Dynamic analysis of a footbridge subjected to human load

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Abstract. The structural branch of engineering is constantly evolving, especially with regard to advances in modeling with the use of improved software, with trends in optimized processes that aim to save time and money. Constructive trends have led to the development of more resistant materials and the design of structures capable of overcoming large spans, using structural elements with smaller sections, that is, thinner, lighter and more flexible. At the same time, the vibration frequencies of these structures tend to become lower, becoming closer to the dynamic excitations, subjecting themselves to oscillations that generate some human discomfort or cause a feeling of insecurity. In this context, the main objective of this research is to verify the performance of a metallic truss footbridge, subject to human loading and to analyze its natural frequencies and mode shapes. According to the chosen structure, the dynamic action that will be applied and analyzed in this study was determined. For this, human excitation was chosen. Newmark's Numerical Integration Method was used to find the system's response to dynamic excitation. Finally, the developed algorithm was validated and the results analyzed, concluding that the footbridge under study would meet the acceleration criteria, therefore, requiring no intervention.

Keywords: Footbridges. Dynamic analysis. Newmark's method.

1 Introduction

Footbridges are essential for safe pedestrian crossing on streets, roads and rivers. They are structures widely used in strategic commercial points in order to allow and facilitate customer access to the site. Currently, several footbridges connecting malls and universities have been built, such as the Bourbon Wallig Footbridge located in the city of Porto Alegre (Brazil), Fig. 1., which served as a model for the present work.

Figure 1. Bourbon Wallig Footbridge

The use of truss steel structures on the footbridges is common, as they add agility and ease of execution, material savings, lightness and high strength. These structures are analyzed and verified so that they support as static loads from their own weight and permanent loads, and dynamic actions proven by the passage of pedestrians and the action of the wind, in order to meet the Ultimate Limit States (ELU) and Limit States of Service (ELS) according to ABNT NBR 8681 [1].

Dynamic actions can cause vibrations in structures, causing material fatigue failure and user discomfort. According to Mello [2], it is essential for the designer to know the methodologies for analyzing and controlling vibrations. And for Varella [3], the increase in vibrations that cause user discomfort has grown over the years with the development of slimmer structures. Thus, this study performs a dynamic analysis of a metallic footbridge subject to human excitation, using the eigenvalues and eigenvectors approach to determine the natural frequencies, and the Newmark's Method for solving the dynamic equilibrium equation. The main objective is to verify the performance of the structure, and analyze the frequencies and modes of vibration.

2 Structure analyzed

The metallic footbridge structure studied in this article was based on the walkway of the Bourbon Wallig from Brazil, Rio Grande do Sul, Porto Alegre. The truss is *Warren* type with a total length of 72 meters, divided into two spans, and formed by steel bars in profile I. As the structure has very similar characteristics in both spans, the present study performed an isolated analysis of only one of the spans, using modeling in two dimensions (2D), as shown in Fig. 2. The dimensions of the metal profiles are the same as the original existing structure.

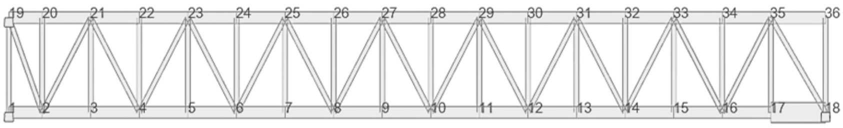

Figure 2. Structure modeled in commercial software

The problem was approached through the finite element method, according to Clough and Penzien [4], Soriano [5] and Hibbeler [6] and the general procedures is shown in the chapters below.

2.1 System Properties

The structure has 36 connections nodes, as shown in Fig. 2, and was defined as a truss model, having two degrees of freedom (GDL) per node. Each GDL represents a coordinate needed to determine the position of all parts of the system at any point in time. Thus, the truss model presents the x and y cartesian coordinates, which are sufficient to determine the rigidity of the system to axial forces. Rotational stiffness is not determined and connecting nodes are classified as hinges. The stiffness and truss mass matrices in local coordinates are presented in eq. (1) and eq. (2), respectively. The transformation matrix from local to global coordinates is shown in eq. (3), while the stiffness and global mass of the structure are shown in eq. (4).

$$K^e = \frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{1}$$

$$M^e = \frac{\rho AL}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}, \tag{2}$$

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}, \tag{3}$$

$$\boldsymbol{K} = T^T K^e T \qquad \boldsymbol{M} = T^T M^e T , \tag{4}$$

where K^e, M^e, K, M are the stiffness and mass of each element, and the global stiffness and mass of the structure, respectively. A, E, L and ρ are the cross-sectional area of the element, the longitudinal modulus of elasticity and the specific density of the steel. T e T^T are the transformation matrix and the transposed transformation matrix, respectively. And θ is the angle of inclination of the bar with reference to the global coordinate system.

For the formation of the Mass Matrix, the present work used the sum of two matrices: the Lumped Matrix, known as Diagonal Matrix, and the Consistent Mass Matrix. The Diagonal Matrix was used to assign the mass of the slabs at each node. For this, the weight of each steel deck slab, used as a deck and covering the structure, was taken from the original design of the footbridge. While the upper slab has a thickness equal to 12cm, with its own weight of 218kg/m², the lower slab, of the deck, has a thickness of 14cm, with the slab's own weight of 266kg/m². From this information, the area mass was transformed into linear mass, and after that, the charge at each node was determined, and the diagonal matrix of nodal mass was added.

2.2 Damping Matrix

According to the Rayleigh's Method, the damping matrix of a structure can be simplified by combining the mass M and stiffness K matrices, multiplied by independent coefficients, α e β, according to eq. (5). These coefficients are determined by the ratio damping ξ of the structure and the two-node vibration modes (ω_i , ω_j). It's also known as the Rayleigh damping matrix. A typical damping ratio of steel structures, 0.5%, was considered. The first and second modes of vibration of the structure are considered.

$$[C] = \alpha[M] + \beta[K], \tag{5}$$

In case of considering the same damping ratio for the considered frequencies, the alpha and beta parameters can be simplified as eq. (6) and eq. (7):

$$\alpha = \xi \frac{2\omega_i\omega_j}{\omega_i+\omega_j}, \tag{6}$$

$$\beta = \xi \frac{2}{\omega_{i+}\omega_j}, \tag{7}$$

2.3 Frequencies and Vibration Modes

The structure analyzed in this article was programmed in *Python 3* language, where all the necessary matrices were formulated, determining the vibration modes and calculating their natural frequencies. In order to compare the developed program, the structure modeling was carried out in commercial software, *Robot Structural Analysis*

(RSA), considering a modal analysis. The difference in the values of the structure frequencies for each vibration mode between the programs is shown in Table 1.

Table 1. Comparison of Natural Frequencies

Vibration Modes	Frequencies (Hz)		Difference (%)
	RSA	*Python 3*	
1º	5.48	5.41	1.22%
2º	12.47	12.39	0.62%
3º	19.27	19.25	0.10%
4º	23.37	22.89	2.07%

The results obtained by the program developed in *Python 3* language can be considered satisfactory, since the differences between the values do not cause any kind of loss. Note that the biggest difference, 2.07%, is related to vibration mode 4, which is responsible for the lateral displacement of the structure, not verified in the present work. The first four vibration modes are shown below, in Fig.3.

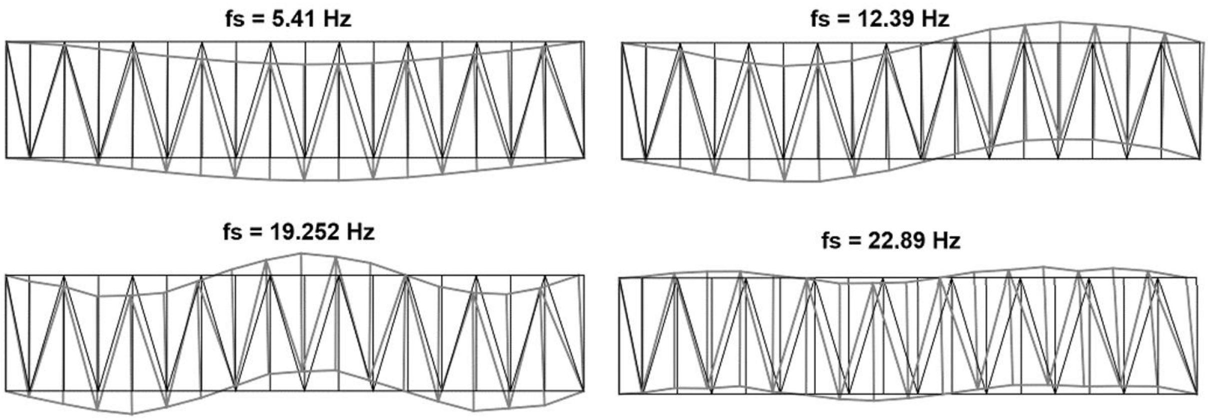

Figure 3. Main Vibration Modes

3 Dynamic Load

This work will address the vibrations produced by human loading, following some guidelines and parameters recommended by Bachmann and Ammann [7]. According to the authors, there are vectors of vertical and horizontal forces produced by human excitation, such as a person's walk. The horizontal loads are not the main ones and were discarded, using only the predominant vertical load applied to the footbridge nodes in the 2D model. Human excitements range from a slow walk, a jog, a march to dancing and jumping. On footbridges, the predominant load is walking, and it can be classified into a range of 3 speeds: slow, normal and fast. For any one of the speeds there are three constants to be known, the excitation frequency, the person's speed and the step length.

For the structure under study, the loading of the normal walking type was used, and its value was determined by eq. 8:

$$F_p(t) = G \, + \, \Delta G_1 \, sin(2\,\pi\, f_s\, t) + \Delta G_2 \, sin(4\,\pi\, f_s\, t - \varphi_2) + \Delta G_3 \, sin(6\,\pi\, f_s\, t - \varphi_3), \qquad (8)$$

where fs is the frequency of human excitation (taken as 2 Hz), t is the period in seconds, φ_2 e φ_3 are the phase angles of the function and G is the average weight of a person in Newton. All formulation variables were studied and analyzed by several authors, including Bachmann and Ammann [7], who recommend the following values: $G \, = \, 800N, \, \Delta G1 \, = \, 0{,}4G, \, \Delta G2 \; e \; \Delta G3 = 0{,}1G, \; \varphi 2 \; e \; \varphi 3 \, = \, \pi/2$, which were taken. Equation 8 consider only one person walking, to consider a crowd, a factor was applied, according to eq. (9):

$$m = \sqrt{\lambda T_0}, \qquad (9)$$

Where m is the crowd factor (taken as 4.7), λ is the mean people flow and T_0 is the total crossing time.

3.1 Validation of the Programmed Algorithm

Miguel *et. al.* [8] analyzed a metallic footbridge subjected to human excitation using MATLAB software, which applied the Finite Differences Method for numerical integration. The present work applied the algorithm developed in *Python 3* programming language, which uses the Newmark's Method, on the footbridge studied by Miguel et. al. [8], as a form of validation. Initially, an algorithm was developed to determine the natural frequencies of the structure, through eigenvalues and eigenvectors. Afterwards, the dynamic load caused by the excitation of human walking was applied, considering a normal load. For this programming, the weight in Newton of a person weighing 80 kg was considered, determining the total time of the crossing, the damping time, as well as the time step and the list of integration time. Table 4 presents the main results of this dynamic analysis and compares with those obtained by Miguel et al[8]. It is noteworthy that the integration method used by Miguel et al.[8] is Finite Differences, while in this study it used up the Newmark's Method.

Table 4 - Validation - Comparison of Results

Response	Current	Miguel et al[8]	Difference %
Displacement (m)	0.0072	0.0074	2.70%
Acceleration (m/s²)	9.46	9.30	1.02%

3.2 Dynamic Analysis

Once the algorithm was validated, a dynamic analysis of the footbridge under study was performed. A series of tests with different integration steps were performed, and a better fit was found at Δt equal to 0.005s. Smaller values did not differ significantly in the results and produced incoherent peaks in acceleration.

Next, in Fig. 4 to 6 , the result of the analysis is presented, which were basically limited to the spatial definition of the load, being the maximum displacement, speed and acceleration.

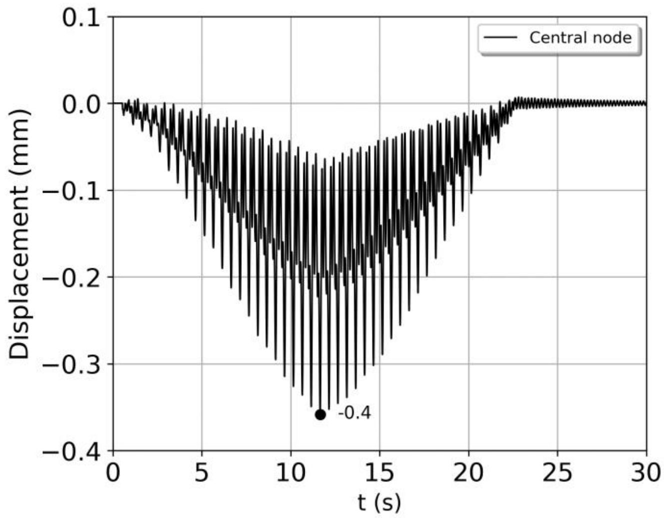

Figure 4. Maximum displacement

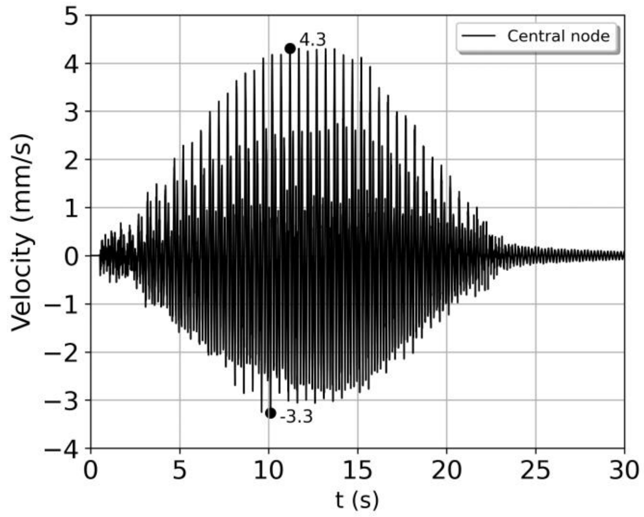

Figure 5. Maximum velocity

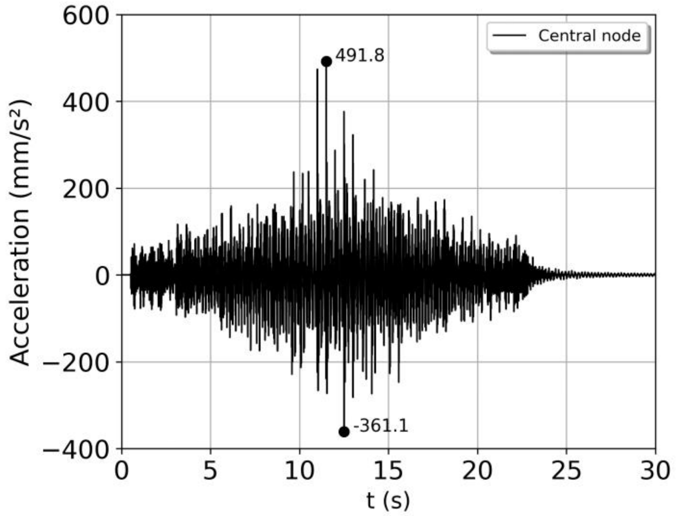

Figure 6. Maximum acceleration

All these results refer to the center of the span, more precisely at node 10. The maximum absolute displacement, velocity and acceleration are 0.4 mm, 4.3mm/s and 491.8 mm/s², respectively.

3.3 Performance Criteria

According to Bachmann and Ammann [7] the maximum admissible acceleration ($\mathrm{Acc_a}$) in m/s², is given by eq. (10):

$$Acc_a = min\begin{cases} 0.5{f_1}^{0.5} \\ 0.25{f_1}^{0.78}, \\ 0.5 \end{cases} \tag{10}$$

Where f_1 is the fundamental frequency of the footbridge, determined as 5.41 Hz. The response, in terms of acceleration, must be fit the criteria of eq. (10). For this analyzed structure, $\mathrm{Acc_a} = 0.5 m/s^2$ or 500 mm/s².

4 Conclusions

The maximum acceleration obtained by the Newmark Integration Method for the dynamic response considering a person in normal walking increased with the crowd effect, as showed in Fig. 6, was 491.8 mm/s², therefore the footbrige under study would meet the criteria and does not require an intervention.

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