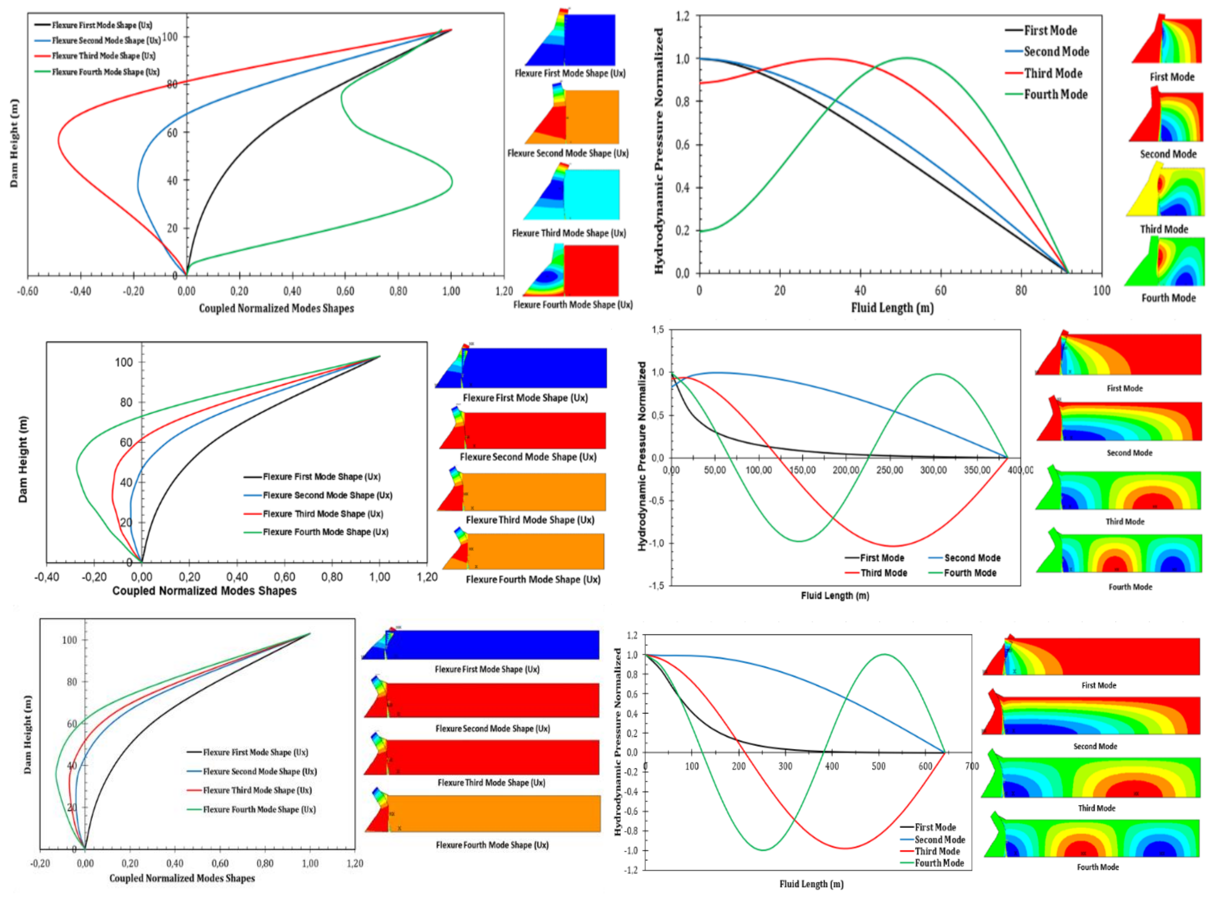


Figure 5. Structure vibration modes and hydrodynamic pressures of the reservoir-dam coupled system for the ratio *L/H*=1, *L/H*=4.2 and *L/H*=7.

Thus, in Figure 5 it is observed that in cavity modes the fluid vibrates with pressure modes and, consequently, the dam is disturbed and follows the effects of the acoustic cavity. Note that the fourth mode of the deformation of the coupled dam in the *L/H*=1 ratio is changed due to the CD transverse mode. Dominant cavity modes in DRI are similar to uncoupled fluid modes. Thus, in the present work, the analysis presented contribute to the understanding of the fluid-structure dynamic coupling between dam-reservoir.

5 Conclusions

This work showed the influence of different reservoir length discretization's on fluid-structure coupled free vibrations in gravity dams. The analysis of the dominant modes shows that the fundamental mode is not altered by the length of the reservoir, but from the second mode onwards investigations in different domains and dimensions of the fluid are necessary. Finally, it is stated that the frequency ranges of the fluid influence the fluid-structure coupled phenomena and that the results were satisfactory in the DRI.

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