

# Influence analysis of suspension parameters on vehicle dynamics through an analytical method

Lucas da S. Gomes<sup>1</sup>, Luiz C. Gertz<sup>2</sup>, Walter J. P. Casas<sup>1</sup>

<sup>1</sup>Department of Mechanical Engineering, Federal University of Rio Grande do Sul  
Rua Sarmiento Leite 425, CEP 90050-170, RS, Porto Alegre, Brasil.  
00327577@ufrgs.br, walter.paucar.casas@ufrgs.br

<sup>2</sup>Department of Mechanical Engineering, Lutheran University of Brazil  
Avenida Farroupilha, 8001, CEP 92425-900, RS, Canoas, Brasil  
luiz.gertz@ulbra.br

**Abstract.** This study was based on a prototype developed by the Automotive Technology Group - ULBRA, whose aim is to evaluate the influence of suspension system parameters on the vehicle's directional behavior. An analytical method was used to calculate the lateral load transfer between the wheels and thus predict the under- or over-steering behavior. A model developed in the CARSIM<sup>®</sup> software served as a comparison and validation of the method. It was possible to define the stiffness of the springs and stabilizer bars to get a vehicle with neutral behavior and it is concluded that the method is effective for understanding the vehicle roll dynamics in curves and the influence of each parameter, such as height and center position gravity, roll center, stiffness, and other design parameters, although it does not consider aspects such as traction or curvature angle variation.

**Keywords:** Directional behavior, vehicle dynamics, suspension system, prototype.

## 1. Introduction

Understanding the dynamic behavior of the vehicle, early in the design stage, allows specifying the stiffness and damping parameters of the suspension system according to the curve behavior determined by the designer.

Based on a two seats prototype developed by the automotive technology group in academic research, this work evaluates the influence of certain design parameters, such as center-of-gravity, instantaneous center of rotation and roll axis, unsprung masses and stiffness of the springs and the front and rear sway bars, in the directional behavior in curves.

Leal *et al.* [1] proposed an analytical calculation method to determine the carbody roll angle, the load transfer between the wheels of each axle and the vertical and lateral loads supported by each tire, being possible to estimate the slip angle of each tire and thus, predict whether the behavior of the vehicle when performing a curvilinear trajectory will be under-steering or over-steering. To verify the method, a model was developed using CARSIM<sup>®</sup> software with the same parameters as the prototype.

Schwartz [2] started the prototype, inspired by Colin Chapman's Lotus Seven, designing a tubular chassis, taking ergonomic and structural issues into account. While Oliveira [3] numerically evaluated a torsional stiffness of 2773.55 Nm/°, satisfying international standards according to the Fiat, Torino 2002 standard, and designed an independent suspension in the rear, type double A, which besides reducing the unsprung mass allows to predict the variation of the camber angle, characteristics that improve the vehicle's behavior on the track.

## 2. Method

Some dynamic effects and concepts are necessary to understand the vehicle's behavior, as well as the definition of some design parameters of the vehicle.

## 2.1 Roll Axis and Center of Gravity

The carbody roll center (CR) is a point where the lateral forces developed by the wheels are transmitted to the sprung mass, according to Gillespie [4], and is directly related to the vehicle's curved behavior. According to Leal *et al.* [1], it is the point around which the carbody starts to turn when is subjected to a lateral force, which is the point at which there is no translational movement at this instant, and the effects of moments and forces can be disregarded for the analysis of tire reactions.

Because of the greater concentration of mass at the front, the center of gravity was considered at 1380 mm from the rear axle and 920 mm from the front axle, resulting in a load distribution of 60% of the weight on the front axle and 40% on the rear axle. The height of the CG was estimated at 425 mm from the ground, as seen in Fig. 1.

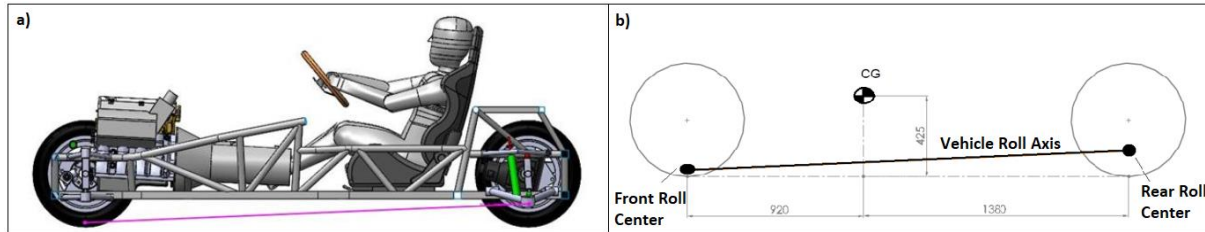


Figure 1. (a) Carbody roll axle (Oliveira, 2016), (b) Estimated position of the center of gravity

## 2.2 Stiffness of springs and stabilizer bars

Gillespie [4] explains that the acceleration transmitted to the sprung mass increases as the natural frequency of the sprung mass grows. It is recommended that the natural frequency of the suspension of a passenger vehicle must be in the range between 1 and 1.5 Hz. But for high-performance cars, it is possible to sacrifice a little comfort for performance by making suspensions stiffer, thus increasing the natural frequency to values between 2 and 2.5 Hz.

The stabilizer bars are circular section steel beams that are fixed to the chassis and the lower ends of independent suspensions. The u-shaped bars increase the transfer of load between the axle wheels, limiting the roll of the carbody in curves. The increase in the load transfer on the shaft consequently causes an increase in the slip angle.

Even though the relationship between the increase in stiffness and the increase in the slip angle is clear, the comfort limits, which consider the natural frequency of the vehicle, were respected according to the literature.

## 2.3 Load transfer

When traveling a curvilinear trajectory, because of centripetal acceleration, a force acts on the vehicle's center of gravity causing a moment, tilting the body sideways and causing that a part of the normal load is transferred from the inner wheel to the outer wheel to the curve, such as seen in Fig. 2(a). According to Leal *et al.* [1], for the same lateral force, the axis that suffers the greatest variation from the normal load will present a greater slip angle. The system of forces acting on the vehicle when it makes a turn is shown in Fig. 2(b).

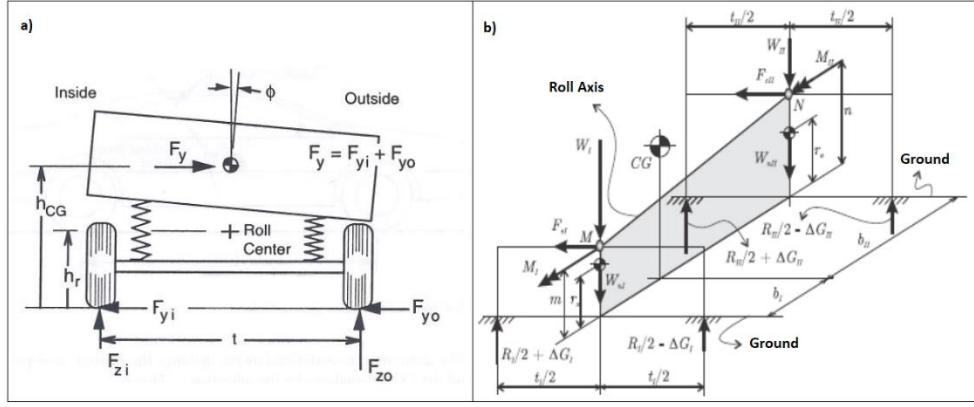


Figure 2. a) Centripetal force acting on the vehicle's CG, Gillespie [4]. b) Systems of forces acting on the vehicle, Leal *et al.* [1]

### 2.3.1 Calculation of the variation of the normal load between the wheels

The transfer of load depends on the type of suspension used, spring stiffness, stiffness and type of stabilizer, and unsprung masses. The total load variation between the front axle wheels (index I) and rear axle (index II), as well as the moment for them, is the sum of four parts that are related to these items, as presented by Leal *et al.* [1].

$$\sum_{j=1}^4 \Delta G_{Ij} = \Delta G_I(1) + \Delta G_I(2) \pm \Delta G_I(3) + \Delta G_I(4) \quad (1)$$

$$\sum_{j=1}^4 \Delta G_{IIj} = \Delta G_{II}(1) + \Delta G_{II}(2) \pm \Delta G_{II}(3) + \Delta G_{II}(4) \quad (2)$$

The first part of the load transfer to an independent suspension, from the inertia of the sprung mass (which is partly absorbed by the springs), is given by eq. (3), where  $\Psi$  is the body roll angle,  $K_I$  is the front springs stiffness and  $t_I$  the front track width. By the action of centripetal force acting on the center of roll of each axis, there is then the second portion of the load transfer, eq. (4) for the front axle and eq. (5) for the rear, where  $W$  is the total vehicle weight,  $b$  is distance between the center of gravity and the axle (I for front and II for rear axle),  $m$  and  $n$  are the distance between the ground and front and rear roll center, respectively, and  $l$  being the wheelbase.

$$\Delta G_I(1) = \Psi K_I \frac{t_I}{2} \quad (3)$$

$$\Delta G_I(2) = \mu_s W \frac{b_{II} m}{l t_I} \quad (4)$$

$$\Delta G_{II}(2) = \mu_s W \frac{b_I n}{l t_{II}} \quad (5)$$

The third part, eq. (6), corresponding to the sway bar (sway bar stiffness is  $K_{EI}$ ), can increase the load transfer with a u-type sway bar, or decrease the mentioned force when the bar is z-type, for this case, a negative signal is used in eq. (1) and eq. (2). The third part referring to the rear axle is obtained replacing the index I to II. The fourth and last portion refers to the charge transfer caused by the inertia of the unsprung masses and can be calculated using eq. (7) and eq. (8), where  $W_n$  is the weight of unsprung mass,  $r_d$  is the wheel dynamic radius and  $p$  is the distance of ground to wheel instant center.

$$\Delta G_I(3) = \Psi K_{EI} \frac{t_I}{2} \quad (6)$$

$$\Delta G_I(4) = 2\mu_s W_{nI} \frac{r_d m}{t_I p_I} \quad (7)$$

$$\Delta G_{II}(4) = 2\mu_s W_{nII} \frac{r_d n}{t_{II} p_{II}} \quad (8)$$

### 2.3.2 Carbody roll angle calculation

The body roll angle,  $\Psi$ , for a vehicle equipped with linear-behavior springs (coil springs) and with independent suspension on both axes can be calculated using eq. (9), where  $\mu_s$  is the lateral coefficient of friction.

$$\Psi = 2\mu_s \frac{W\left(h_m - \frac{b_{I1}n + b_{II}m}{l}\right) + r_d \left[ w_{nI} \left(1 - \frac{m}{p_I}\right) + w_{nII} \left(1 - \frac{n}{p_{II}}\right) \right]}{t_I^2(K_I + K_{EI}) + (K_{II} + K_{EII})} \quad (9)$$

To calculate the roll angle as a function of the vehicle speed,  $v$ , and the curve radius,  $\rho$ , it is necessary to rewrite the equation:

$$\Psi = \frac{2v^2}{\rho g} \left\{ \frac{W\left(h_m - \frac{b_{I1}n + b_{II}m}{l}\right) + r_d \left[ w_{nI} \left(1 - \frac{m}{p_I}\right) + w_{nII} \left(1 - \frac{n}{p_{II}}\right) \right]}{t_I^2(K_I + K_{EI}) + (K_{II} + K_{EII})} \right\} \quad (10)$$

### 2.4 Slip angle calculation

According to Karnopp [5], the angle formed between the wheel's median plane and the resulting velocity vector, after the application of lateral force, is called the slip angle,  $\alpha$ . The slip angle on each axle is nothing more than the average of the slip angles of the axle wheels. This angle will define the vehicle's behavior in a curve. When the axes have equal slip angles, the behavior will be neutral, Fig. 3(a). For a greater slip angle on the front axle, the vehicle will understeer, it will follow a straight path in the curve such a Fig. 3(b), and when the rear axle slip angle is larger, it will have an oversteer behavior as shown in Fig. 3(c).

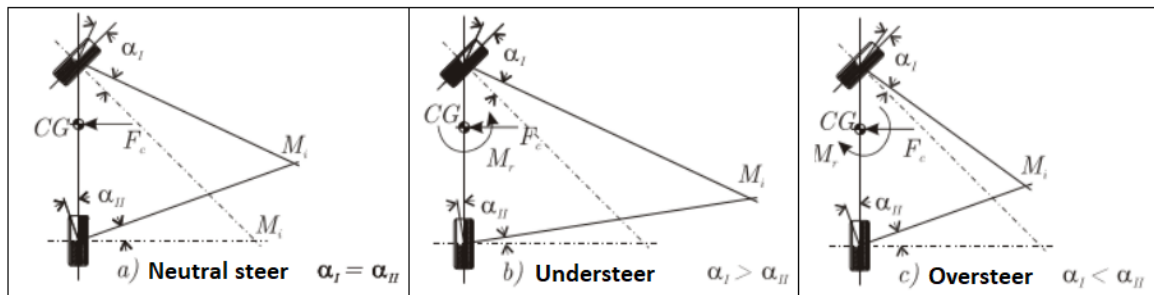


Figure 3. Turning vehicle behavior, Nicolazzi [1]

After calculating the load transfer,  $\Delta G$ , on both axes, the value of the normal load,  $G$ , applied to each wheel is known. One can then calculate the value of the lateral force supported by the tire, eq. (11) corresponds to the outer wheel and eq. (12) to the inner wheel to the curve. The diagram shown in Fig. 4 refers to the lateral force as a function of the normal load, at different slip angles, for a given type of tire. From this diagram, we can then, using interpolation, find the approximate value of the slip angle on each axis,  $\alpha_I$  e  $\alpha_{II}$ .

$$S_{Ie} = \mu_s G_{Ie} \quad (11)$$

$$S_{Ii} = \mu_s G_{Ii} \quad (12)$$

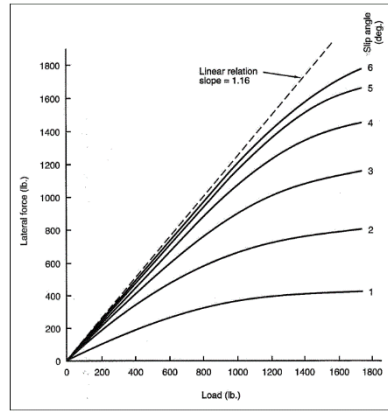


Figure 4. Lateral versus vertical force, Milliken [6]

### 2.5 Simulation test

A model with the same parameters was developed in CARSIM<sup>®</sup>. The chosen test was in the handling and stability category, which follows the ISO4138 standard, being one test used to know the dynamic behavior of a vehicle on the road. In the test, the vehicle travels a flat circular track with a radius equal to 100 m, starting at 6 km/h until it loses grip completely, that is, the friction of the tires changes from static to dynamic.

## 3. Results

The prototype parameters, used in the calculations, are described in Table. 1.

Table 1. Prototype parameters

Parameter	Symbol	Unit	Value
Sprung mass	$m_s$	kg	650
Front unsprung mass	$m_{nI}$	kg	50
Rear unsprung mass	$m_{nII}$	kg	50
CG height	$h_m$	m	0.425
CG distance from the front axle	$b_I$	m	0.92
CG distance from the rear axle	$b_{II}$	m	1.38
CR height of front suspension	$m$	m	0.034
CR height of rear suspension	$n$	m	0.134
wheelbase	$l$	m	2.3
Dynamic radius	$r_d$	m	0.3
CI height of front suspension	$p_I$	m	0.162
CI height of rear suspension	$p_{II}$	m	0.365
Front track width	$t_I$	m	1.37
Rear track width	$t_{II}$	m	1.37

It was verified, through the two methods, that by adding a "u" type stabilizer bar in one of the axles, the load transfer in this axle is increased and, because of the decrease in the rolling angle of the body, the load transfer on the other axis. Therefore, changing a parameter in one of the suspensions has an influence on the behavior of the other suspension.

The function of the stabilizer bars in the project was to get the same performance or even superior to a model with more rigid springs, that is, with a greater natural frequency of the suspended mass, as seen in tests 1, 2 and 3 in the table 2. Making the vehicle more comfortable for the occupants. Following Gillespie's [4] recommendation, the vehicle's natural frequency is 2 Hz.

Table 2. Evaluated variables and directional behavior

		Analytical method			
Variables	Unit	Test 1	Test 2	Test 3	Test 4
Front spring rigidity	N/m	44000	68000	44000	44000
Rear spring rigidity	N/m	24000	37000	37000	37000
Front roll bar	N/m	0	0	11745	11745
Rear roll bar	N/m	0	0	7850	7850
Sprung mass natural frequency	Hz	2	2,4	2	2
Directional behavior	-	Understeer	Understeer	Understeer	Understeer
		CARSIM			
		Test 1	Test 2	Test 3	Test 4
Tractor axle		Rear	Rear	Rear	Front/Rear
Directional behavior		Neutral	Neutral	Neutral	Understeer

The differences in the results in directional behavior between the load transfer and CARSIM® calculations are caused because the equations do not consider which axis is pulling. In test 4 (Table 2), traction on both axes was simulated, therefore, eliminating this variable, the same behavior predicted by the analytical method was observed. The greater load on the front axle, or in other words, the center of gravity shifted forward, is the most responsible for the understeer behavior of the vehicle.

## 4. Conclusions

To achieve a near-to-neutral behavior, it was determined that the stiffness of each front-rear suspension spring must be 44000 N/m, and for the back-rear, the stiffness of each spring must be 24000 N/m. The "u"-shaped stabilizer bar, inserted at the front and rear of the prototype, has rigidity, respectively, equal to 11745 N/m and 7826 N/m, so that the load transfer between the wheels of each axle remains proportional to the vehicle without the bars, and consequently maintain the neutral behavior.

It is concluded that the analytical method was extremely important to understand the effects of each plot (CG height, CR height, springs and stabilizers) and suspension characteristics on load transfer, that is, it is a great tool for both learning, how much to be used in projects and suspension settings.

**Authorship statement.** The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors or has the permission of the owners to be included here.

## References

- [1] L. Leal, E. Rosa and L. Nicolazzi "Uma introdução à modelagem quase-estática de automóveis". Departamento de Engenharia Mecânica, Universidade Federal de Santa Catarina, 2012.
- [2] R. Shavartz, "Projeto e análise estrutural de um chassi automotivo". Trabalho de Conclusão, Universidade Luterana do Brasil, 2013
- [3] A. Oliveira, "Análise estrutural de protótipo automotivo". Trabalho de conclusão, Universidade Luterana do Brasil, 2016.
- [4] T. Gillespie, "Fundamentals of vehicles dynamics". Warrendale, Pa: Society Of Automotive Engineers, 1992.
- [5] D. Karnopp, "Vehicle stability". Davis, Ca: Marcel Dekker, 2004.
- [6] W. Milliken and D. Milliken, "Race car vehicle dynamics". Warrendale, Pa: Society Of Automotive Engineers, 1995.