Topology Optimization of Binary Structures Subjeted to Self-weight Loads

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Abstract. The study of structures subject to self-weight loads is particularly important for the fields of civil, aeronautical, and aerospace engineering. Topology optimization emerges as a crucial tool in this analysis providing structures with non-intuitive conceptual designs and greater material savings. Binary methods are among the most established methods, where the design variables assume discrete values 0 and 1 for the void and for the solid material, respectively. In previous studies, it has been reported that topology optimization of structures subject to self-weight loads using binary methods is almost impossible to be employed without the RAMP material model. This article shows that binary topology optimization for self-weight loads depends on the formulation and not only on the material interpolation. To illustrate that, the classic SIMP material model is used together with the recently developed Topology Optimization of Binary Structures (TOBS) method for topology optimization of structures subject to self-weight loads. The algorithm was tested and verified to analyze two bidimensional benchmarking problems. The effect of penalty variation on the final topology was discussed for modified SIMP model. From the results, it was demonstrated that the modified SIMP model combined with TOBS allows to efficiently optimize structures subject to self-weight loads.

Keywords: Topology optimization, Integer Linear Programming, Binary variables, Modified SIMP model, Self-weight.

1 Introduction

Structural topology optimization presents a large field of application in engineering, where the idea is to find the optimal material distribution minimizing an objective function subjected to constraints. However, according to Han et al [1] and Xu et al [2], most contributions on topology optimization in the past decades are focused on minimizing the compliance or other global response functions subject to volume fraction constraint under fixed external loads. From Han et al [1], another important kind of load condition, i.e. body force loads, which are design-dependent and include self-weight, inertial forces, and centrifugal forces, has not been studied extensively. This paper will focus only in self-weight loads.

Bruneel and Duysinx [3] initiated topology optimization problems researches of structures subjected to self-weigth loads. In their paper, three challenges were observed: non-monotonous behavior of the compliance, inactive volume constraint, and parasitic effect for low density region using the SIMP model. Following Brunneel and Duysinx [3] study, Yang et al [4] and Ansola et al [5] proposed the modified ESO method to solve the topology optimization problem including self-weight loads. Huang and Xie [6] developed a new BESO method using the RAMP material model to solve the self-weight problem. Recently, Han et al [1] used the same method of the previous author to solve topology optimization problems of a structure subjected to body force loads (self-weight, centrifugal forces, and inertial forces). In their works, Han et al [1] and Huang and xie [6] stated that is almost impossible to obtain a {0,1} design using the power law material interpolation scheme. From this, the TOBS method arises as an alternative to overcome this statement. TOBS presents computational performance similar to standard methods such as SIMP, that is, the solution of the linear problem discretized by finite elements is the computational bottleneck. Solving the ILP problem takes less than 1 s per iteration for 2D toy problems. A more complete study on the computational performance of the method can be verified in Sivapuram and Picelli [7].

The TOBS method combines four numerical ingredients: sequential problem linearization, relaxation of constraints (motion limits), sensitivity filtering, and an integer programming solver. The method presents an improvement over the previously available discrete topology optimization methods, namely, the popular BESO method by Huang and Xie [6] and those by Svanberg and Werne [8] and Beckers [9]. Furthermore, the use of design variables {0,1} gives the TOBS method an advantageous potential in solving design-dependent physical problems. Several researches demonstrate the capabilities of the TOBS method: Sivapuram and Picelli [10] performed volume minimization subject to compliance constraints and Sivapuram et al. [11] extended it to material design, including several restrictions. Sivapuram and Picelli [7] performed topology optimization of structures including design-dependent fluid pressure and thermal expansion loads, and recently, Picelli et al [12] devised a technique that considers separate physics analysis and optimization within the context of fluid-structure interaction (FSI) systems.

By exploring the TOBS capabilities, the aim of this paper is to show that topology optimization including self-weight loads depends of the formulation and not only the material interpolation. The problems are solved by using the TOBS method with a modified SIMP model. Several classic bidimensional examples are presented.

2 Topology Optimization of Binary Structures (TOBS)

The TOBS method, proposed by Sivapuram and Picelli [10], employs binary design variables {0,1}. This methodology linearizes the objective and constraint functions associated with integer linear programming (Williams [13]). Therefore, the linearized optimization problem to be solved is given by:

$$\begin{aligned} \underset{\Delta\mathbf{x}^k}{\text{Minimize}} \quad & \left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\mathbf{x}^k} \Delta\mathbf{x}^k \\ \text{Subject to} \quad & \left.\frac{\partial g_i}{\partial \mathbf{x}}\right|_{\mathbf{x}^k} \Delta\mathbf{x}^k \leq \bar{g}_i - g_i^k \quad i \in [1, N_g] \\ & ||\Delta\mathbf{x}^k||_1 \leq \beta N_d \\ & \Delta x_j \in \{-x_j, 1 - x_j\} \quad j \in [1, N_d] \end{aligned} \tag{1}$$

where $f(\mathbf{x})$ is the objective function, bounded by $g_i(\mathbf{x}) \leq \bar{g}_i$. $i \in [1, N_g]$, where N_g and N_d are respectively the number of inequality constraints and elements in the vector of design variables. β is the flip limits and $||\Delta\mathbf{x}^k||_1$ is the truncation error. The term g_i^k is the value of the constraint g_i in the k_{th} optimization iteration. The ILP solver is used to find the optimal change $\Delta\mathbf{x}$ for the integer design variables $\mathbf{x}$. After each iteration, the design variables are updated as $\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta\mathbf{x}^k$.

3 Topology Optimization Problem and Sensitivity Analysis

The formulation of the binary optimization problem from eq. (1) is related to minimizing the mean compliance of the structure under body forces subject to a given volume constraint. Differing from the topology optimization problem with fixed external forces, the applied force vector, $\mathbf{f}$, includes design-dependent gravity loads. The optimization problem is expressed as:

$$\begin{aligned} \underset{x}{\text{Minimize}} \quad & C(x) \\ \text{Subject to} \quad & V_i(x) \leq \overline{V}_i, \quad i \in [1, N_g] \\ & x_j \in [0, 1], \quad j \in [1, N_d] \end{aligned} \tag{2}$$

where $C(x)$ is the structural compliance or total deformation energy, V_i is the volume fraction of the structure, and $\overline{V}_i$ is the constrained volume fraction. The examples presented in this work are solved with one volume constraint, i.e., with N_g = 1.

From Huang and Xie [14] and Han et al. [1] the adjoint method can be used to determine the sensitivity of displacement and force vectors by introducing a Lagrangian multiplier vector λ. The sensitivity of the objective function can be expressed as:

$$\frac{dC}{dx_j} = \frac{\partial \mathbf{f}^T}{\partial x_j}\mathbf{u} - \frac{1}{2}\mathbf{u}^{\mathbf{T}}\frac{\partial \mathbf{K}}{\partial x_j}\mathbf{u} \tag{3}$$

It is seen that the sensitivity of the compliance can be both positive and negative. It may even change sign when the value of the design variable is changed. Therefore, this problem exhibits non-monotonous behavior.

4 Material Models

In this work, the modified SIMP and RAMP material interpolation models will be used. In the modified SIMP material model, proposed by Sigmund [15], the intermediate material stiffness can be penalized with the power-law as:

$$E(x_j) = E_{min} + x_j^p(E_0 - E_{min}) \tag{4}$$

where p is the penalization, E_{min} is the stiffness of soft (void) material, and E_0 is the Young's modulus of solid material. Later, Felix et al [16] proposed the use of an η variable to express the ratio between the defined E_{min} and E_0. From this, the eq. (4) becomes:

$$E(x_j) = [\eta + x_j^p(1-\eta)]E_0 \tag{5}$$

Alternatively, the RAMP material model was proposed by Stolpe and Svanberg [17]. In this model, the density and Young's modulus of the material are expressed by:

$$\rho_(x_j) = \rho_0 x_j \tag{6}$$

$$E(x_j) = E_0\frac{x_j}{1+q(1-x_j)} \tag{7}$$

where ρ_0 and E_0 denote the density and Young's modulus of the solid material, respectively and q is the penalty factor which is larger than 0 for topology optimization problems.

When the structure is meshed with finite elements using four-node quadrangular elements, the elemental load vector subjected to its self-weight can be obtained as the following equation assuming that the gravity is aligned with the global y direction:

$$\mathbf{f_j} = \frac{1}{4}V_j\rho_j g\overline{\mathbf{f}} = \frac{1}{4}V_j\rho_j g[0,\ -1,\ 0,\ -1,\ 0,\ -1,\ 0,\ -1]^T \tag{8}$$

where g is the gravity acceleration. Thus, the variation of the external forces is

$$\frac{\partial \mathbf{f}}{\partial x_j} = \frac{1}{4}V_j\rho_j g\overline{\mathbf{f}} \tag{9}$$

From eq. (3), the sensitivity for the modified SIMP material model with respect to the design variable x_j is given by:

$$\frac{dC}{dx_j} = \frac{1}{4} V_j \rho_j g \overline{\mathbf{f}}^T \mathbf{u_j} - \frac{1}{2} p x_j^{p-1} (1-\eta) \mathbf{u}_j^T \mathbf{K}_j \mathbf{u}_j \tag{10}$$

Likewise, from eq. (3), the sensitivity for the RAMP material model can be expressed as:

$$\frac{dC}{dx_j} = \frac{1}{4} V_j \rho_j g \overline{\mathbf{f}}^T \mathbf{u_j} - \frac{1+q}{2[1+q(1-x_j^2)]^2} \mathbf{u}_j^T \mathbf{K}_j \mathbf{u}_j \tag{11}$$

4.1 Sensitivity Filter

To avoid the well known problem of obtaining checkerboard solutions, generally a mesh-independent filter is used (Sivapuram and Picelli [10], Huang and Xie [14]). The filtered sensitivity of an element e is obtained using a weighted average of element sensitivity over the neighborhood of e defined by a radius r_{min}. The filtered sensitivity filter is given as:

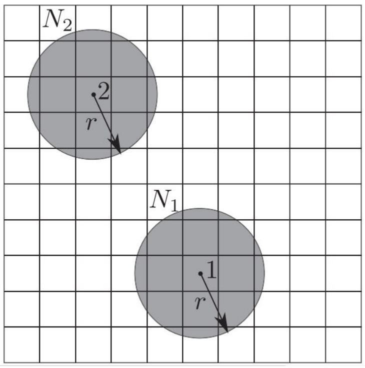

Figure 1. Filtering: areas of averaging for filtered elemental sensitivities (Elements 1 and 2) [10].

$$\frac{\partial f}{\partial x_e} = \frac{\sum_{m \in N} H_{em} \frac{\partial f}{\partial y_m}}{\sum_{m \in N} H_{em}} \tag{12}$$

where $\partial f / \partial x_e$ is the filtered element-based sensitivity, N is the neighborhood around element e as shown in Fig. (1) from where elemental sensitivities are considered for weighted averaging, and the weights H_{em} are given by:

$$H_{em} = \texttt{max}(0, r_{min} - \texttt{dist}(x_e, x_m)) \tag{13}$$

where r is the radius of circular (2D) neighborhood region N. The weights are defined such that the nodes nearer to the element contribute higher to its corresponding filtered sensitivities compared to farther nodes. `dist` function computes the Euclidean distance between two nodes in eq. (7). In some cases, seeking to improve time stabilization, the filtered sensitivity field is calculated in two consecutive iterations as:

$$\frac{\partial f}{\partial x_e} = \frac{\frac{\partial f}{\partial x_e}^k + \frac{\partial f}{\partial x_e}^{k-1}}{2} \tag{14}$$

5 Examples

5.1 Self-Weight Arch Example

The design domain of a rectangular plate is illustrated in Fig. 2. This model is a benchmarking example for topology optimization with self-weight loads and it was studied by some other researchers (Bruneel and Duysinx [3], Ansola et al [5] and Huang and Xie [6]). The following material properties are assumed: Young's modulus of 200 GPa, Poisson's ratio of 0.3, and density of 7850 kg/m^3. Due to the symmetry, only half of the design domain is analysed using a mesh of 50 x 50 four-node quadrilateral elements. The parameters for the TOBS are: penalty = 5, ϵ = 0.5%, β = 1%, convergence tolerance (τ) = 0.001, x_{min} = 10^{-3}, η = 10^{-9}, and r_{min} = 0.3 m. Plane stress condition is assumed.

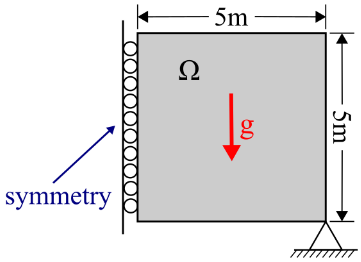

Figure 2. Design domain of self-weight arch example.

Figure 3 shows that it was possible to achieve at a valid optimal topology using the TOBS method combined with both RAMP and modified SIMP material models. In addition, the result obtained by Huang and Xie [6] using the BESO soft-kill method combined with RAMP is presented for comparison. All topology solutions were quite similar. These results already shows that the TOBS method allows solving problems of structures subject to self-weight using a power-law interpolation.

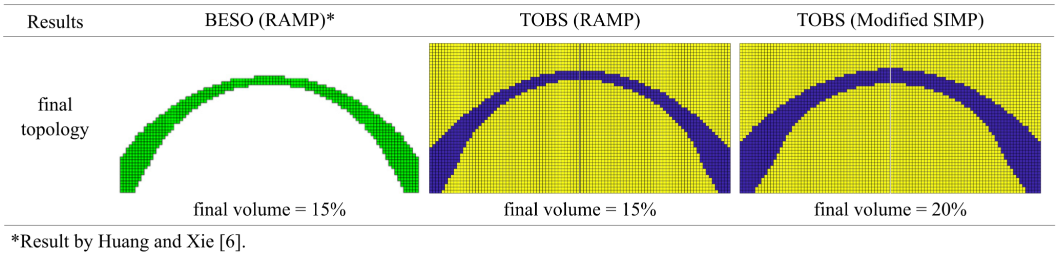

Figure 3. Comparison of final topologies for arch structure example using TOBS method with RAMP and modified SIMP material models.

The effect of penalty variation was studied for arch example. For this study, the material properties and TOBS parameters are the same except for the penalty values. In this case, only the modified SIMP material model was used. The final volume was defined as 25% for all cases. Figure 4 presents the final topology for different penalty values.

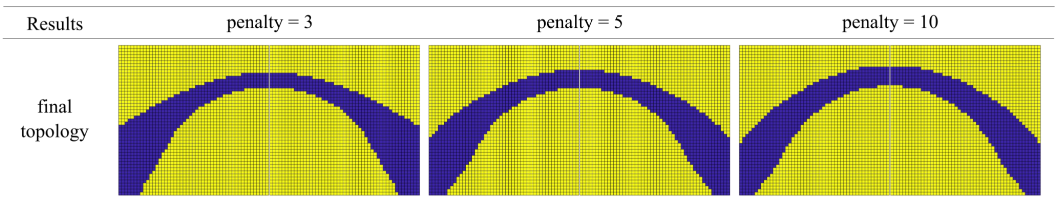

Figure 4. Comparison of different penalty values in final topology using TOBS method with SIMP material model.

This result is important because it shows that the TOBS method also allows optimizing structures subject to self-weight with lower penalty values (penalty = 3), since the value commonly used in this type of study is penalty

= 5. The final structure topology using penalty = 3 differs a little from the other conditions in the region of the supports, where it is observed that the optimizer sought to place more material in that location.

5.2 MBB Beam Example

The design domain of the MBB beam is illustrated in Fig. 5. The Young's modulus is 200 GPa, Poisson's ratio is 0.3, and density is 7850 kg/m^3. Due to the symmetry, only half of the design domain is analysed using a mesh of 100 x 50 four-node quadrilateral elements. The parameters for the TOBS are: penalty = 5, ϵ = 0.5%, β = 1%, convergence tolerance (τ) = 0.001, $x_{min} = 10^{-3}$, $\eta = 10^{-9}$, and r_{min} = 0.3 m. The final volume is assumed as 40% for all cases. Plane stress condition is assumed.

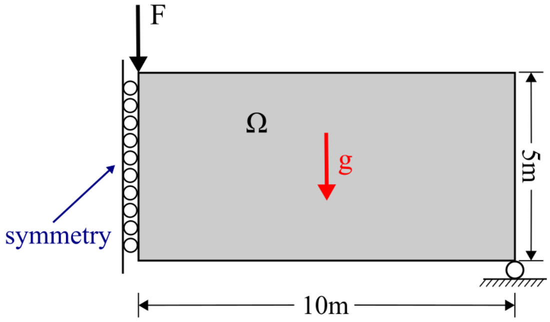

Figure 5. Design domain of MBB beam example.

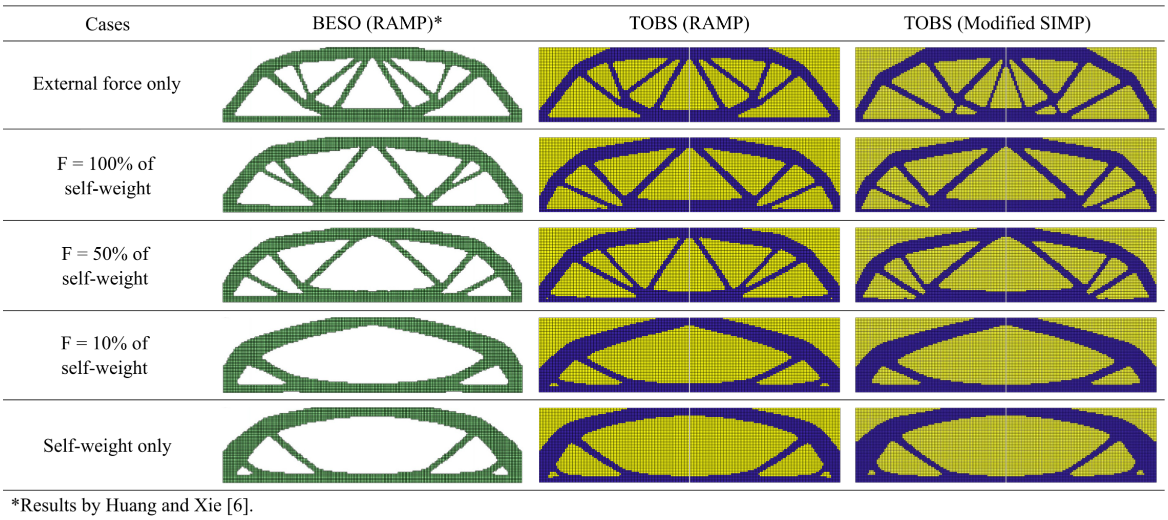

Figure 6. Comparison of final topologies for MBB beam example using TOBS method with RAMP and modified SIMP material models.

Figure 6 presents the optimal topology results achieved using the TOBS method combined with the RAMP and modified SIMP material models for the MBB beam case. At the same time, the results obtained by Huang and Xie [6] using the BESO method combined with the RAMP are shown in a comparative way. It can be seen that the topologies using the TOBS method converged to points of local minima different from the Huang and Xie [6] results, but they still look very similar. In general, it can be seen that, when the effect of the external force is reduced, the optimizer tends to remove the central supports and make the topology closer to an arch. Again, these results show that it is possible to use the TOBS method to solve problems of including self-weight loads using a power-law interpolation.

6 Conclusions

A study of topology optimization of structures subjected self-weight loads using TOBS method combining with RAMP and SIMP material models is proposed. The results demonstrated that both the RAMP and modified SIMP material models can be used in combination with TOBS efficiently. Moreover, from the results using the modified SIMP material model, it is proved that the topology optimization of self-weighted structures can be solved using a power-law interpolation. Besides, it is shown that the optimization will also depend on the optimizer and not only on the material interpolation model.

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References

[1] Y. Han, B. Xu, Q. Wang, and Y. Liu. Bi-directional evolutionary topology optimization of continuum structures subjected to inertial loads. *Advances in Engineering Software*, vol. 155, pp. 102897, 2021.

[2] H. Xu, L. Guan, X. Chen, and L. Wang. Guide-weight method for topology optimization of continuum structures including body forces. *Finite Elements in Analysis and Design*, vol. 75, pp. 38–49, 2013.

[3] M. Bruyneel and P. Duysinx. Note on topology optimization of continuum structures including self-weight. *Structural and Multidisciplinary Optimization*, vol. 29, n. 4, pp. 245–256, 2005.

[4] X. Yang, Y. Xie, and G. Steven. Evolutionary methods for topology optimisation of continuous structures with design dependent loads. *Computers & structures*, vol. 83, n. 12-13, pp. 956–963, 2005.

[5] R. Ansola, J. Canales, and J. A. Tarrago. An efficient sensitivity computation strategy for the evolutionary structural optimization (eso) of continuum structures subjected to self-weight loads. *Finite elements in analysis and design*, vol. 42, n. 14-15, pp. 1220–1230, 2006.

[6] X. Huang and Y. Xie. Evolutionary topology optimization of continuum structures including design-dependent self-weight loads. *Finite Elements in Analysis and Design*, vol. 47, n. 8, pp. 942–948, 2011.

[7] R. Sivapuram and R. Picelli. Topology design of binary structures subjected to design-dependent thermal expansion and fluid pressure loads. *Structural and Multidisciplinary Optimization*, vol. 61, n. 5, pp. 1877–1895, 2020.

[8] K. Svanberg and M. Werme. Topology optimization by sequential integer linear programming. In *IUTAM Symposium on Topological Design Optimization of Structures, Machines and Materials*, pp. 425–436. Springer, 2006.

[9] M. Beckers. Topology optimization using a dual method with discrete variables. *Structural Optimization*, vol. 17, n. 1, pp. 14–24, 1999.

[10] R. Sivapuram and R. Picelli. Topology optimization of binary structures using integer linear programming. *Finite Elements in Analysis and Design*, vol. 139, pp. 49–61, 2018.

[11] R. Sivapuram, R. Picelli, and Y. M. Xie. Topology optimization of binary microstructures involving various non-volume constraints. *Computational Materials Science*, vol. 154, pp. 405–425, 2018.

[12] R. Picelli, S. Ranjbarzadeh, R. Sivapuram, R. Gioria, and E. Silva. Topology optimization of binary structures under design-dependent fluid-structure interaction loads. *Structural and Multidisciplinary Optimization*, vol. 62, n. 4, pp. 2101–2116, 2020.

[13] H. P. Williams. *Integer Programming, Logic and Integer Programming*. Springer US, Boston, MA, 2009.

[14] X. Huang and Y. Xie. Convergent and mesh-independent solutions for the bi-directional evolutionary structural optimization method. *Finite Elements in Analysis and Design*, vol. 43, n. 14, pp. 1039–1049, 2007.

[15] O. Sigmund. Morphology-based black and white filters for topology optimization. *Structural and Multidisciplinary Optimization*, vol. 33, n. 4-5, pp. 401–424, 2007.

[16] L. Félix, A. A. Gomes, and A. Suleman. Topology optimization of the internal structure of an aircraft wing subjected to self-weight load. *Engineering Optimization*, vol. 52, n. 7, pp. 1119–1135, 2020.

[17] M. Stolpe and K. Svanberg. An alternative interpolation scheme for minimum compliance topology optimization. *Structural and Multidisciplinary Optimization*, vol. 22, n. 2, pp. 116–124, 2001.