A geometry trimming approach for topology optimization of acoustic problems

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Abstract. One challenging scientific problem in Topology Optimization (TO) is how to set its framework to account for different physics, such as acoustics. In this case, the dynamic acoustic pressure oscillations and numerical issues when interpolating acoustic and solid material properties become a burden. In this paper, we first investigate the use of Topology Optimization of Binary Structures with Geometry Trimming (TOBS-GT) for solving acoustic problems. The governing equations are solved via the Finite Element Method and sensitivities are computed with the adjoint method by using automatic differentiation. In order to verify the proposed methodology, a 2D acoustic problem was investigated. The objective is to minimize the average sound pressure level on a certain part of a 2D rectangular room by trimming out the design domain along the ceiling. The obtained results are similar to the ones found in the literature solved with different TO methods. The potential advantages here are obtaining designs without gray scale and with well-defined boundaries. This indicates that the TOBS-GT approach is a promising tool for solving acoustic problems.

Keywords: Topology Optimization, Acoustic problems, TOBS, Sound pressure level minimization.

1 Introduction

Topology Optimization is a computational tool developed in the late eighties by Bendsøe and Kikuchi [1], which is used to provide an optimal geometry design for a given physical problem. The idea is to distribute material within a defined domain by fulfilling prescribed constraints in order to optimize an objective function. The final goal is to obtain a set of binary $\{0, 1\}$ pseudo-densities, also called design variables, where usually 0 means the absence of material (void) and 1 represents the presence of solid material.

Despite the outstanding achievements of TO during the last couple decades, several problems are still in development, the acoustics field being one of them, as TO only began to be extended to acoustic problems in the 2000s. Sigmund and Jensen [2] present various results of applying topology optimization to structures and devices that are subjected to acoustic waves. Duhring et al. [3] developed a method for the optimization of both room acoustics and noise barriers, while Wadbro and Berggren [4] applied TO to an acoustic horn.

One challenge in TO is how to obtain the final structural layout represented by a binary design, which can be of great relevance for problems where explicit boundary description is important, as acoustics. In the matter of acoustics, the material interpolation is inverted from the original structural TO, with 0 meaning a solid region and 1 indicating the existence of an acoustic fluid. One common approach is to relax the binary constraint by allowing intermediate design variables (between solid and acoustic fluid), as done so in the Solid Isotropic Material with Penalization (SIMP) method. In this case, some techniques were proposed to reduce or eliminate intermediate density (gray scale) elements in the final solutions, such as projection methods. Another approach consists of boundary-description based methods, e.g. the level-set method (LSM), in which an implicit function is used to describe the structure with clearly defined boundaries, with the disadvantage of the final solution being strongly affected by the initial design configuration. This method was recently used in the work of by Noguchi and Yamada [5] for two-layered acoustic metasurfaces .

A different way of approaching the problem is the use of discrete TO methods, with Bi-directional Evolutionary Structural Optimization (BESO), initially proposed by Xie and Steven [6], being the most established one. BESO consists in the update of the design variables as a fixed change in volume fraction every iteration, with the disadvantage of not being based on mathematical optimization, thus not guaranteeing each iteration as an optimal step.

Within this context, Sivapuram and Picelli [7] proposed a binary method solved with formal mathematical programming (TOBS). This paper investigates the use of TOBS with a geometry trimming technique (TOBS-GT) for acoustic problems, which is the first method to trim out the solid region of the acoustics topology optimization and to use only the acoustic domain in the analysis. Consequently, there is no vibration in the "rigid" solid. This method does not need continuation schemes as $\{0, 1\}$ solutions are always obtained due to the use of binary variables.

The model studied in this paper pursues a noise reduction in a certain part of a room. This type of problem is relevant for many applications where the construction of acoustically better environments is desirable, as industrial machinery noise control in closed spaces, reducing engine noise in car cabins and so on.

The remainder of this paper is organized as follows. Section 2 describes the acoustic model governed by Helmholtz equation. In Section 3 the TOBS method with geometry trimming is outlined. Section 4 presents and discusses numerical results and Section 5 concludes the paper.

2 Acoustic Analysis

The model adopted was inspired by the work of Duhring et al. [3] and is illustrated in Fig. 1. The room is described by a domain Ω initially filled with air. Sinusoidal sound waves of angular frequency ω are emitted from a source that is vibrating with volume velocity Q_S. The governing equation for steady-state linear acoustic problems with sinusoidal sound waves is the Helmholtz equation [8]

$$\nabla \cdot (\rho^{-1} \nabla \hat{p}) + \omega^2 K^{-1} \hat{p} = 0, \tag{1}$$

where $\hat{p}$ is the complex sound pressure amplitude, ρ is the density, K is the bulk modulus of the acoustic medium and they all depend on the position r. The aim is to find the optimal solid material distribution in the ceiling (Ω_d). The appearance of the material parameters in their inverse form in eq. (1) motivated the choice of interpolating the inverse density and bulk modulus linearly as

$$\begin{aligned} \rho^{-1} &= \rho_2^{-1} + \theta_p(\rho_1^{-1} - \rho_2^{-1}), \\ K^{-1} &= K_2^{-1} + \theta_p(K_1^{-1} - K_2^{-1}). \end{aligned} \tag{2}$$

Here $\rho_1 = 1.204$ kg/m^3 and $K_1 = 141.921$ kPa are the properties of air while ρ_2 and K_2 are the properties of a solid material, which are arbitrarily chosen values (e.g. aluminum properties). The properties of the rigid material are not relevant, as in TOBS-GT it is trimmed out of the domain, as it will be further explained in section 3.3. The design variable θ is introduced to control these properties, such that it represents a solid material when it is equal to zero, and air when it is equal to one. This variable θ is penalized with SIMP as

$$\theta_p = \theta_{min} + (1 - \theta_{min})\theta^p, \tag{3}$$

with $\theta_{min} = 0.001$ and $p = 3$. The boundary conditions are a point source at r_0 with volume velocity Q_S and a perfectly reflecting surface in the rigid walls of the room, respectively

$$\nabla \cdot (\rho^{-1} \nabla \hat{p}) + \omega^2 K^{-1} \hat{p} = -i\omega Q_s \delta(r - r_0), \quad \mathbf{n} \cdot (\rho^{-1} \nabla \hat{p}) = 0. \tag{4}$$

Here **n** is the normal unit vector pointing out of the domain. Finite element analysis is used to solve the problem.

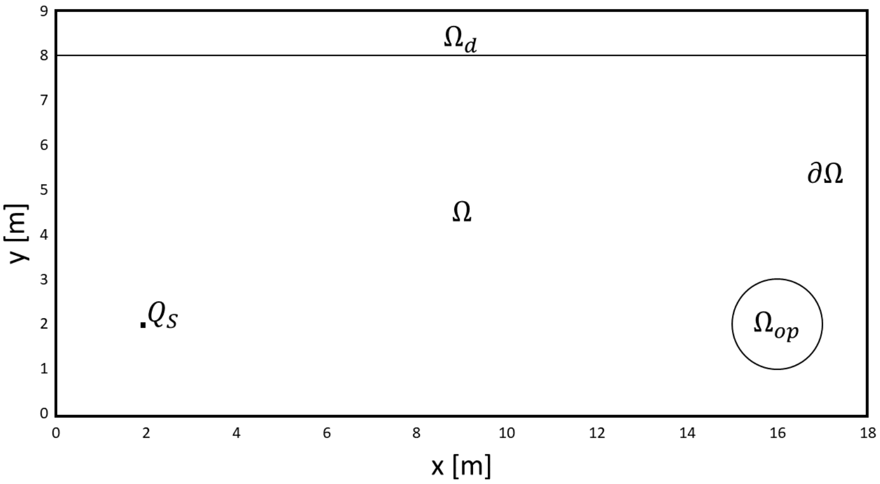


Figure 1. Dimensions of the rectangular room in 2D with design domain Ω_d, output domain Ω_{op} and a point source with volume velocity Q_S.

3 Topology Optimization Framework

3.1 Optimization Formulation

The goal of the topology optimization is to find the optimal material distribution in the design domain Ω_d (the ceiling of the room) that satisfies the prescribed volume constraint and minimizes objective function Φ, which is the average of the Sound Pressure Level (SPL) in the output domain Ω_{op}.The sound pressure level is given by

$$SPL = 20 \log\left(\frac{p_{rms}}{p_{ref}}\right), \tag{5}$$

where $p_{ref} = 20\mu$ Pa is the pressure reference for air and p_{rms} is the root mean square pressure. Under those circumstances the formulation of the optimization problem takes the form

$$\begin{aligned} &\underset{\theta}{\text{Minimize}}\ \Phi(r, \theta(r)) = \frac{1}{\int_{\Omega_{op}} dr} \int_{\Omega_{op}} SPL\ dr, \\ &\text{Subject to } V(\theta) \geq \overline{V}, \\ &\qquad\qquad \theta \in \{0, 1\}, \end{aligned} \tag{6}$$

where $V(\theta)$ is the air volume fraction in the design domain and $\overline{V}$ is its constrained value.

3.2 TOBS Method

The TOBS method, first proposed by Sivapuram and Picelli [9] in 2018, was formulated in order to align the use of binary design variables with formal mathematical programming. An overview of the method will be given within this section. General topology optimization problems are highly nonlinear and nonconvex, therefore the method proposes the use of sequential approximation of the optimization problem. In this case we use sequential linearization of the objective and constraint functions. Considering a Taylor's series expansion and truncating its first term (linear part) and representing the design variable in its discrete form as $\boldsymbol{\theta}$, the objective function and volume constraint can be expressed as follows, for the optimization iteration n,

$$\begin{aligned}\Phi(\boldsymbol{\theta}) &\approx \Phi(\boldsymbol{\theta}^n) + \frac{d\Phi(\boldsymbol{\theta}^n)}{d\boldsymbol{\theta}} \cdot \Delta\boldsymbol{\theta}^n + O(\left|\left|\Delta\boldsymbol{\theta}^n\right|\right|_2^2),\\ V(\boldsymbol{\theta}) &\approx V(\boldsymbol{\theta}^n) + \frac{dV(\boldsymbol{\theta}^n)}{d\boldsymbol{\theta}} \cdot \Delta\boldsymbol{\theta}^n + O(\left|\left|\Delta\boldsymbol{\theta}^n\right|\right|_2^2),\end{aligned} \tag{7}$$

where the truncation error is given as $O(\left|\left|\Delta\boldsymbol{\theta}^n\right|\right|_2^2)$, and $\Delta\boldsymbol{\theta}^n$ is the vector that represents the changes in the design variable. These changes should be restricted in order to keep the design variable with integer (i.e., binary) values. For example, by considering an element that contains a solid material ($\theta_j = 0$), the changes in the design variable can be restricted as $\Delta\theta_j \in \{0, 1\}$, meaning that this element may either turn into air ($\theta_j = 1$) or keep its value ($\theta_j = 0$) in the optimization iteration. The same procedure is analogous for an element that contains air ($\theta_j = 1$). The bound constraints for $\Delta\theta_j$ can then be expressed as,

$$\begin{cases} 0 \leq \Delta\theta_j^n \leq 1 & \text{if } \theta_j^n = 0,\\ -1 \leq \Delta\theta_j^n \leq 0 & \text{if } \theta_j^n = 1,\end{cases} \tag{8}$$

or, in a unified form,

$$\Delta\theta_j^n \in \{-\theta_j^n, 1 - \theta_j^n\}, \tag{9}$$

where $\Delta\theta_j^n \in \{-1, 0, 1\}$. In order to maintain the linear approximation from eq. (7) valid, the truncation error $O(\left|\left|\Delta\boldsymbol{\theta}^n\right|\right|_2^2)$ should be reasonably small. One can control the truncation error by

$$\left|\left|\Delta\boldsymbol{\theta}^n\right|\right|_1 \leq \beta N_d. \tag{10}$$

Here β is an additional constraint added to the subproblem to restrict the the number of elements that may turn from air to solid and vice-versa is to a fraction of the total number of elements (N_d), thus maintaining the truncation error small. By considering the sequential linear approximations from Eq. (7), and the extra constraints from eq. (9) and eq. (10), one can write the approximate integer linear subproblem as

$$\begin{aligned}\underset{\Delta\boldsymbol{\theta}^k}{\text{Minimize}}\ & \frac{d\Phi(\boldsymbol{\theta}^n)}{d\boldsymbol{\theta}} \cdot \Delta\boldsymbol{\theta}^n,\\ \text{Subject to}\ & \frac{dV_\theta(\boldsymbol{\theta}^n)}{d\boldsymbol{\theta}} \cdot \Delta\boldsymbol{\theta}^n \geq \overline{V} - V_\theta\big(\boldsymbol{\theta}^n\big) := \Delta V_\theta^k,\\ & \left|\left|\Delta\boldsymbol{\theta}^n\right|\right|_1 \leq \beta N_d,\\ & \Delta\theta_j^n \in \{-\theta_j^n, 1 - \theta_j^n\},\ j \in [1, N_d].\end{aligned} \tag{11}$$

While the truncation error constraint (eq. (10)) restrains the topology from undergoing great changes, this might lead to infeasibility of some of the constraints V_i in the current iteration n, when the bound $\Delta V_\theta^n = \overline{V} - V_\theta\big(\boldsymbol{\theta}^n\big)$ is used. This undesirable effect may be avoided by modifying the bound of the constraint (ΔV_θ^n). This approach also helps in generating feasible subproblems when the initial guess of the design variable is distant from feasibility. Therefore, the constraint bounds are modified by considering

$$\Delta V_\theta^n = \begin{cases} -\epsilon V_\theta\big(\boldsymbol{\theta}^n\big) & : \overline{V} < (1-\epsilon) V_\theta\big(\boldsymbol{\theta}^n\big),\\ \overline{V} - V_\theta\big(\boldsymbol{\theta}^n\big) & : \overline{V} \in [(1-\epsilon) V_\theta\big(\boldsymbol{\theta}^n\big), (1+\epsilon) V_\theta\big(\boldsymbol{\theta}^n\big)],\\ \epsilon V_\theta\big(\boldsymbol{\theta}^n\big) & : \overline{V} > (1+\epsilon) V_\theta\big(\boldsymbol{\theta}^n\big),\end{cases} \tag{12}$$

where ϵ is the relaxation parameter corresponding to the constraint given by V_θ.

The integer optimization subproblems generated by using sequential linearizations (Eq. (11)) can be solved through Integer Linear Programming (ILP), which is essentially the same as a Linear Programming (LP) problem, but imposing additional constraints to ensure that the design variables can only achieve integer values. It is also a naturally understandable choise, since we aim to achieve a binary $\{0, 1\}$ solution. In this work, the ILP problem is solved by using the branch-and-bound algorithm from the CPLEX® optimization library, which is developed by IBM®. The branch-and-bound method consists of an algorithm based on a tree data structure, in which the ILP problem is first solved without any integer constraints (by using a linear optimization technique such as the Simplex method); then, branches of LPs are created with additional inequality constraints being imposed on the design variables in order for the solution to be yielded as integer (Land and Doig [10], Vanderbei [11]).

3.3 Geometry trimming and meshing

The proposed methodology is based on the decoupling of the optimization grid and the FEA mesh. The process is illustrated in Fig. 2. The acoustic and sensitivity analyses are carried out using COMSOL Multiphysics®.

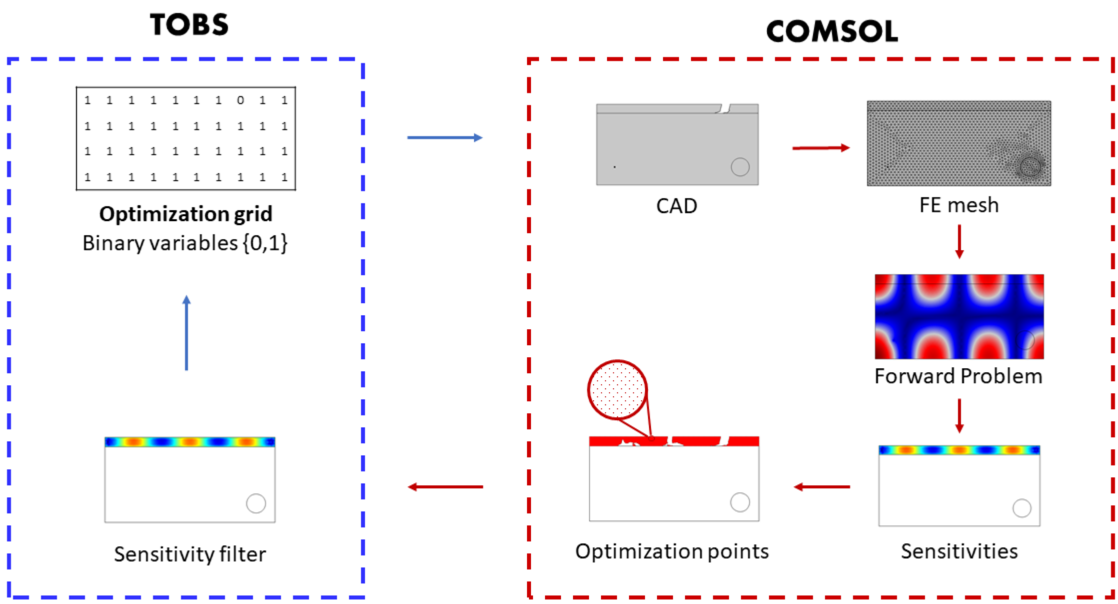


Figure 2. TOBS-GT procedure.

First, a CAD file is created with the analysis domain. The geometry is then freely meshed by using the option `physics controlled` in COMSOL Multiphysics®. The option takes into account some built-in physics requirements when meshing. The forward and adjoint problems are solved, computing acoustic pressures and sensitivities. These entities are computed using nodal variables at the finite element software. The sensitivities are obtained via automatic differentiation. After that, optimization grid is created and the sensitivities are interpolated at the TOBS optimization points. The finite element shape functions can be used to interpolate the sensitivities at such points. Standard spatial filtering is applied at the sensitivities defined at the optimization grid as suggested by Picelli et al. [12]. The TOBS-ILP solver is then used to find a new set of binary design variable values. The new topology is then used at the next iteration.

Besides the analysis domain, the contours of the holes defined by the binary variables are saved as `.dxf` and provided to the FEA package. A new CAD file can be created by trimming out the holes from the analysis domain. This is carried in the geometry building section from COMSOL Multiphysics® with the command `difference`. In this way, the modeled solid regions are not considered in the simulation. This procedure eliminates the influence of the emulated solid, which is not actually truly solid in the standard density-based approaches. Another benefit of decoupling the optimization variables and the FEA is that the optimization grid can be refined in order to obtain crispier topologies while the FEA mesh can be maintained in a certain size with a reasonable computational cost.

4 Numerical Results

The average of the sound pressure level is minimized in the output domain Ω_{op} by distributing material in the design domain Ω_d in the rectangular room represented in Fig. 1. The initial design is a domain filled with air ($\theta = 1$) and volume restriction chosen was $V(\theta) \geq 0.85$. The parameters used were $r = 0.3$ m, $\epsilon = 0.02$ and $\beta = 0.03$. The optimization is performed to a single frequency $f = 34.56$ Hz, which is close to a natural frequency for the initial design. The optimization grid was set to 720×40 elements.

The optimized design was found in 24 iterations and the objective function was reduced from 104.91 to 85.71 dB as can be observed in Fig. 3. Figure 4 shows the SPL for the inital and optimized designs, making it clear that the material distributed in the ceiling is influencing the SPL in the room, such that it has a lower value in the output domain, with a nodal line going through it.

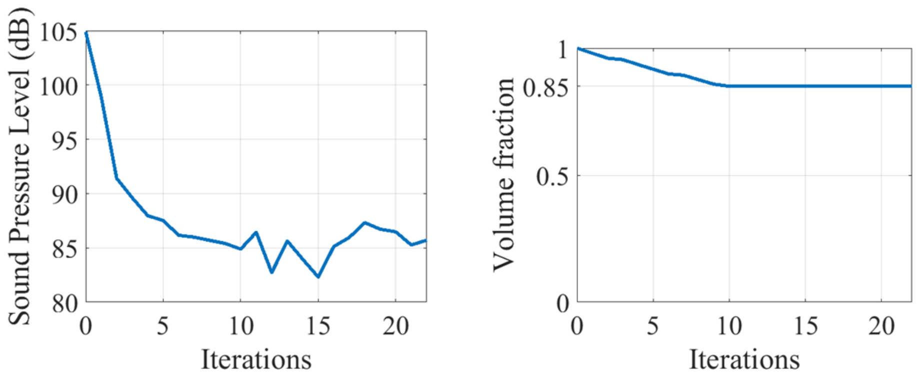
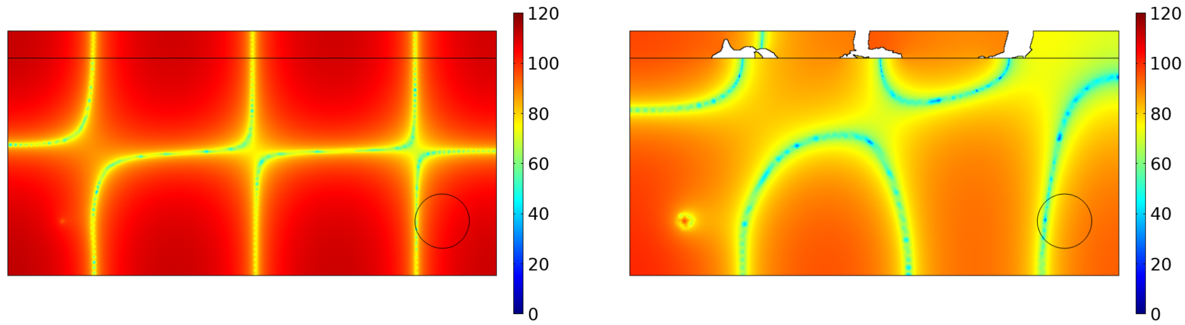


Figure 3. Topology optimization convergence.



(a) Initial Design

(b) Final Design

Figure 4. Sound Pressure Level plots in dB.

In Fig. 5 we have a comparison between the frequency response for the initial and optimized designs, where the SPL is plotted as a function of the frequency ω. It became evident that the natural frequencies, included the one used as the driving frequency for the optimization, changed to a lower value.

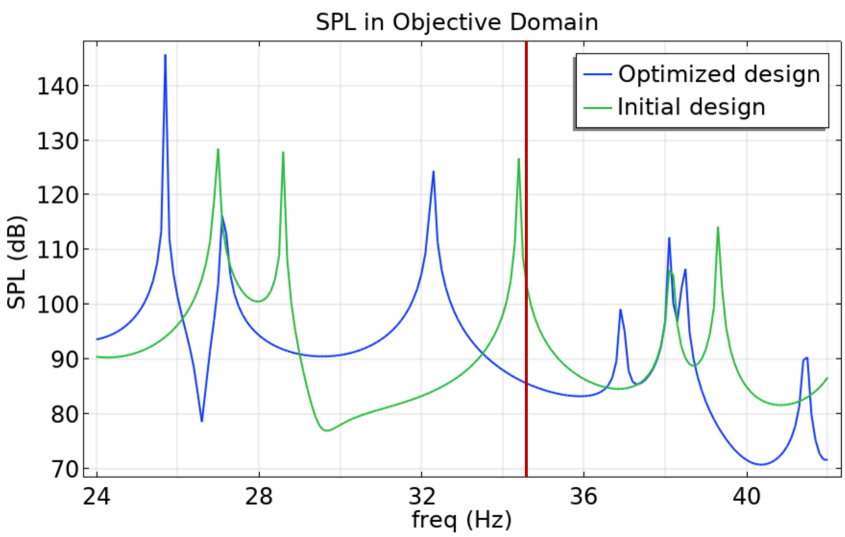


Figure 5. Frequency response comparison for the initial and optimized designs, with the chosen frequency for the optimization marked in the vertical red line.

5 Conclusions

In this paper it was shown that TOBS-GT can be employed to minimize the average of the SPL in a part of a room in 2D. The obtained results are similar to the ones found in the literature solved with different TO methods. The potential advantages here are obtaining designs without gray scale and with well-defined boundaries. This indicates that the TOBS method with the geometry trimming approach is a promising tool for solving acoustic problems. Ongoing work should aim to reduce the oscillations in the optimization.

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