Coupling Model for Viscoelastic Sandwich Beams Used for Vibration Control

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Abstract. Dynamic neutralizers are simple devices used to minimize the levels of vibration and noise radiated from a mechanical structure in a certain frequency range. They operate by inserting a high mechanical impedance in the region of interest, applying reaction forces, and dissipating vibratory energy. Composite sandwich beams with viscoelastic materials are used to control vibration in structures with high modal density, such as plates, slender buildings and electrical transmission lines due to their dynamic characteristics. In addition to the high damping factor, the use of these neutralizers with multiple degrees of freedom (MDOF) is favorable for the control of these structures, especially when working in a wide frequency band. This study presents a novel coupling model using angular and translational degrees of freedom between the structure to be controlled (primary system) and the MDOF auxiliary system attachment point. The dynamic behavior of the MDOF sandwich beam under study is modeled using the commercial finite element software ANSYS®. Then, in the Matlab® software, the compound system is assembled using the primary system modal parameters coupled to the dynamic behavior of the sandwich beam using translational and rotation equivalent springs. The finite element method was validated with experimental data of a single degree of freedom viscoelastic neutralizer. The numerical and experimentally dynamic behavior showed an excellent approximation. Additionally, the transfer functions of a fixed-free beam with a sandwich beam attached to the free end are analyzed studying the coupling of translational and rotation DOF. For MDOF neutralizers, the coupling model accuracy is extremely important to ensure the proper design of the auxiliary system physical parameters for vibration control.

Keywords: Sandwich beams, constrained layers, multiple degrees of freedom coupling, finite elements, viscoelastic materials.

1 Introduction

Vibration or oscillation is characterized by the oscillatory movement of a system, manifesting itself in the transfer of potential energy into kinetic energy in a cyclic way. Such behavior is triggered by the action of dynamic loads. To reduce the response of a system under such conditions, different types of passive and active/adaptive controls are used. Among the passive controls, one can mention the addition of damping, isolation, and the use of dynamic neutralizers (DNs). DNs, called auxiliary systems, are simple devices that aim to reduce the vibration levels of the structure in which they are fixed to acceptable values. According to SUN et al. [1], these devices operate in the form of a reaction force and vibratory energy dissipation through a high injection of mechanical impedance in the structure to which they are fixed. Generally speaking, NDs are made up of mass, stiffness, and damping elements.

These devices can be divided according to their construction model: systems with one degree of freedom (DOF) and systems with multiple degrees of freedom (MDOF). Among the MDOF models, sandwich beam and constrained layer devices stand out. These systems are composed of a base beam, a layer of viscoelastic material fixed on the base beam and finally, a constricting layer, generally metallic, fixed on the viscoelastic material.

Through this construction, the dissipative capacity of the viscoelastic material is transmitted to the base beam through the shear imposed by the constricting layer. Silva [2] studied the optimal design of viscoelastic sandwich beams for vibration control using the equivalent dynamic stiffness method. In the compound model equations, the author replaced the MDOF ND with a dynamic equivalent spring, whose stiffness is a function of frequency. In this context, this paper proposes to expand the models used in Silva [2], by modeling the auxiliary system as a translational and a rotational equivalent spring.

This paper will present a methodology that can be applied to describe the dynamic behavior of a compound structure composed of a primary system, representing the system to be controlled, and a constrained layer structure, represents the MGL auxiliary system, which representing the MGL control device. The translational and rotational coupling is capable of representing more accurately the real composite system and aims to improve and strengthen the methodologies currently used in the optimal design of MGL NDVs.

2 Theoretical Modeling of the Compound System

2.1 Viscoelastic Materials

Viscoelastic materials exhibit both viscous and elastic properties, capable of storing and dissipating mechanical energy. Among these models, we can mention the Maxwell, Kevin-Voigt, or Zenner models described by Banks [3], which are based on linear springs and viscous dampers. One model utilizes the constitutive equation of these materials based on fractional-order derivatives, as described by Cruz [4], and, combined with the Zenner model and applying Fourier transform it is possible to obtain the complex shear modulus, given by

$$G_c(\Omega) = \frac{G_0 + G_\infty b_1 (i\Omega)^\beta}{1 + b_1 (i\Omega)^\beta}, \tag{1}$$

where $G_0(\Omega)$ represents the lowest asymptotic value of the modulus, that is, at frequencies close to zero. On the other hand, G_∞ represents the upper asymptotic value of the modulus, for frequencies tending to infinity, β is the fractional order of the derivative and b_1 is the constant related to the material's relaxation time.

The effects of temperature and working frequency can be combined using just one variable, the reduced frequency. This is possible through the principle of frequency-temperature superposition, where, from experimental data, an accurate dynamic characterization is obtained. This principle establishes that the different dynamic properties curves can be superimposed, through a reference temperature and through frequency shifts, generating two fundamental curves (G and η_G). So, mathematically, we have:

$$G_{R0}(\Omega_{\text{red}}) = \frac{T_0 \rho_0}{T\rho} G_R(\Omega, T); \tag{2}$$

$$\eta_{R0}(\Omega_{\text{red}}) = \eta_G(\Omega, T)\,, \tag{3}$$

where $\Omega_{\text{red}} = \alpha_T(T)\Omega$ represents the reduced frequency, α_T the displacement factor, T_0 the reference temperature, in absolute scale, ρ the density and ρ_0 the density at the reference temperature.

The values of α_T are estimated so that the partial curves shift in frequency so that complete superpositions are achieved at the reference temperature. This model, proposed by Williams, Landel and Ferry in 1980, called the WLF equation, is used to estimate α_T:

$$\log_{10} \alpha_T(T) = \frac{-\theta_1 (T - T_0)}{\theta_2 + T - T_0}, \tag{4}$$

where θ_1 and θ_2 are characteristic parameters of each material and are determined experimentally.

2.2 A primary system with MDOF

The equation of motion that can be used to describe its dynamic behavior in the domain of the frequency is given by:

$$[-\Omega^2 M + i\Omega C + K]Q(\Omega) = F(\Omega), \quad (5)$$

where M, C, and K represent the mass, damping stiffness matrix respectively, Q represents the displacements DOF of the system and F corresponds to the external force. As demonstrated by Ewins [5], the equations of motion of the MGL system can also be written in the frequency domain using the Fourier transform. Additionally, the solution can be performed in modal space, employing a coordinate transformation. The response of the system, to a given force $F(\Omega)$, can be evaluated using the transfer functions obtained when the eigenvalue problem assembled in the modal space is solved.

2.3 Equivalent dynamic stiffness for 1 DOF model

The theory introduced by Espíndola and Silva [6] of equivalent generalized parameters (PGE) is presented. Through PGE it is possible to establish the dynamic equivalence of models used in the design of auxiliary systems. A classic 1 DOF system with viscoelastic material is shown in Figure 1 and its model with PGE of equivalent stiffness.

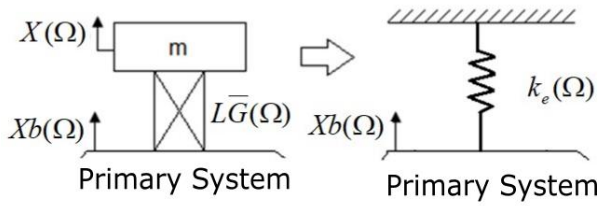

Figure 1. PGE model for a 1 DOF system

The model using PGE does not introduce additional degrees of freedom in the composite system model, being embedded in the equivalent dynamic stiffness, naturally, as functions of frequency. It is important to note here that the temperature was omitted for simplicity and will be considered constant. As described in Doubrawa [7], the equivalent dynamic stiffness can be calculated analytically for systems with one DOF by the following equation:

$$k_{eq}(\Omega) = \frac{-\Omega^2 m_a \mathrm{r}(\Omega)[{\varepsilon_a}^2 - \mathrm{r}(\Omega)\{1+\eta^2(\Omega)\} + \mathrm{i}\Omega m_a \Omega_a \mathrm{r}(\Omega)\eta(\Omega){\varepsilon_a}^3}{[{\varepsilon_a}^2 - \mathrm{r}(\Omega)]^2 + [\mathrm{r}(\Omega)\eta(\Omega)]^2}. \quad (6)$$

2.3.1 Coupling model for 1 DOF systems

The behavior of the composite system can be described as a function of the equations of motion of the primary system, applying PEG. Using the equivalent stiffness model, the composite system can be modeled by modifying eq. 5:

$$\left[-\Omega^2 M + i\Omega C + \widetilde{K}\right]Q(\Omega) = F(\Omega), \quad (7)$$

where the matrix $\widetilde{K}$ represents the stiffness matrix of the composite system. This, in turn, can be obtained by inserting the concentrated stiffness $k_{eq}(\Omega)$ in the matrix of the primary system, in the index corresponding to the degree of freedom of the fixation point. Such procedure is described through eq.8:

$$\widetilde{K} = K + \begin{bmatrix} 0 & & & & \\ & \ddots & & & \\ & & k_{eq}(\Omega) & & \\ & & & \ddots & \\ & & & & 0 \end{bmatrix} = K + K_{eq}. \quad (8)$$

The eq. 7 can be used to calculate the response of the composite system to any excitation force, in a manner analogous to that described for MGL systems. This methodology can be generalized to add p independent auxiliary systems, inserting the concentrated stiffness $k_{eq_j}(\Omega)$, with $j = 1\ a\ p$, in their respective positions on the diagonal of the matrix K_{eq}.

2.3.2 Coupling model for MDOF auxiliary systems

To obtain the dynamic equivalent stiffness of the MDOF auxiliary system, the concepts described previously need to be expanded. The base of the mechanical system with 1 DOF is punctual, massless, and is assumed to only move in one direction. As for the MGL system, the base is continuous, can move and rotate in any of the three directions, and is located somewhere in the coupling region between the systems.

Thus, for the sandwich beam proposed, the base for calculating the equivalent dynamic stiffness was defined as the nodes inserted in the contact line between the primary system and the auxiliary system, along the width of the base beam. These nodes belong to the auxiliary system FEM mesh. The equivalent dynamic stiffness on the basis of the finite element model of the auxiliary system $\boldsymbol{K}_{b_{eq}}(\Omega)$ is given by:

$$K_{b_{eq}}(\Omega) = \begin{bmatrix} [K_{eq_1}] & & & 0 \\ & [K_{eq_2}] & & \\ & & \ddots & \\ 0 & & & [K_{eq_j}] \end{bmatrix} \tag{9}$$

where $\boldsymbol{K}_{eq_j}$ is the equivalent dynamic stiffness matrix of the jth node inserted in the base. This matrix is full and has grade 6, containing information on the equivalent stiffness of all degrees of freedom. For the sandwich beam proposed, in the coordinate system adopted, only the displacements along z and rotations along x were of interest. Therefore, the matrices K_{eq_j} was truncated, containing only the dynamic stiffness in the degrees of freedom in the direction of interest. Only the elements referring to the z-axis translation and θ_x rotation were considered. With these simplifications, K_{eq_j} is given by:

$$\breve{K}_{eq_j}(\Omega) = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & K_{eq_{zz}} & K_{eq_{z\theta_x}} & & \\ & & K_{eq_{\theta_x z}} & K_{eq_{\theta_x\theta_x}} & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix}, \tag{10}$$

additionally, applying the definition of dynamic stiffness, we have that the elements of the matrix $\breve{K}_{eq_j}(\Omega)$ are:

$$K_{eq_{zz}}(\Omega) = F_{reaçao_z}/X_z\,; \tag{11}$$

$$K_{eq_{z\theta_x}}(\Omega) = F_{reaçao_z}/X_{\theta_x}\,; \tag{12}$$

$$K_{eq_{\theta_x z}}(\Omega) = M_{reaçao_{\theta_x}}/X_z\,; \tag{13}$$

$$K_{eq_{\theta_x\theta_x}}(\Omega) = M_{reaçao_{\theta_x}}/X_{\theta_x}, \tag{14}$$

where X_z and X_{θ_x} represent the displacements in the z and θ_x direction respectively. Two different conditions must be applied, separately, to obtain the matrix $\breve{K}_{eq_j}(\Omega)$:

- First, the following contour conditions, in the form of unitary displacements, are applied to all nodes located in the base of the auxiliary system: $Xj(\Omega) = \{0,0,1,0,0,0\}$. Evaluating the reaction forces and reaction moments, the equations 11 and 13 can be solved.
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Similar to the development of equation 8 for 1 DOF auxiliary systems, for MDOF, the equation of motion can be described as:

$$\breve{K} = K + \begin{bmatrix} 0 & & & & & \\ & \ddots & & & & \\ & & [\breve{K}_{eq_1}] & & & \\ & & & \ddots & & \\ & & & & [\breve{K}_{eq_j}] & \\ & & & & & \ddots & \\ & & & & & & 0 \end{bmatrix} = K + \breve{K}_{eq}. \tag{15}$$

The matrix $\breve{K}_{eq}$ must be assembled with caution so that the matrices $\left[\breve{K}_{eq_j}\right]_{6x6}$ are inserted in the correct position in the primary system.

3 Numerical model

3.1.1 Viscoelastic FEM with 1 DOF

Since the finite element software used (ANSYS®) does not support the viscoelastic model chosen, the material's mechanical properties were imported as experimental data. To validate this approach, a simple 1 DOF viscoelastic auxiliary system was analyzed (Figure 2, left). This configuration is widely used in vibration control and was described in great lengths by Doubrawa [7]. The viscoelastic material used was the butyl rubber BT 806/55, and its proprieties can be found on Silva [8]. The equivalent dynamic stiffness was calculated analytically, using eq.15, numerically using the software ANSYS®, and measured experimentally. The FEM is presented by (Figure 2, right). The equivalent dynamic stiffness is presented in Figure 3.

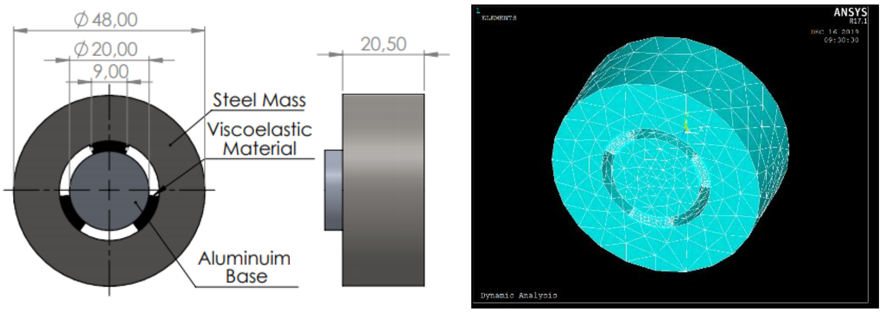

Figure 2. One DOF auxiliary system dimensions (left) and FEM (right)

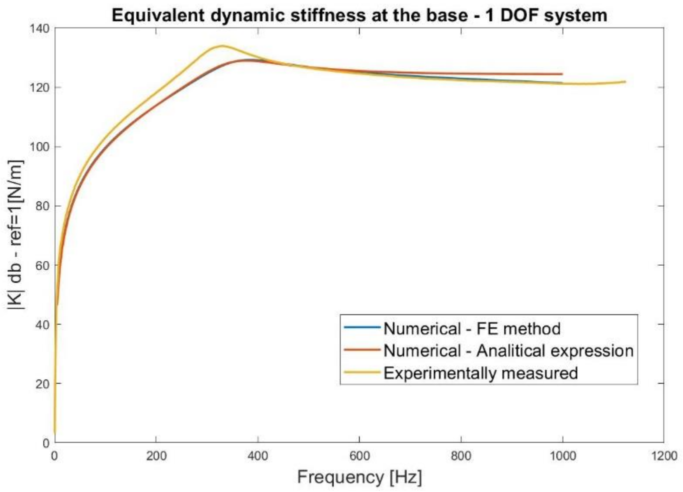

Figure 3. Comparison between equivalent dynamic stiffness methods for 1 DOF system

3.1.2 Compound system using MDOF auxiliary system

For the MDOF primary system, a steel cantilever beam with a rectangular cross-section was chosen, the properties are described in Table 1. The compound system was composed of the steel cantilever beam with the sandwich beam attached to its free end. The same viscoelastic material, butyl rubber BT 806/55, was used for the sandwich beam, composed of an aluminum base and constrained layer.

Figure 4 presents a mechanical model of the MDOF compound system. In the left, a schematic complete FEM is presented, and on the right, the auxiliary MGL system was substituted by the equivalent dynamic stiffness. The presented model makes use of a displacement and a rotational spring.

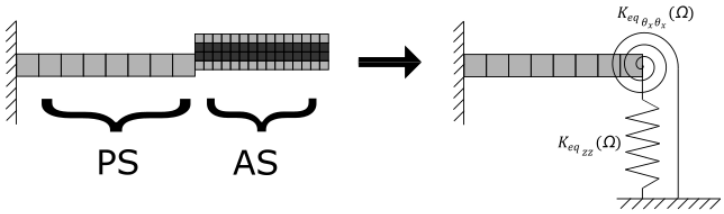

Figure 4. Equivalent models of the compound system

Table 1. Material proprieties and dimensions of the primary and auxiliary systems

	Length [mm]	Width [mm]	Thickness [mm]	Young modulus [GPa]	Poisson coefficient
Primary system beam	500	50.8	12.7	210	0.3
Sandwich base beam	150	25.4	6	69	0.33
Sandwich viscoelastic	150	25.4	4.2	*	*
Sandwich Constraint Layer	150	25.4	6	69	0.33

*The viscoelastic proprieties vary in function of the frequency and temperature and can be found on Silva [8].

Two models were compared, the first describes the dynamic behavior of the compound system utilizing the equivalent translational and rotational springs inserted in the motion equations, and in the second, the entire compound system was modeled using FEM. In the first model, the matrix $\breve{K}_{eq}$ was obtained using the FEM of the sandwich beam, and the motion equations of the composite system were assembled using the modal parameters of the primary system, similar as described by Silva [2] The response to a unitary excitation in the region of coupling between the systems was obtained. In the second model, the primary system was modeled using 960 elements (8 nodes shell elements, with 6 DOF), and the sandwich beam was modeled using 160 elements in the base (8 nodes shell elements, with 6 DOF) and 480 elements in the viscoelastic and constraint layer (20 nodes solid elements, with 3 DOF). The coupling between both systems was assured using constraint equations merging the nodal displacements in the region where the sandwich beam was attached. The comparison between the response of the compound system obtained by both analyses is presented in Figure 5.

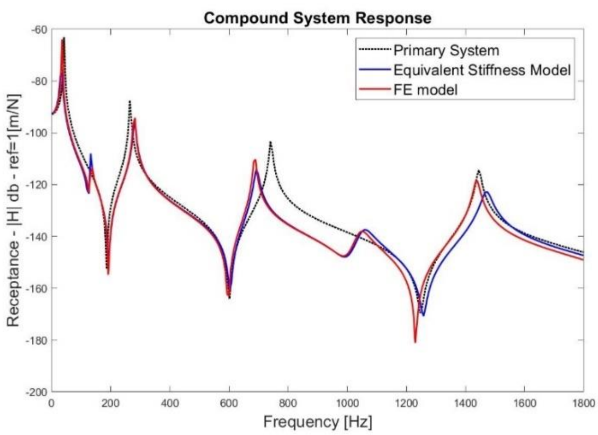

Figure 5. Compound system response

It is noteworthy the similarity between the resonant frequencies of the different models. In general, the composite system model via dynamic stiffness at the base generated a satisfactory approximation of the behavior of the steel beam attached to a viscoelastic sandwich beam.

4 Conclusions

A methodology to predict the dynamic behavior of the system composed of a steel beam and another one composed of viscoelastic material was proposed, using the concept of equivalent dynamic stiffness. This way, the compound structure can be represented by the primary system modal parameters and an embedded spring. This spring was considered in both angular and linear displacement. This approach for coupling ensures the auxiliary system can exert influence in the primary system similar to what happens in the physical coupling.

The model proposed offered robustness and accuracy in predict the behavior of compound systems attached to constrained layer viscoelastic beams. The approach using equivalent stiffness is suitable for use in the optimal design of MDOF viscoelastic dynamic neutralizers.

To obtain the equivalent dynamic stiffness of systems with viscoelastic materials via FEM method, a methodology to import the viscoelastic behavior as experimental data in the FE software was proposed. A simple 1 DOF system was studied, comparing the FEM method, analytical and experimental data. Additionally, the transfer functions of a fixed-free metallic beam with a viscoelastic sandwich beam attached to the free end were studied, considering a coupled model with displacement and angular DOF.

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